Key Stage 3 *National Strategy*

Supplement of examples
Years 7, 8 and 9

4

Supplement of examples
Years 7, 8 and 9

USING AND APPLYING MATHEMATICS TO SOLVE PROBLEMS

Pupils should be taught to:

Solve word problems and investigate in a range of contexts

As outcomes, Year 7 pupils should, for example:

Solve word problems and investigate in contexts of number, algebra, shape, space and measures, and handling data; compare and evaluate solutions.

Problems involving money

For example:

- A drink and a box of popcorn together cost 90p.
  Two drinks and a box of popcorn together cost £1.45.
  What does a box of popcorn cost?

- Six friends went to a Chinese restaurant.
  The total cost for the set menu was £75.
  How much would the set menu cost for eight people?

- This is what a stationery shop charges for printing a book.

  Print charges
  3p per page
  75p for the cover

  Jon paid £4.35 for his book, including its cover.
  How many pages were there in his book?

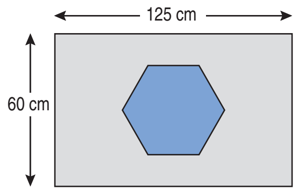
See Y456 examples (pages 84–5).

Problems involving percentages

For example:

- The value of a £40 000 flat increased by 12% in June.
  Its new value increased by a further 10% in October.
  What was its value in November?

- 20% of this flag is blue.
  What area of the flag is blue?



- Which of these statements is true?
  A. 75% of £6 > 60% of £7.50
  B. 75% of £6 = 60% of £7.50
  C. 75% of £6 < 60% of £7.50

- The price of a pair of jeans was decreased by 10% in a sale.
  Two weeks later the price was increased by 10%.
  The final price was not the same as the original price.
  Explain why.

See Y456 examples (pages 32–3).

As outcomes, Year 8 pupils should, for example:

Solve more demanding problems and investigate in a range of contexts; compare and evaluate solutions.

Problems involving money

For example:

- At Alan's sports shop, GoFast trainers usually cost £40.95, but there is ⅓ off in the sale.
  At Irene's sports shop, GoFast trainers normally cost £40, but there is a discount of 30% in the sale.
  Which shop sells the trainers for less in the sales?

- A supermarket sells biscuits in these packets:

  15 biscuits for 56p
  24 biscuits for 88p
  36 biscuits for £1.33

  Which packet is the best value for money?
  Why do you think this is the case?

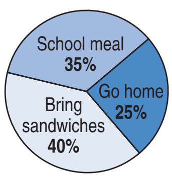
- Tina and Jill each gave some money to a charity. Jill gave twice as much as Tina, and added £4 more. Between them, the two girls gave £25 to the charity. How much did Tina give?

Problems involving percentages

For example:

- Chloe and Denise each bought identical T-shirts from the same shop.
  Chloe bought hers on Monday when there was 15% off the original price.
  Denise bought hers on Friday when there was 20% off the original price.
  Chloe paid 35p more than Denise.
  What was the original price of the T-shirt?

- The pie chart shows the lunch choices of class 8NP.



  7 pupils in class 8NP have a school meal.
  How many go home for lunch?

- The floods in Mozambique in February 2000 affected 600 000 people, one twenty-eighth of the population of Mozambique.
  350000 people had to leave their homes.
  What percentage of the population had to leave their homes during the floods?

As outcomes, Year 9 pupils should, for example:

Solve increasingly demanding problems; explore connections in mathematics across a range of contexts; generate fuller solutions.

Problems involving money

For example:

- Two families went to the cinema.
  The Smith family bought tickets for one adult and four children and paid £19.
  The Jones family bought tickets for two adults and two children and paid £17.
  What was the cost of one child's ticket?

- Thirty years ago the money used in Great Britain was pounds, shillings and pence.
  There were 20 shillings in £1.

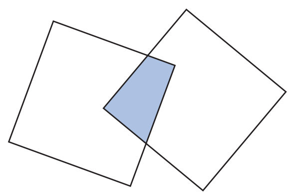
  A gallon of petrol cost 7 shillings thirty years ago.
  Today it costs about £3.80.
  How much has the cost of petrol risen in the last thirty years?

Problems involving percentages

For example:

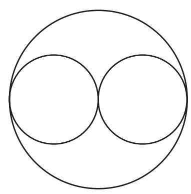
- After an advertising campaign costing £950, a firm found that its profits rose by 15% to £7200.
  From a financial point of view, was the advertising worthwhile? Justify your answer.

- The diagram shows two overlapping identical squares.



  20% of each square is shaded.
  What fraction of the whole diagram is shaded?

- The diagram shows two identical small circles in a big circle.



  What percentage of the big circle is filled by the two small circles?

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Solve word problems and investigate in a range of contexts (continued)

Problems involving ratio and proportion

For example:

- In a country dance there are 7 boys and 6 girls in every line.
  42 boys take part in the dance.
  How many girls take part?
- Some children voted between a safari park and a zoo for a school visit. The result was 10 : 3 in favour of the safari park.
  130 children voted in favour of the safari park.
  How many children voted in favour of the zoo?
- The risk of dying in any one year from smoking is about 1 in 200 of the population.
  The population of the UK is about 58 million.
  How many people are likely to die this year from smoking?
- A boy had £35 in birthday money.
  He spent some of the money.
  He saved four times as much as she spent.
  How much did he save?
- There are 3 chocolate biscuits in every 5 biscuits in a box.
  There are 30 biscuits in the box.
  How many of them are chocolate biscuits?
- A recipe for mushroom soup uses 7 mushrooms for every ½ litre of soup.
  How many mushrooms do you need to make 2 litres of soup?
  How much soup can you make from 21 mushrooms?
- 10 bags of crisps cost £3.50.
  What is the cost of 6 bags of crisps?
- £1 = 12.40 Danish kroner.
  How much is £1.50 in Danish kroner?

See Y456 examples (pages 26–7).

As outcomes, Year 8 pupils should, for example:

Problems involving ratio and proportion

For example:

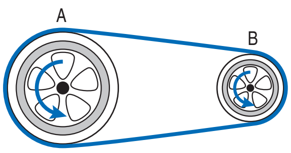
- To make orange paint, you mix 13 litres of yellow paint to 6 litres of red paint to 1 litre of white paint.
  How many litres of each colour do you need to make 10 litres of orange paint?

- Flaky pastry is made by using flour, margarine and lard in the ratio 8 : 3 : 2 by weight.
  How many grams of margarine and lard are needed to mix with 200 grams of flour?

- A square and a rectangle have the same area.
  The sides of the rectangle are in the ratio 9:1.
  Its perimeter is 200cm.
  What is the length of a side of the square?

- The chocolate bars in a full box weighed 2kg in total. Each bar was the same size.
  Eight of the bars were eaten.
  The bars left in the box weighed 1.5kg altogether.
  How many chocolate bars were in the original box?

- A and B are two chain wheels.



  For every 2 complete turns that wheel A makes, wheel B makes 5 complete turns.
  Wheel A makes 150 turns.
  How many turns does wheel B make?

- A 365g packet of coffee costs £2.19.
  How much per 100g of coffee is this?

- £1 is worth 3 Australian dollars (A$).
  A girl changed £50 into Australian dollars, went on holiday to Australia and spent A$96.
  At the end of the holiday she changed the Australian dollars back into £.
  How much did she get?

- The cost of a take-away pizza is £6.
  A pie chart is drawn to show how the total cost is made up. The cost of labour is represented by a sector of 252° on the pie chart.
  What is the cost of labour?

- Cinema tickets for one adult and one child cost a total of £11.55.
  The adult's ticket is one-and-a-half times the price of the child's ticket.
  How much does each ticket cost?

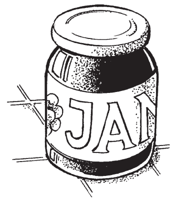
As outcomes, Year 9 pupils should, for example:

Problems involving ratio and proportion

For example:

- 2 parts of red paint mixed with 3 parts of blue paint make purple paint.
  What is the maximum amount of purple paint that can be made from 50 ml of red paint and 100ml of blue?

- A recipe for jam uses 55g of fruit for every 100g of jam.



  I want to make ten 454 g jars of jam.
  How many grams of fruit do I need?

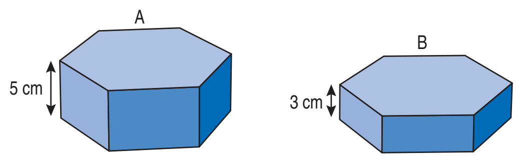
- A boy guessed the lengths of some objects.

	Guess	Actual length
A.	5 cm	7 cm
B.	40 cm	53 cm
C.	3 m	2 m
D.	10 m	7.8 m
E.	20 m	25 m

  He divided each guessed length by the measured length to produce an accuracy ratio.

  The accuracy ratio for object A is 0.71:1 (to 2 d.p.).
  Find the accuracy ratio for each other guess.
  Which guess was the most accurate?
  Which guess was the least accurate?

- Prisms A and B have the same cross-sectional area.



  Prism A is 5 cm high and has a volume of 200cm^3.
  What is the volume of prism B?

- Take a two-digit number. Reverse the digits.
  Is it possible to make a number that is one-and-a-half times as big as the original number?
  Justify your answer.
  Find a two-digit number that is one-and-three-quarters times as big when your reverse its digits.

- The Queen Mary used to sail across the Atlantic.
  Its usual speed was 33 miles per hour.
  On average, it used fuel at the rate of 1 gallon for every 13 feet sailed.
  Calculate, to the nearest gallon, how many gallons of fuel the ship used in one hour of travelling at its usual speed.
  (There are 5280 feet in one mile.)

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

Problems involving number and algebra

For example:

- I think of a number, add 3.7, then multiply by 5.
  The answer is 22.5. What is the number?

- Use each of the digits 1, 2, 3, 4, 5 and 8 once to make this sum correct:
  □□ + □□ = □□

- Use only the digits 2, 3, 7 and 8, but as often as you like.
  Make each sum correct.

□□ + □□ = 54	□□ + □□ = 155
□□ + □□ = 69	□□ + □□ = 105
□□ + □□ = 99	□□ + □□ = 110

- Here is a subtraction using the digits 3, 4, 5, 6, 7.

  365 – 47

  Which subtraction using all the digits 3, 4, 5, 6, 7 has the smallest positive answer?

- Two prime numbers are added. The answer is 45.
  What are the numbers?

- What is the largest multiple of 6 you can make from the digits 6, 7 and 8?
  What is the largest multiple of 7 you can make from the digits 5, 6 and 7?

- $^{4}/_{15}$ and $^{24}/_{9}$ are examples of three-digit fractions.
  There is only one three-digit fraction which equals $1\frac{1}{2}$.
  What is it?
  Find all the three-digit fractions that equal $2\frac{1}{2}$.
  Explain how you know when you have found them all.

- What operation is represented by each ✻ ?

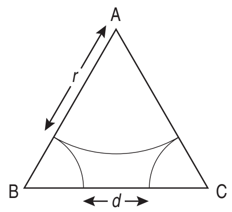
a. 468 ✻ 75 = 543	c. 468 ✻ 75 = 393
b. 468 ✻ 75 = 6.24	d. 468 ✻ 75 = 35 100

- Find two consecutive numbers with a product of 702.

- A function machine changes the number n to the number 3n + 1.
  What does it do to these numbers?
  2 5 9 21 0
  What numbers must be input to get these numbers?
  10 37 100

- Triangle ABC is an equilateral triangle.
  The length of a side is a cm.
  Arcs with centres A, B and C are drawn as shown.

  Express d in terms of a and r.



See Y456 examples (pages 78–9, 82–3).

As outcomes, Year 8 pupils should, for example:

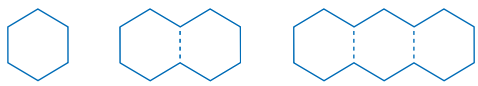
Problems involving number and algebra

For example:

- What fraction is half way between $\frac{3}{5}$ and $\frac{5}{7}$?
- Find the missing digits.
  The ✻ need not be the same digit in each case.

  a. $\frac{✻}{✻✻} + \frac{✻}{8} = \frac{3}{✻✻}$

  b. $(✻✻)^3 = ✻✻✻7$
- Find four consecutive odd numbers with a total of 80.
  Find three consecutive numbers with a product of 21 924.
  Find two prime numbers with a product of 6499.
- The product of two numbers is 999.
  Their difference is 10.
  What are the two numbers?
- Calculate these, without a calculator:
  4^2 4^3 4^4 4^5 4^6
  What digit does 4^{20} end in?
  What digit does 4^{21} end in?
- This is a pattern of hexagons.
  Each side is 1 cm in length.



Find the three values marked by '**?**' in this table.

No. of hexagons	1	2	3	4	...	9	...	?	...	25
Perimeter	6	10	14	18	...	?	...	66	...	?

Explain why the perimeter of a pattern of hexagons could not be 101cm.

- Each inside edge of two cube-shaped tins is 2s cm and s cm respectively.
  The larger tin is full of water; the smaller tin is empty.
  How much water is left in the larger tin when the smaller tin has been filled from it?
- A rectangle is 7k cm long and 3k cm wide.
  Find the area of a square whose perimeter is the same as that of the rectangle.
  Which has the larger area, the square or the rectangle? By how much?
- A boy gets 2 marks for each sum that he gets right and $^{-}1$ mark for each that he gets wrong.
  He did 18 sums and got 15 marks.
  How many sums did he get right?

As outcomes, Year 9 pupils should, for example:

Problems involving number and algebra

For example:

- 7^3 is 343. Without using a calculator, work out the units digit of 7^{12}.
- A number is a multiple of 21 and 35, and has four digits. What is the smallest number it could be?
- Here is a multiplication using the digits 3, 4, 5, 6, 7.

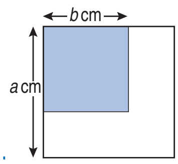
  34 × 56 × 7

  Which multiplication using all of the digits 3, 4, 5, 6, 7 has the smallest answer?
  Justify your answer.
- Put the digits 1, 2, 3, 4, 5 and 6 in place of the stars.
  Use each digit once.

  ✻ × ✻✻ = ✻✻✻
- Find the smallest number greater than 50 that has the same number of factors as 50.
  Justify your answer.
- Start with a two-digit number (TU). Find 3U + T.
  Repeat this with the new number to form a sequence.
  Try other two-digit numbers. What happens?
  What number gives the shortest sequence?
  What number gives the longest sequence?
- This fraction sum is made from four different digits.

  $\frac{3}{6} + \frac{1}{2}$

  The fraction sum is 1.
  Find as many as possible fraction sums made from four different digits and with a sum of 1.
- The diagram shows a square of side a cm, from which a square of side b cm has been removed. Use the diagram to show that $a^2 - b^2 = (a - b)(a + b)$.



- I think of a number. All except two of 1 to 10 are factors of this number. The two numbers that are not factors are consecutive. What is the smallest number I could be thinking of?
- Two satellites circle round the Earth.
  Their distance from the centre of the Earth is:
  Satellite A 1.53×10^7 miles
  Satellite B 9.48×10^6 miles

  What is:
  a. the minimum distance apart,
  b. the maximum distance apart,
  the satellites could be?

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

More problems involving number and algebra

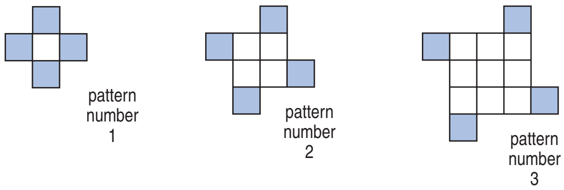
For example:

- Here is a sequence of five numbers.

  2 ☐ ☐ ☐ 18

  The rule is to start with 2 then add the same amount each time. Write in the missing numbers.

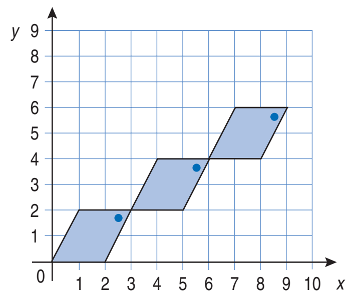
- This is a series of patterns with white and blue tiles.



  How many white tiles and blue tiles will there be in pattern number 8? Explain how you worked this out.
  Will one of the patterns have 25 white tiles?
  Give a mathematical reason for your answer.

- Does the line y = x – 5 pass through the point (10, 10)? Explain how you know.

- Lucy has some blue tiles, each with a marked corner. She sets them out as shown.



  Lucy carries on laying tiles. She says: '(21, 17) will be the coordinates of one of the marked corners.'

  Do you think Lucy is right? Explain your answer.

- This formula tells you how tall a boy is likely to grow.

  > Add the mother's and father's heights.
  > Divide by 2.
  > Add 7 cm to the results.
  > A boy is likely to be this height, plus or minus 10cm.

  Marc's mother is 168 cm tall and his father is 194cm tall. What is the greatest height that Marc is likely to grow?

- A girl at a fair tries the hoopla.
  She pays a £2 coin for 4 goes and is given change.
  The cost of each go is c pence.
  Which of these expressions gives her change in pence?
  4c – 2 4c – 200 2 – 4c 200 – 4c

See Y456 examples (pages 78–9, 82–3).

As outcomes, Year 8 pupils should, for example:

More problems involving number and algebra

For example:

- The next number in the sequence is the sum of the two previous numbers.
  Fill in the missing numbers.

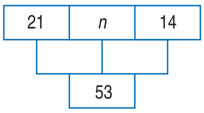
  ☐ ☐ ☐ 1 0 1 1 2 3 5 8

- Here is a sequence:

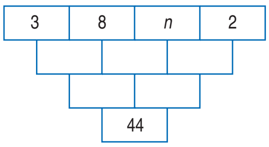
  6 19 32 45 58 71 …

  The sequence continues in the same way.
  a. Write the next three terms of the sequence.
  b. Explain the rule for finding the next term.
  c. What is the 30th term of the sequence?
  d. What is the *n*th term of the sequence?

- In this 'pyramid' each number is equal to the sum of the two numbers immediately above it.
  Find the missing number *n* which makes the bottom number correct.

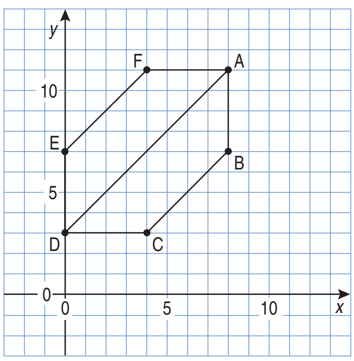


  What about this one?



- Elin played a number game. She said:
  'Multiplying my number by 4 then subtracting 5 gives the same answer as multiplying my number by 2 then adding 1.'
  Work out the value of Elin's number.

- Look at this diagram.



  The line through points A and F has the equation $y = 11$.
  What is the equation of:
  a. the line through points A and B?
  b. the line through points F and E?
  c. the line through points B and C?

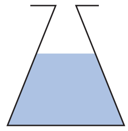
- A line with equation $y = mx + 9$ passes through the point (10, 10). What is the value of *m*?

As outcomes, Year 9 pupils should, for example:

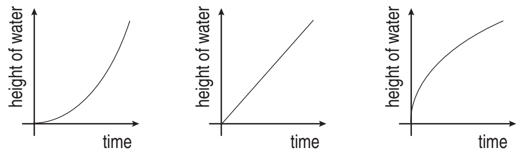
More problems involving number and algebra

For example:

- This bottle is being steadily filled with water.



  Which graph shows the relationship between time and the height of water in the bottle?

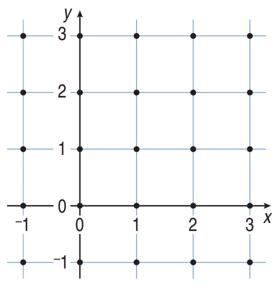


- Show that if $a(b - 4) = 0$, then either $a = 0$ or $b = 4$.

- The length of an edge of a solid cube is *a* cm.
  Its surface area is *S* cm^2, and its volume is *V* cm^3.
  Prove that $S^3 = 216V^2$.

- Find a pair of numbers satisfying $7x - 2y = 38$, such that one is three times the other.
  Is there more than one answer?

- The point (*x*, *y*) on the dotted grid is satisfied by the inequalities:
  $x > 0$ and $y > 0$
  $x + y < 4$
  $x > y$
  What are the coordinates of the point?

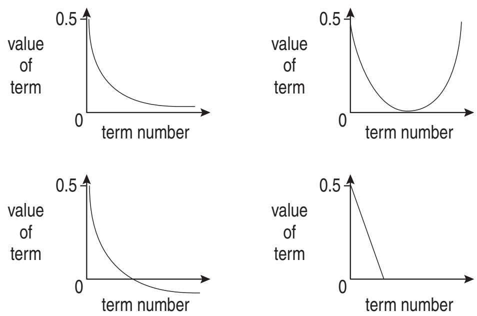


- Look at these expressions:

  $n - 2$ $2n$ n^2 $n/2$ $2/n$

  Which expression gives the greatest value:
  a. when *n* lies between 1 and 2?
  b. when *n* lies between 0 and 1?
  c. when *n* is negative?

- The *n*th term of an infinite sequence is $n/(n^2+1)$.
  The first term of the sequence is ½.
  Which of these four graphs has the same shape as the line joining the terms of the sequence?



Pupils should be taught to:

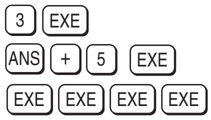
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

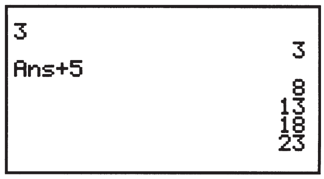
Problems to solve with a graphical calculator

For example:

- Enter these instructions (or their equivalent) on your graphical calculator.

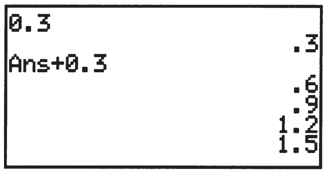


Describe the sequence you see. What is the rule? How many additions are needed to exceed 60?



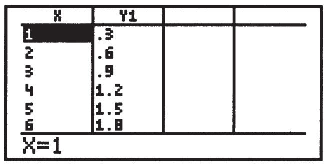
Generate these sequences on your calculator.
64 32 16 8 …
1 3 9 27 …
1 $^{-}1$ 1 $^{-}1$ 1 $^{-}1$ …
Write down the rules that you used.

- Generate a sequence starting with 0.3 and adding 0.3.



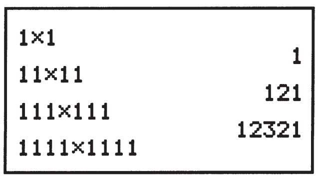
Predict what will come next.
Will 4.2 be in the sequence? How do you know?
What sequence would have terms where each term is 10 times bigger than those on the screen? 100 times bigger?

Use the table facility with 0.3x for y_1.



Give the next two terms.
Is 3 the same as 3.0?
What is 10 × 0.3? What are 2.4 ÷ 0.3 and 2.4 ÷ 8?
Write down other relationships you can deduce from the sequence.
What number is half way between 0.3 and 0.6? Between 0.6 and 0.9? Between 0.9 and 1.2?
Generate the sequence of 'halfway values'.

- Enter these calculations on your calculator.



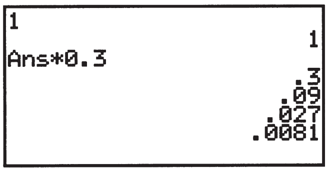
Predict the next term.
What would be the result of multiplying 111 111 by 111111?
What if you replace every digit 1 by a digit 3?

As outcomes, Year 8 pupils should, for example:

Problems to solve with a graphical calculator

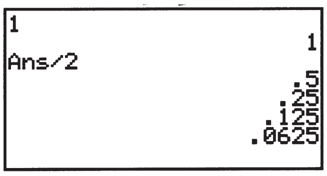
For example:

- Multiply repeatedly by 0.3.

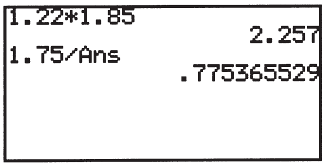


What comes next?
Are the numbers getting bigger or smaller?
What fractions are equivalent to these decimals?
Express each term using powers.

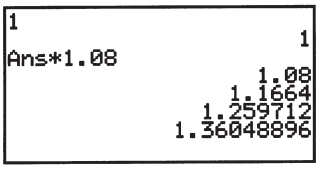
Now try dividing repeatedly by 2.
Will terms ever become zero?
Or negative?



- Find the increase in depth of oil in a rectangular tank 1.22 m wide and 1.85m long when 1750 litres of oil are added.

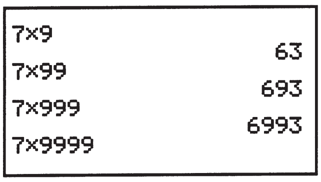


- Explore an annual growth rate of 8% linked to compound interest or population change.



How many years will it take for the initial amount to double?
What happens if the initial input is varied?
How does the time to double vary with the percentage growth rate?

- Use your calculator to explore these calculations:
  7 × 9, 7 × 99, 7 × 999, …
  Predict the answer to 7 × 99999.
  What about 7 × 999………999? Generalise.



Explore 9 × 9, 9 × 99, 9 × 999, …
Find a general rule.

As outcomes, Year 9 pupils should, for example:

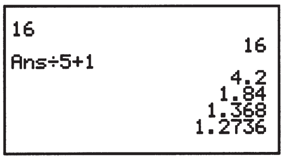
Problems to solve with a graphical calculator

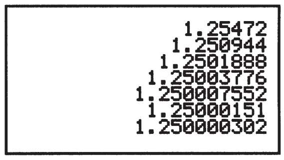
For example:

- Choose a starting number.
  Divide the number by 5.
  Add 1 to the answer.
  This is now your new starting number.
  Keep repeating the process.

  Enter these instructions on your graphical calculator.

  1 6 EXE
  ANS ÷ 5 + 1 EXE
  EXE EXE EXE EXE

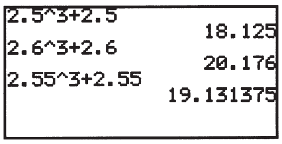


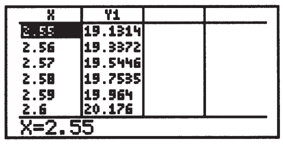


What happens when you keep repeating the process? What value do the terms approach?
What if you start with 6, or 300, or 3275?
What if you divide by 4 instead of 5?
What if you add 2 instead of 1?

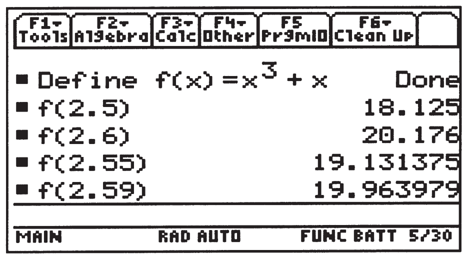
What happens if you divide the starting number by p, then add q?
What value do the terms approach?

- Find an approximate solution to $x^3 + x = 20$.





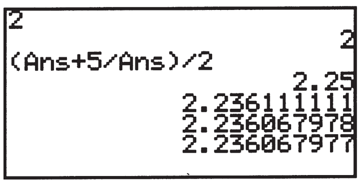
or:



- Find an approximate value of √5.

  Let the first estimate be 2.
  Consider a rectangle with sides 2 and $\frac{5}{2}$.

  Explain mathematically why the mean of these two values gives a better approximation to √5,
  i.e. $\frac{1}{2}\left(2 + \frac{5}{2}\right)$



Pupils should be taught to:

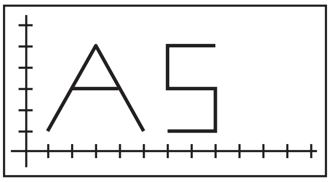
Solve word problems and investigate in a range of contexts (continued)

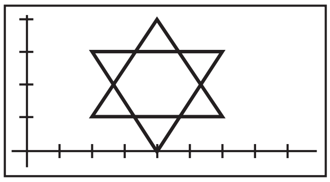
As outcomes, Year 7 pupils should, for example:

More problems to solve with a graphical calculator

For example:

- Use 'plot' and 'line' on a graphical calculator to draw these shapes.



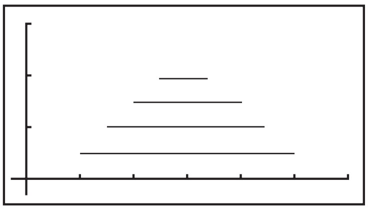
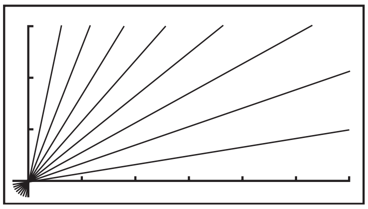


- Plot the points (3, 1) and (6, 1).
  These two points are the ends of the base of a square.
  Plot the other two points of the square.

  Clear the screen.
  Plot the points (3, 1) and (6, 1).
  These two points are now the ends of the base of a trapezium. Plot the other two points of the trapezium.
  Now plot two different trapezia with the points (3, 1) and (6, 1) as ends of their base.
  What if the base of the trapezium is (3, 1) and (5, 2)?

  Draw the line joining (3, 1) to (6, 1).
  Draw a parallelogram standing on this line.
  Now draw a hexagon standing on the line.
  Draw another shape standing on the line.
  What if the starting points are (3, 1) and (4, 5)?

- Use your graphical calculator to create displays like these by joining points.

- Use your graphical calculator to draw the outline of a skeleton cube.

- Use your graphical calculator to draw the line y = x.
  Now draw the lines y = x + 1, y = x + 2, and so on.
  Describe what happens.

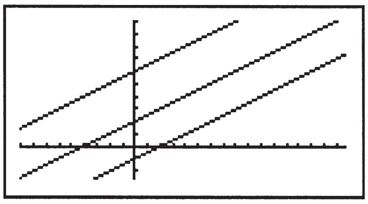
  Now draw the lines y = x, y = 2x, y = 3x, and so on.
  Describe what happens.

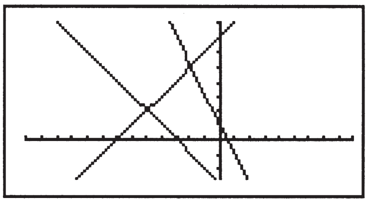
As outcomes, Year 8 pupils should, for example:

More problems to solve with a graphical calculator

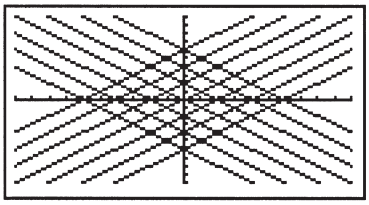
For example:

- Suggest possible equations for these straight-line graphs.

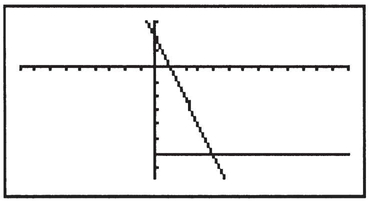




- Create a display like this with your graphical calculator.



- Draw some more straight-line graphs that pass through the point $(4, {}^{-}6)$.

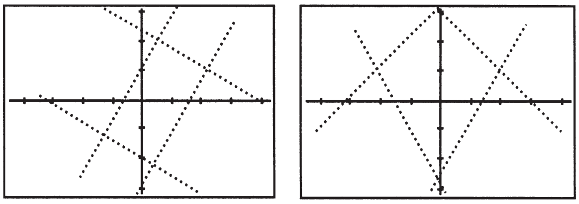


As outcomes, Year 9 pupils should, for example:

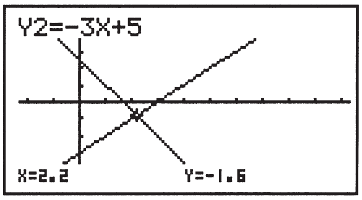
More problems to solve with a graphical calculator

For example:

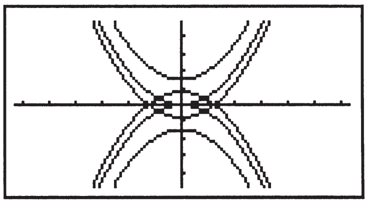
- Use a graphical calculator to draw these quadrilaterals.



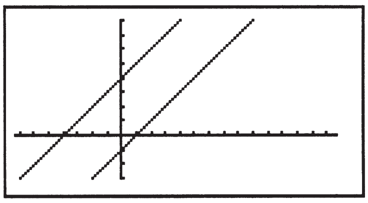
- Solve graphically the simultaneous equations:
  $y = 2x - 6$
  $y = {}^{-}3x + 5$



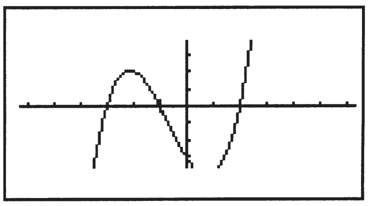
- Create a display like this with your graphical calculator.



- Suggest possible equations for these two lines. Find the shortest distance between them.



- Find a possible equation for this curve.



- Use your calculator to draw the curves $y = x^n$ for n = 1, 2, 3, 4, and so on. Describe what happens.

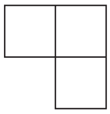
Pupils should be taught to:

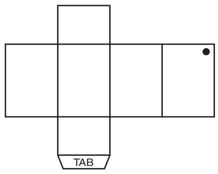
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

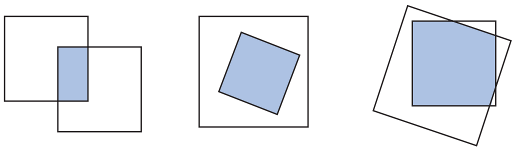
Problems involving shape and space

For example:

- This shape is called an L-triomino.
  Draw shapes made from two L-triominoes touching edge to edge.
  
  a. Draw two different shapes, each with only one line of symmetry.
  b. Draw two different shapes, each with rotation symmetry of order 2.
  c. Draw a shape with two lines of symmetry and rotation symmetry of order 2.

- Here is a net with a tab to make a cuboid.
  There is a dot (●) in a corner of one of the faces.
  

  Imagine folding the net up.
  Write T on the edge that the tab will be stuck to.
  Draw a dot on each of the two corners that will meet the corner with the ●.

- Two squares can be overlapped to make different shapes.
  The squares can be the same size or different sizes.
  For example, you could make these shapes.
  
  Which of these shapes can be made by overlapping two squares: rhombus, isosceles triangle, pentagon, hexagon, octagon, decagon, kite, trapezium?
  If you think a shape cannot be made, explain why.

- Here are some instructions to draw a regular pentagon on a computer:

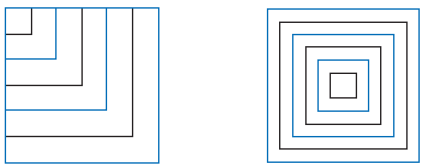
  REPEAT 5 (FORWARD 10, LEFT TURN 72°)

  Complete the instructions to draw a regular hexagon:

  REPEAT 6 (FORWARD 10, LEFT TURN …)

  How would you draw a regular octagon, decagon, …?

- Which letters of the alphabet have parallel lines in them?
  Use **Logo** to draw:
  a. some of the letters of the alphabet;
  b. a parallelogram.

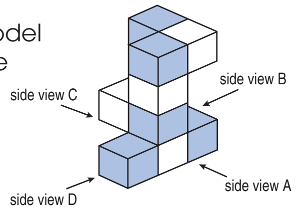
- Use **Logo** to draw squares inside each other so that there are equal distances between the squares.
  

As outcomes, Year 8 pupils should, for example:

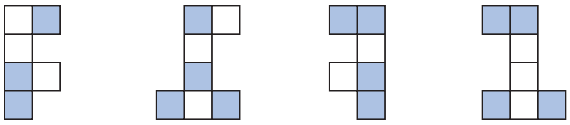
Problems involving shape and space

For example:

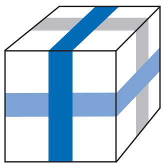
- The diagram shows a model with nine cubes, five blue and four white.



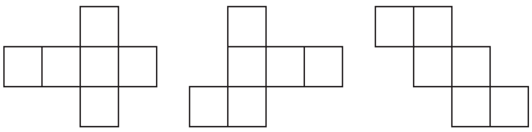
The drawings below show the four side views of the model. Which view does each drawing show?



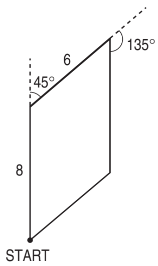
- This parcel is tied with three bands of ribbon.



Draw the bands on these three nets so that they will make the same parcel when folded up.

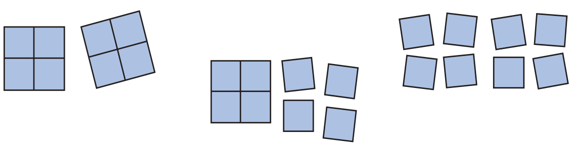


- The instructions to draw this shape are:
  - REPEAT 2
    - [FORWARD 8
    - TURN RIGHT 45°
    - FORWARD 6
    - TURN RIGHT 135°]



Write the instructions to draw a kite with one right angle and two angles of 110°.

- A 4 by 2 rectangle can be cut into squares along its grid lines in three different ways.



In how many different ways can a 6 by 3 rectangle be cut into squares along its grid lines?

Can a 5 by 3 rectangle be cut into 7 squares? 8 squares? 9 squares?
Give mathematical reasons for your answers.

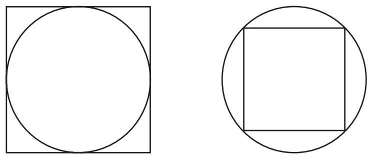
As outcomes, Year 9 pupils should, for example:

Problems involving shape and space

For example:

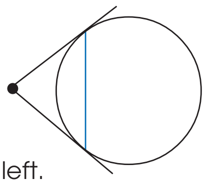
- Triangle T has vertices at (1, 2), (2, 4) and (3, 4).
  a. Draw T on squared paper.
  b. Triangle R is obtained by reflecting T in the x-axis. Draw R.
     What are the coordinates of its vertices?
  c. Triangle S is obtained by reflecting R in the y-axis. Draw S. What are its coordinates?
  d. There is a transformation that takes triangle T directly to triangle S. Describe this transformation as precisely as you can.

- Use **Logo** or **dynamic geometry software**.
  Draw a circle inside a square.
  Draw a square inside a circle.



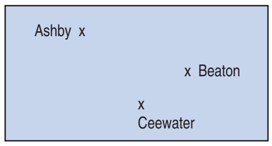
If the circles have equal diameters, what is the ratio of the areas of the squares?

- In this diagram, tangents to the circle are drawn at each end of the chord. The tangents intersect at the black dot.
  Imagine the chord moves to the left.
  What happens to the black dot?
  What if the chord moves to the right?



- The plan below shows the positions of three towns on an 8 cm by 10cm grid.

Scale: 1cm to 10km.



The towns need a new radio mast. It must be:
– nearer to Ashby than Ceewater, and
– less than 45km from Beaton.
Construct on the plan the region where the new mast can be placed.

- A blue counter and a grey counter are placed some distance apart.

Where could yellow counters be placed so that:
  a. the centre of each yellow counter is equidistant from the centres of the blue and the grey counters?
  b. the centre of each yellow counter is twice as far from the centre of the blue counter as from the centre of the grey counter?

Pupils should be taught to:

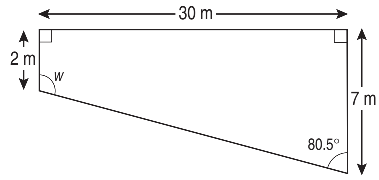
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

More problems involving shape and space

For example:

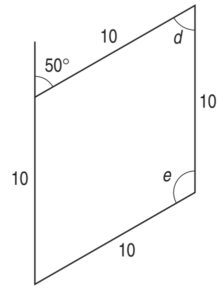
- Use this diagram to calculate angle w.



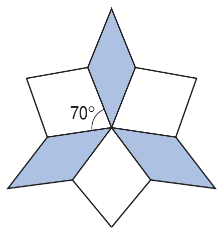
  What facts about angles did you use?

- The diagram shows a rhombus.

  Calculate the sizes of angles d and e.



- The shape shown below has three identical white tiles and three identical blue tiles.
  The sides of each tile are all the same length.
  Opposite sides of each tile are parallel.



  One of the angles in the white tile is 70°.
  Find the sizes of the other angles in the white tile.
  Find the sizes of all the angles in a blue tile.

- Draw and label axes on squared paper, choosing a suitable scale. Plot these points.

A ($^{-}1$, 1)	B (2, $^{-}1$)	C ($^{-}3$, $^{-}1$)	D (5, $^{-}1$)
E (2, 2)	F (1, $^{-}2$)	G (5, 2)	H (2, 1)

  a. Name the four points that are the vertices of:
     i. a square;
     ii. a non-square parallelogram;
     iii. a non-square trapezium.

  b. Name three points that are the vertices of:
     i. a right-angled triangle;
     ii. a non-right-angled isosceles triangle.

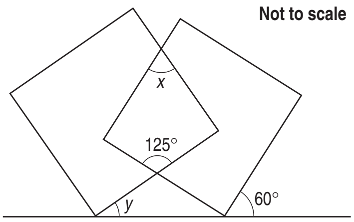
See Y456 examples (pages 110–11).

As outcomes, Year 8 pupils should, for example:

More problems involving shape and space

For example:

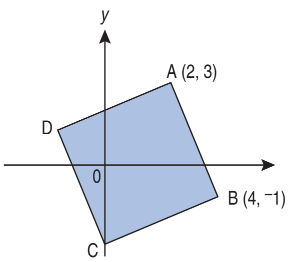
- The diagram shows two overlapping squares and a straight line.



  Calculate the values of angles x and y.

- The shaded shape ABCD is a square.

  What are the coordinates of D?

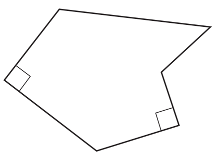


- Use ruler and compasses to construct a rhombus and a square which have the same area.

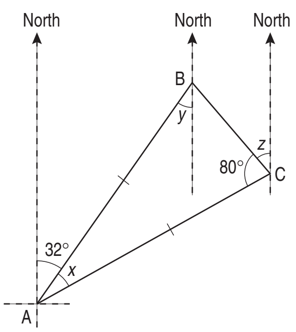
- Use ruler and compasses to construct a regular hexagon.
  Draw two diagonals of the hexagon to form a right-angled triangle.
  Explain why it is a right-angled triangle.

- What is the maximum number of right angles you can have in a hexagon?

  What about other polygons?



- The diagram shows the positions of three points, A, B and C. The distances AB and AC are equal.



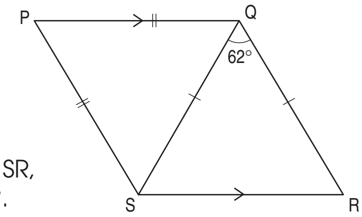
  Calculate the sizes of the angles marked x, y and z.

As outcomes, Year 9 pupils should, for example:

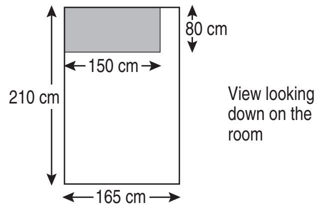
More problems involving shape and space

For example:

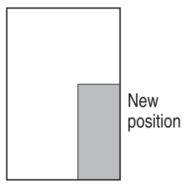
- In the diagram:
  PQ = PS,
  QR = QS,
  PQ is parallel to SR,
  angle SQR is 62°.
  Calculate the sizes of the other angles.



- In a small room, a cupboard is in the position shown.

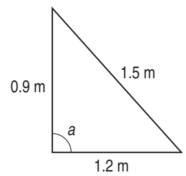


  Is the room wide enough to move the cupboard to the new position shown?



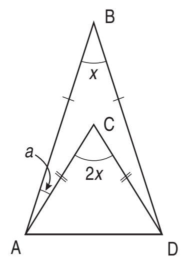
  Give a mathematical justification for your answer.

- Show that angle a is 90°.

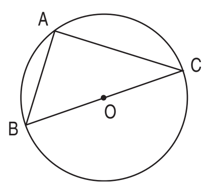


- Two isosceles triangles have the same base AD. Angle ACD is twice the size of angle ABD.

  Call these angles 2x and x. Prove that angle a is always half of angle x.



- BC is a diameter of a circle centre O. A is any point on the circumference. Prove that angle BAC is a right angle.



  Hint: Join AO.

Pupils should be taught to:

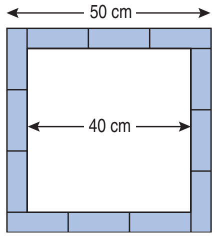
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

Problems involving perimeter and area

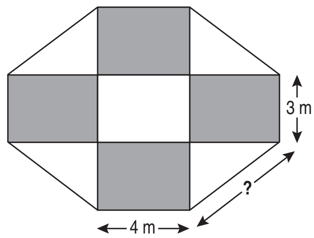
For example:

- Twelve rectangles, all the same size, are arranged to make a square outline, as shown in the diagram.



Calculate the area of one of the rectangles.

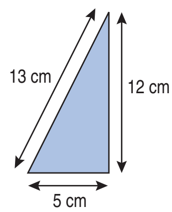
- This plan of a garden is made of rectangles and triangles.



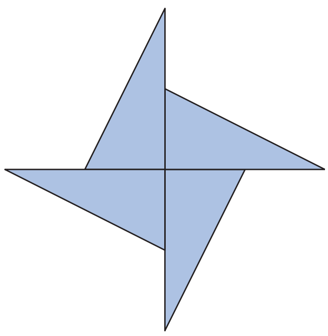
The area of each rectangle is 12 square metres.
What is the area of the whole garden?

The perimeter of the garden is 34 metres.
What is the longest side of each triangle?

- Triangles like this



are used to make a star.



What is the area of the star?
What is its perimeter?

- Use **Logo** or **dynamic geometry software** to draw a square.
Now draw a square which is one quarter of the area of the first square.
Explain why the area of the second square is one quarter of the area of the first square.

See Y456 examples (pages 96–7).

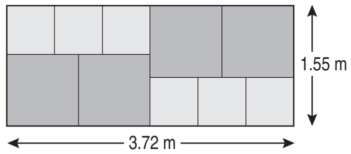
As outcomes, Year 8 pupils should, for example:

Problems involving perimeter, area and volume

For example:

- What is the smallest perimeter for a shape made of 8 regular hexagons each of side a?
  9 regular hexagons? 10 regular hexagons?

- Two sizes of square paving stones are used to make a path 3.72 metres long.



  Calculate the width of a small paving stone.

- The length of a rectangle is 4 cm more than its width. Its area is 96cm². What is its perimeter?

- A boy has some small cubes.
  The edge of each cube is 1.5 centimetres.
  He makes a large cube out of the small cubes.
  The volume of the larger cube is 216 cm³.
  How many small cubes does he use?

- Boxes measure 2.5 cm by 4.5cm by 6.2cm.
  Work out the largest number of boxes that can lie flat in a 9cm by 31cm tray.

- How many different cuboids can be made using exactly one million cubes?

- The end faces of a square prism are identical squares.
  What square prism do you end up with if you keep cutting square prisms from an 8 by 6 by 4 cuboid?

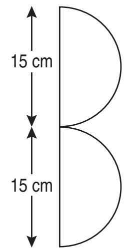
- A cuboid container holds 1 litre of water.
  Find the minimum surface area of your cuboid.
  You may find it helpful to use a **spreadsheet**.

As outcomes, Year 9 pupils should, for example:

Problems involving circumference, area and volume

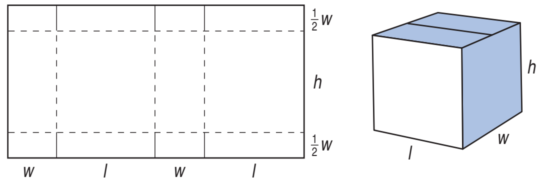
For example:

- A letter B is made out of a piece of wire.
  It has a straight side and two equal semicircles.



  Calculate the total length of the wire.

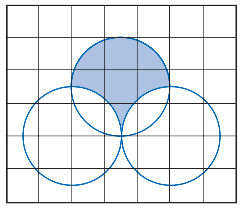
- Some boxes are made from rectangular sheets of thick card. Edges are joined with sticky tape.
  This is how to mark the cut and fold lines.
  Cuts are solid lines and folds are dotted lines.



  The net is used to make a box from a sheet of card 100 cm long and 40cm wide. The box is 20cm high. Calculate the volume of the box.

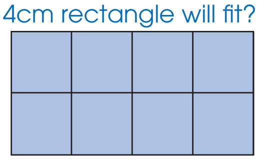
- Cover a 5 by 5 pinboard with shapes of area two square units, with no two shapes the same.

- A design is made from three circles on a 1 cm grid.



  Calculate the perimeter and area of the shaded part of the design. Give each answer correct to one decimal place.

- What is the area of the smallest circle into which this 2 cm by 4cm rectangle will fit?



- A cylinder has a diameter of 7.4cm and a height of 10.8cm.
  Find the diameter and height of some other cylinders with the same volume.
  Which of these cylinders has the smallest surface area?
  You may find it helpful to use a **spreadsheet**.

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

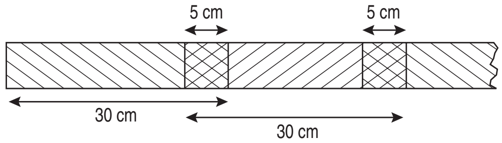
Problems involving measures

For example:

- A pupil paced the length and width of a corridor.
  She found it was 5 paces wide by 142 paces long.
  She measured one pace. It was 73 cm.
  What was the approximate length of the corridor in metres?

- A map has a scale of 1cm to 6km.
  A road measured on the map is 6.6cm long.
  What is the length of the road in kilometres?

- Strips of paper are each 30cm long.
  The strips are joined together to make a streamer.
  They overlap each other by 5cm.

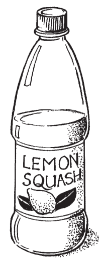


  How long is a streamer made from only two strips?

  A streamer is made that is 280cm long.
  How many strips does it use?

- A box of figs costs £2.80 per kilogram.
  A fig from the box weighs 150g.
  Find the cost of the fig.

- A lemon squash bottle holds 750ml.
  The label says: Dilute with 3 parts water.
  Asmat likes her drink to be strong.
  She adds between 2 and 2.5 parts water to the squash to make a drink.
  How many litres of lemon drink can she make with one bottle of squash?



- A number of bars of soap are packed in a box that weighs 850 g. Each bar of soap weighs 54g.
  When it is full, the total weight of the box and the soap is 7.6kg. How many bars of soap are in the box?

- The maximum load in a small service lift is 50kg.
  60 tins of food must go up in the lift. Each tin weighs 840g.
  Is it safe to load all of these tins into the lift?

  The tins are to go in a cupboard which is 1.24m high.
  Each tin is 15cm high.
  How many layers of tins will fit in the cupboard?

- Every morning Ramesh catches a school bus at 8:05 a.m.
  It arrives at the school at about 8:40 a.m.
  Each Friday, the bus takes longer and it arrives at 8:55 a.m.
  How long does Ramesh spend coming to school over a school week, a term of 16 weeks, a year of 39 weeks…?

See Y456 examples (pages 86–9).

As outcomes, Year 8 pupils should, for example:

Problems involving measures

For example:

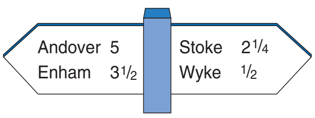
- A school expects between 240 and 280 parents for a concert. Chairs are to be put out in the hall. Each chair is 45 cm wide, and no more than 8 chairs must be put in a row together.
  There must be a corridor 1m wide down the middle of the hall and 0.5m of space between the rows for people to get to their seats.
  What is the minimum space needed to set out the chairs?

- The Wiro Company makes 100000 wire coat-hangers each day.
  Each coat-hanger uses 87.4cm of wire.
  How many km of wire are used each day?
  What is the greatest number of coat-hangers that can be made from 100m of wire?

  The company decides to reduce the amount of wire used for each coat-hanger to 85cm.
  How many more coat-hangers can be made from 100m of wire?

- Mr Green sells apples at 40p per kilogram.
  Mrs Ball sells apples at 24p per pound.
  Who sells the cheaper apples?
  Explain how you worked it out.

- The distance to Andover is given as 5 miles.



  Do you think that this is correct to:
  P. the nearest yard,
  Q. the nearest quarter mile,
  R. the nearest half mile, or
  S. the nearest mile?

  Use your answer to state:
  a. the shortest and longest possible distances to Andover;
  b. the shortest possible distance between Stoke and Enham.

- Using a map, measure the bearings and distances 'as the crow flies' between the centres of York, Sheffield, Derby and Nottingham.
  Make a scale drawing of the positions of these four cities using a scale of 1 : 500000.

As outcomes, Year 9 pupils should, for example:

Problems involving measures

For example:

- Here is an old recipe for egg custard with raisins.
  Change the amounts into metric measures.

  Egg custard with raisins
  ¼ pound of raisins
  1 pint of milk
  3 eggs

  Put the raisins in an 8 inch bowl.
  Mix the eggs and milk, and pour over the raisins.
  Bake in an oven at 320° Fahrenheit for about 1 hour.

- A plank of wood weighed 1.4 kg.
  25 centimetres of the plank were cut off its length.
  The plank then weighed 0.8kg.
  What was the length of the original plank?

- The diameter of a red blood cell is 0.000714cm and the diameter of a white blood cell is 0.001243cm.

  Work out the difference between the diameter of a red cell and the diameter of a white cell.
  Give the answer in millimetres.

  Calculate how many white cells would fit across a full stop which has a diameter of 0.65mm.

- A world-class sprinter can run 100 m in about 10 seconds.
  A bus takes 15 minutes to go 3 miles to the next town.
  Which average speed is faster, that of the sprinter or the bus?

- Alice walked 800m to school.
  She timed herself with a stop watch.
  It took her 10 minutes 27.6 seconds.
  What was her average speed?
  How accurately do you think it is sensible to give your answer?

- Pluto takes 248 years to circle the Sun at a speed of 1.06×10^4 miles per hour.
  Assuming Pluto's orbit is a circle, how far is it from the Sun?

- Liquid for cleaning jewellery is stored in a cuboid container with a base of 5 cm by 4cm.
  A gold brooch weighing 48.25g is to be cleaned by immersing it in the liquid.
  The density of gold is $19.3 g/cm^3$.
  By how much will the liquid in the container rise?

Pupils should be taught to:

Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

Problems involving probability

For example:

- Samir spins a fair coin and records the results.
  In the first four spins, 'heads' comes up each time.

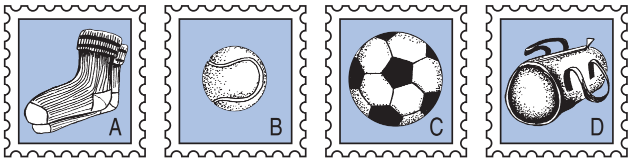
1st spin	2nd spin	3rd spin	4th spin
head	head	head	head

  Samir says:
  'A head is more likely than a tail.'
  Is he correct? Give a reason for your answer.

- There are six balls in a bag.
  The probability of taking a red ball out of the bag is 0.5.
  A red ball is taken out of the bag and put to one side.
  What is the probability of taking another red ball out of the bag?

- In each box of cereal there is one free card.
  You cannot tell which card will be in a box.
  Each card is equally likely.

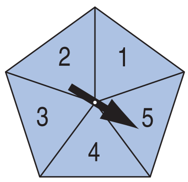
  Altogether there are four different cards.
  When you have them all, you can send for free sports socks.



  Zoe needs card A. Paul needs cards C and D.
  They buy one box of cereal.
  What is the probability that:
  a. the card is one that Zoe needs?
  b. the card is one that Paul needs?

  Their mother opens the box.
  She tells them the card is not card A.
  What is the probability now that:
  c. the card is one that Zoe needs?
  d. the card is one that Paul needs?

- A fair spinner has five sections numbered 1, 2, 3, 4, 5.
  What is the probability of getting a prime number from one spin?



  What about a fair spinner with 6 sides? 7 sides?

  Draw a bar-line graph to show the probability of getting a prime number from one spin of a spinner with 4 to 15 sides.

- Some children choose six tickets numbered from 1 to 200.
  Kay chooses numbers 1, 2, 3, 4, 5 and 6.
  Zak chooses numbers 14, 45, 76, 120, 137 and 182.
  Mary then picks a number at random from 1 to 200.
  Is Kay or Zak more likely to have Mary's number? Explain why.

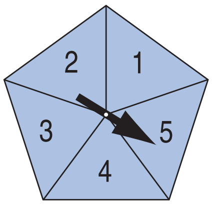
See Y456 examples (pages 112–13).

As outcomes, Year 8 pupils should, for example:

Problems involving probability

For example:

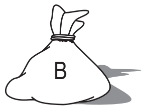
- Here is a spinner with five equal sections.



Jane and Sam play a game.
They spin the pointer many times.
If it stops on an odd number, Jane gets 2 points.
If it stops on an even number, Sam gets 3 points.
Is this a fair game?
Explain your answer.

- Bag A contains 12 red counters and 18 yellow counters.

Bag B contains 10 red counters and 16 yellow counters.



I am going to take one counter at random from either bag A or bag B.
I hope to get a red counter.
Which bag should I choose? Justify your choice.

- All the cubes in a bag are either red or black.
The probability of taking out a red cube at random is $\frac{1}{5}$.
One cube is taken at random from the bag and placed on the table. The cube is red.
What is the smallest number of black cubes there could be in the bag?

Another cube is taken from the bag and placed beside the first cube. The second cube is also red.
From this new information, what is the smallest number of black cubes there could be in the bag?

- The names of all the pupils, all the teachers and all the canteen staff of a school are put in a box.
One name is taken out at random.
A pupil says:
'There are only three choices.
It could be a pupil, a teacher or one of the canteen staff.
The probability of it being a pupil is $\frac{1}{3}$.'

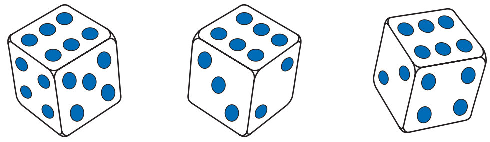
The pupil is wrong. Explain why.

As outcomes, Year 9 pupils should, for example:

Problems involving probability

For example:

- Some pupils threw three fair dice.

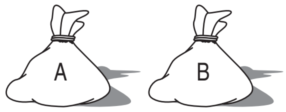


They recorded how many times the numbers on the dice were the same.

Name	No. of throws	Results		
		all different	two the same	all the same
Morgan	40	20	12	2
Sue	140	81	56	3
Zenta	20	10	10	4
Ali	100	54	42	0

Write the name of the pupil whose data are most likely to give the best estimate of the probability of getting each result. Explain your answer.

- Two bags, A and B, contain coloured cubes.



Each bag has the same number of cubes in it.
The probability of taking a red cube at random out of bag A is 0.5.
The probability of taking a red cube at random out of bag B is 0.2.

All the cubes are put in an empty new bag.
What is the probability of taking a red cube out of the new bag?

What if bag A has twice the number of cubes that are in bag B?

- Karen and Huw each have three cards, numbered 2, 3 and 4. They each take one of their own cards.
They then add together the numbers on the four remaining cards.
What is the probability that their answer is an even number?

- John makes clay pots.
Each pot is fired independently. The probability that a pot cracks while being fired is 0.03.

  a. John fires two pots.
  Calculate the probability that:
  i. both pots crack;
  ii. only one of them cracks.

  b. John has enough clay for 80 pots.
  He gets an order for 75 pots.
  Does he have enough clay to make 75 pots without cracks? Explain your answer.

Pupils should be taught to:

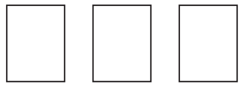
Solve word problems and investigate in a range of contexts (continued)

As outcomes, Year 7 pupils should, for example:

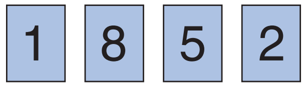
Problems involving handling data

For example:

- Write a different number in each of these boxes so that the mean of the three numbers is 9.



- James has four number cards.
  Their mean is 4.
  James takes another card.
  The mean of his five cards is now 5.
  What number is on the new card?

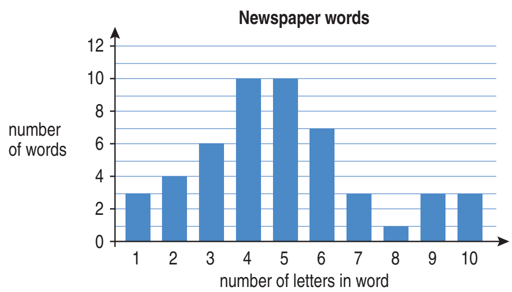


- Jeff has played two of the three games in a competition.

	Game A	Game B	Game C
Score	62	53	

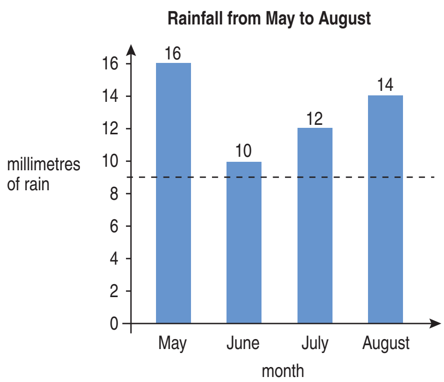
  To win, Jeff needs a mean score of 60.
  How many points does he need to score in game C?

- Kelly chooses a section of a newspaper. It has 50 words in it.
  She draws a bar chart of the number of letters in each word.



  Kelly says:
  '23 of the 50 words have fewer than 5 letters. This shows that nearly half of all the words used in the newspaper have fewer than 5 letters in them.'
  Explain why she could be wrong.

- Here is a bar chart showing rainfall.



  Kim draws a dotted line on the bar chart. She says:
  'The dotted line on the chart shows the mean rainfall for the four months.'
  Use the chart to explain why Kim cannot be correct.

See Y456 examples (pages 114–17).

As outcomes, Year 8 pupils should, for example:

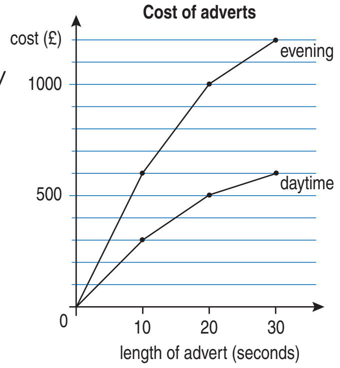
Problems involving handling data

For example:

- Imran and Nia play three different games in a competition. Their scores have the same mean.
  The range of Imran's scores is twice the range of Nia's scores.
  Fill in the missing scores in the table below.

Imran's scores		40	
Nia's scores	35	40	45

- This chart gives the cost of showing advertisements on TV at different times.



An advertisement lasts 25 seconds. Use the graph to estimate how much cheaper it is to show it in the daytime compared with the evening.

An advertisement was shown in the daytime and again in the evening. The total cost was £1200.
How long was the advertisement in seconds?

- The cost of exporting 350 wide-screen TV sets was:

Item	Cost
freight charge	£371
insurance	£49
port charges	£140
packing	£280

a. Find the mean cost of exporting a wide-screen TV.
b. Construct a pie chart showing how the export costs in the table make up the total cost.

- Fifteen pupils measured an angle.
  Here are their results.

Angle measured as	Number of pupils
45°	5
134°	3
135°	4
136°	3

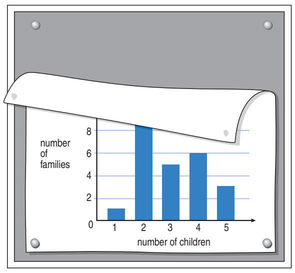
Use the results to decide what the angle is most likely to measure.
Give your reasons.

As outcomes, Year 9 pupils should, for example:

Problems involving handling data

For example:

- A class collected information about the number of children in each of their families.
  The information was displayed in a frequency chart, but you cannot see it all.



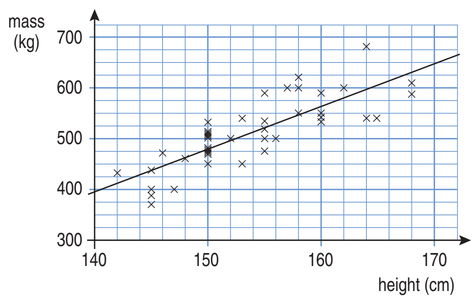
Let n be the number of families with 2 children.
a. Show that the total number of children in all families is 55 + 2n.
b. Write an expression for the total number of families.
c. The mean number of children is 3.
   What is the value of n?

- Here are some data about the population of the regions of the world in 1950 and 1990.

Regions of the world	Population in millions in 1950	Population in millions in 1990
Africa	222	642
Asia	1558	3402
Europe	393	498
Latin America	166	448
North America	166	276
Oceania	13	26
World	**2518**	**5292**

a. In 1990, for every person who lived in North America, how many people lived in Asia?
b. A pupil thinks that from 1950 to 1990 the population of Oceania went up by 100%.
   Is the pupil right? Explain your answer.

- This scatter graph shows heights and masses of some horses. It also shows a line of best fit.



a. What does the scatter graph show about the relationship between the height and mass of horses?
b. The mass of a horse is 625 kg. Estimate its height.
c. Laura thinks that the length of the back leg of a horse is always less than the length of its front leg. If this were true, what would the scatter graph look like? Draw a sketch.

Pupils should be taught to:

Identify the information necessary to solve a problem; represent problems mathematically in a variety of forms

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
investigate, explore, solve, explain… true, false…
problem, solution, method, answer, results, reasons, evidence...

Identify the necessary information; represent problems mathematically, making correct use of symbols, words, diagrams, tables and graphs.
For example, solve:

- Birthday candles
  Mrs Sargent is 71 years old.
  Every year since she was born she has blown out the corresponding number of candles on her birthday cake.
  How many birthday candles has she blown out altogether?

  Related objectives:
  Explain and justify methods and conclusions, orally and in writing.

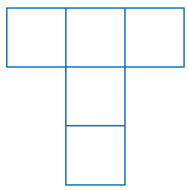
  Link to expressing functions in words then symbols (pages 160–1); simple mappings (pages 160–1).

- Challenging calculators
  Find at least two different solutions to these problems.
  a. The 7 key doesn't work.
     Make your calculator display 737.
  b. None of the keys that are odd numbers work.
     Make the calculator display 975.

  Related objectives:
  Break a complex calculation into simpler steps; choose and use appropriate and efficient operations, methods and resources, including ICT.

  Link to rapid recall of number facts, including complements to 100 and multiplication and division facts (pages 88–9); using the order of operations, and brackets (pages 86–7).

- Tiles
  A T-shape is made from 5 square tiles.
  The length of a side of a tile is w cm.
  Write an expression for the area of the T-shape.
  If the area of the T-shape is 720 cm^2, what is the length of its perimeter?

  Another T-shape is made from 5 square tiles.
  The length of a side of a tile is v cm.
  Write an expression for the perimeter of the T-shape.
  If the perimeter of the T-shape is 120 cm, what is its area?

  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to using the formula for the area of a rectangle; calculating the perimeter and area of compound shapes made up of rectangles (pages 232–3).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
best estimate, degree of accuracy…
justify, prove, deduce… conclude, conclusion…
counter-example, exceptional case…

Identify the necessary information; represent problems in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.
For example, solve:

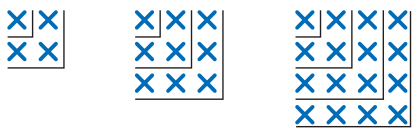
- Estimating lengths
  Who is best at estimating 10 centimetres?
  Decide how to find out. For example, each pupil could cut 20 pieces of string to an estimated 10 cm, then measure each piece to find the 'error' to the nearest half centimetre. Find the mean error for each pupil.

  Confirm by estimating 20 different lengths of ribbon held up at the front of the class, including four lengths of 10 cm.

  Related objectives:
  Give solutions to an appropriate degree of accuracy in the context of the problem.

  Link to comparing two distributions using the range and one or more of the measures of average (pages 272–3);
  interpreting tables, graphs and diagrams for both discrete and continuous data and drawing inferences; relating summarised data to the questions being explored (pages 268–71).

- Crosses
  How many crosses are there in each pattern?



  a. Find the value of:
     i. $1 + 3$ ii. $1 + 3 + 5$ iii. $1 + 3 + 5 + 7$
  b. Draw the next pattern in the sequence.
     What is the value of $1 + 3 + 5 + 7 + 9$?
  c. What is the sum of:
     i. the first 6 odd numbers?
     ii. the first 20 odd numbers?
     iii. the first n odd numbers?
  d. Add the crosses in each pattern along diagonals. Prove from the third pattern that
     $1 + 2 + 3 + 4 + 3 + 2 + 1 = 4^2$
     Generalise this result.

  Related objectives:
  Solve problems and explore pattern and symmetry in a range of contexts.

  Link to generating sequences from practical contexts and finding the general term in simple cases (pages 154–7).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
trial and improvement…
generalise…

Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem. For example, solve:

- Five coins
  A game involves tossing five coins for a 10p stake.
  If you score exactly two heads you win 20p and get your stake back; otherwise you lose.
  Give mathematical reasons to justify whether this is a sensible game to play.

  Related objectives:
  Present a proof, making use of symbols, diagrams and graphs and related explanatory text;
  justify generalisations and choice of presentation, explaining selected features.

  Link to identifying all the mutually exclusive outcomes of an experiment; knowing that the sum of probabilities of all mutually exclusive outcomes is 1 (pages 278–81);
  comparing experimental and theoretical probabilities (pages 284–5);
  appreciating the difference between mathematical explanation and experimental evidence (pages 284–5).

- Painted cubes
  A cube of side 3 cm is made up of 27 individual centimetre cubes. The cube is dipped into a pot of paint, so that all the exterior sides are covered in the paint. The cube is then broken up into the individual 27 centimetre cubes.

  How many of the cm cubes have 3 sides painted? 2 sides painted? 1 side painted? 0 sides painted?
  Explore for different sized cubes.

  Generalise. Justify your generalisation.
  Explore further.

  Related objectives:
  Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples, explaining why; pose extra constraints and investigate whether particular cases can be generalised further.

  Link to generating sequences using term-to-term and position-to-term definitions, on paper and using ICT (pages 148–51);
  finding the next term and nth term of a sequence and exploring its properties (pages 152–3).

Pupils should be taught to:

Break problems into smaller steps or tasks; choose and use efficient operations, methods and resources

As outcomes, Year 7 pupils should, for example:

Break a complex calculation into simpler steps, choosing and using appropriate and efficient operations, methods and resources, including ICT. For example, solve:

- Square shuffle
  This activity is based on a slider puzzle.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

  Seat 15 pupils in a 4 by 4 array of chairs.
  The aim is to move the pupils across the chairs so that the person in the back left seat (labelled 13) is moved to the front right seat (labelled 4) in as few moves as possible.

  The rules are:
  - The person in front of, behind or alongside the empty chair can move into it.
  - No one else can move.
  - Diagonal moves are not allowed.

  Explore for different sizes of squares.

  Related objectives:
  Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing.

  Link to generating and describing simple integer sequences (pages 144–7); expressing simple functions in words then symbols, and representing simple mappings (pages 160–3).

- Number problems
  a. Make up word problems to reflect statements such as:
     36 × 18 × 45 = 29 160
     (36 + 45) ÷ 2 = 40.5
     20% of 450 = 90

  b. What operation is represented by each ✻ ?
     905 ✻ 125 = 1030   905 ✻ 125 = 780
     905 ✻ 125 = 7.24   905 ✻ 125 = 113 125

  Related objectives:
  Represent problems mathematically, making correct use of symbols and words.

  Link to written calculations (pages 104–7); understanding operations (pages 82–5).

See Y456 examples (pages 74–5).

As outcomes, Year 8 pupils should, for example:

Solve more complex problems by breaking them into smaller steps, choosing efficient techniques for calculation, algebraic manipulation and graphical representation, and resources, including ICT.
For example, solve:

- Missing digits
  Find the missing digits represented by the ✻ in examples such as:
  $(\ast 5)^2 = \ast\ast\ast$ $(\ast\ast)^2 = \ast\ast 25$
  $(\ast\ast)^3 = \ast\ast\ast 7$ $(\ast\ast\ast)^2 = \ast 44 \ast 44$
  $(3 \times \ast\ast)^2 = 54\ast 56$

  Make up some examples of your own.
  Make sure that someone else can solve them!

  Related objectives:
  Solve problems in a range of contexts (number).

  Link to factors, powers and roots (pages 52–5).

- Hand luggage
  An airline specifies that hand baggage must meet this requirement:

  length + width + depth must be less than 1 m

  What dimensions for the hand baggage would give the most space for the contents?

  Related objectives:
  Solve problems in a range of contexts (measures).

  Link to knowing the formula for the volume of a cuboid; calculating volumes and surface areas of cuboids and shapes made of cuboids (pages 238–41).

- Everything 15% off!
  In a gift shop sale everything is reduced by 15%.
  A quick way of calculating the sale price is to multiply the original price by a number.
  What is the number?
  Give a mathematical reason to justify your answer.

  After two weeks, the sale price is reduced by a further 15%. Show that this means the original price has been reduced by 27.75%.

  Related objectives:
  Use logical argument to establish the truth of a statement; give solutions to an appropriate degree of accuracy in the context of the problem.

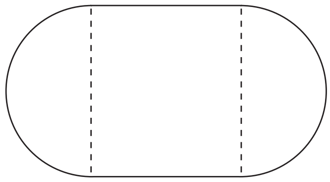
  Link to expressing a given number as a percentage of another; using the equivalence of fractions, decimals and percentages to compare proportions; calculating percentages and finding the outcome of a given percentage increase or decrease (pages 70–7).

As outcomes, Year 9 pupils should, for example:

Solve substantial problems by breaking them into simpler tasks, using a range of efficient techniques, methods and resources, including ICT; use trial and improvement where a more efficient method is not obvious. For example, solve:

- Running track
  Design a running track to meet these constraints:

  – The inside perimeter of the track has this shape.

  – Both straights must be at least 80 metres.
  – Both ends must be identical semicircles.
  – The total inside perimeter must be 400 m.
  What is the greatest area the running track can enclose?

  What if the track has to be 8 metres wide?
  What is the smallest rectangular field needed to contain it?

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to using the formulae for the circumference and area of a circle (pages 234–7).

- Number puzzle
  A number plus its square equals 30.
  How many different solutions can you find?

  What if a number plus its cube equals 30?

  Related objectives:
  Suggest extensions to problems, conjecture and generalise, identify exceptional cases or counter-examples, explaining why;
  pose extra constraints and investigate whether particular cases can be generalised further.

  Link to simplifying algebraic expressions (pages 116–17); expanding the product of two linear expressions (pages 118–21); using systematic trial and improvement methods and ICT tools to find approximate solutions of equations (pages 132–5).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Present and interpret solutions, explaining and justifying methods, inferences and reasoning

Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing. For example, solve:

- Number cell puzzles

2	5			

The empty cells are filled by adding the two preceding numbers, for example:

2	5	7	12	19

Using the same rule, find the missing numbers for these cells:

6				24

10				23

What about these?

10				24

				24

Related objectives:
Suggest extensions to problems by asking 'What if ...?'; understand the significance of a counter-example when looking for generality.

Link to adding/subtracting simple fractions (pages 66–9); calculating fractions of quantities and measurements (whole-number answers) (pages 66–9); multiplying a fraction by an integer (pages 66–9); extending mental methods of calculation to include decimals, fractions and percentages (pages 92–101).

- Square totals
  Start with a 1–100 square.
  Choose a 2 by 2 square, for example:

1	2
11	12

Because the first number in the square is 1, call the sum of the four numbers in the square S_1.
$S_1 = 1 + 2 + 11 + 12$, so $S_1 = 26$.
What about S_2, S_3, S_4, …?
What do you notice?
Why does this happen?

Try a 2 by 3 rectangle, or a 3 by 3 square. Explore further.

Related objectives:
Suggest extensions to problems by asking 'What if…?'

Link to generating simple linear sequences (pages 144–7); expressing functions in words then symbols (pages 160–1); simple mappings (pages 160–1).

As outcomes, Year 8 pupils should, for example:

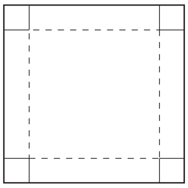
Use logical argument to establish the truth of a statement; give solutions to an appropriate degree of accuracy in the context of the problem.
For example, solve:

- Consecutive sums
  Prove that the sum of any five consecutive numbers is always divisible by 5.

  Related objectives:
  Represent problems in algebraic, geometric or graphical form.

  Link to multiples, factors and primes, and tests of divisibility (pages 52–5).

- Max box
  Open-top boxes can be made from paper by cutting identical squares from each corner and folding up the sides.



  Start with a 20 cm square.
  Plan to make an open-top box with the greatest possible capacity.
  What are its dimensions?

  Explore for other sizes of squares.

  Related objectives:
  Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples.

  Link to ordering decimals (pages 40–1); constructing linear functions arising from real-life problems and plotting their graphs (pages 172–3); interpreting graphs arising from real situations (pages 174–7).

- Pizza
  These are the ingredients for a pizza for 4 people.

½ oz dried yeast	2 oz mushrooms
½ pint water	2 tomatoes
1 lb of plain flour	4 oz cheese
½ teaspoon of salt	6 black olives
8 oz ham	

  Adapt the recipe for 6 people.
  Convert the recipe to metric measurements.

  Related objectives:
  Solve problems in a range of contexts.

  Link to solving simple word problems involving ratio and direct proportion (pages 78–81); converting imperial to metric measures (pages 228–9).

As outcomes, Year 9 pupils should, for example:

Present a concise, reasoned argument, using symbols, diagrams, graphs and text; give solutions to an appropriate degree of accuracy; recognise limitations on accuracy of data and measurements; give reasons for choice of presentation, explaining features, showing insight into the problem's structure.
For example, solve:

- Perimeter
  The perimeter of a triangle is 48 cm. The length of the shortest side is s cm, and of another side is 2s cm. Prove that $12 > s > 8$.

  Related objectives:
  Represent problems in algebraic, geometric or graphical form.

  Link to solving problems using properties of triangles (pages 184–9).

- Round table
  At Winchester there is a large table known as the Round Table of King Arthur.

  The diameter of the table is 5.5 metres.
  A book claims that 50 people can sit around the table. Do you think this is possible?
  Explain and justify your answer.
  State all the assumptions that you make.

  Related objectives:
  Solve substantial problems by breaking them into simpler tasks, using efficient techniques, methods and resources, including ICT; use trial and improvement where a more efficient method is not obvious.

  Link to using circle formulae (pages 234–7).

- Seeing the wood for the trees
  Estimate the number of trees that are needed each day to provide newspapers for the UK.

  Related objectives:
  Solve increasingly demanding problems; explore connections in mathematics across a range of contexts.

  Link to discussing how data relate to the problem, identifying possible sources; identifying possible bias and planning to minimise it (pages 250–1); communicating results using selected tables, graphs and diagrams in support, using ICT as appropriate (pages 272–5); examining results critically, recognising the limitations of any assumptions and their effect on conclusions drawn (pages 272–5).

Pupils should be taught to:

Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples

As outcomes, Year 7 pupils should, for example:

Suggest extensions to problems by asking 'What if…?'; begin to generalise and to understand the significance of a counter-example.

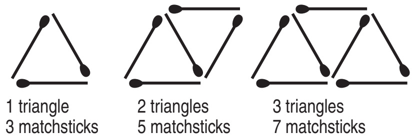
Carry out simple investigations, explain the approach and results, and generalise outcomes. For example:

- Factors
  Find the smallest number with exactly 3 factors.
  Now find the smallest number with exactly 4 factors.
  What about other numbers of factors? For example, can you find a number with exactly 13 factors?

  Related objectives:
  Choose and use efficient operations, methods and resources.

  Link to multiples, factors and primes, and tests of divisibility (pages 52–5).

- Matchstick shapes
  Rows of triangles can be generated using matchsticks.

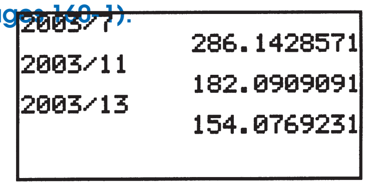


  One triangle can be made from three matchsticks, two from five matchsticks and so on. How many matchsticks are needed for 5 triangles? For 10 triangles? For 50 triangles?
  Find the rule for the number of matchsticks in any number of triangles in a row.
  Explore rows of other shapes made from matchsticks.

  Related objectives:
  Identify the necessary information; represent problems mathematically.
  Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing.

  Link to generating sequences (pages 144–57); expressing functions in words then symbols (pages 160–1) and representing mappings (pages 160–1).

- Prime numbers
  Is 2003 a prime number?
  How can you be sure?
  Explore other four-digit numbers.



  Related objectives:
  Explain and justify methods and conclusions, orally and in writing.

  Link to multiples, factors and primes, and tests of divisibility (pages 52–5).

As outcomes, Year 8 pupils should, for example:

Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples.

Carry out investigations, explain the approach and results, and generalise outcomes. For example:

- Multiplication squares
  Start with a multiplication square.
  Choose a 2 by 2 square of numbers, for example:

12	16
15	20

  Multiply the two pairs of numbers on opposite corners, i.e.

  $12 \times 20 = 240$
  $15 \times 16 = 240$

  Are both products always equal?
  Prove that for all such squares, the products of the numbers in opposite corners are equal.
  Try bigger squares and rectangles.
  Explore further.

  Related objectives:
  Use logical argument to establish the truth of a statement; present problems and solutions in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.

  Link to multiples and factors (pages 52–5).

- Triangles in cubes
  Triangles are made by joining three of the vertices of a cube. How many different shapes can the triangles have? How many of the shapes are isosceles triangles? How many are equilateral triangles?

  Which of the triangles has the largest area? Justify your answer.

  Related objectives:
  Identify the necessary information to solve a problem; represent problems and solutions in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.

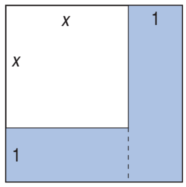
  Link to solving geometrical problems using side and angle properties of triangles, explaining reasoning with diagrams and text (pages 184–9); using the formula for the area of a triangle (pages 234–7).

As outcomes, Year 9 pupils should, for example:

Suggest extensions, conjecture and generalise; identify exceptional cases or counter-examples, with explanation; justify generalisations, arguments or solutions; pose extra constraints and investigate whether particular cases can be generalised further.

Carry out investigations, explain the approach and results, and generalise outcomes. For example:

- Difference of two squares
  Find two consecutive whole numbers whose squares differ by 37.
  Use the diagram to help.



  Use a diagram to simplify $(n + 1)^2 - (n - 1)^2$.
  Find two consecutive odd numbers whose squares differ by 72.

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to simplifying expressions (pages 116–21); generating sequences, on paper and using ICT (pages 148–53).

- Making statements
  Adding two numbers and then squaring is the same as squaring each number and then adding.
  Is this always true, never true, or sometimes true?
  Justify your answers.

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to using formulae; substituting numbers into expressions and formulae; deriving a formula and changing its subject, extending to more complex formulae (pages 138–43).

- Splitting 17
  If the sum of two numbers is 17, what is the greatest product they can have?
  What if there are three numbers? Explore.

  Related objectives:
  Solve substantial problems by breaking them into simpler tasks, using a range of methods and resources, including ICT; use trial and improvement where a more efficient method is not obvious.

  Link to using index notation for integer powers and the index laws (pages 56–9).

Pupils should be taught to:

Suggest extensions to problems, conjecture and generalise; identify exceptional cases or counter-examples (continued)

As outcomes, Year 7 pupils should, for example:

- Perimeter and area

  The larger the perimeter of a rectangle, the larger its area.

  Is this statement true?

  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to using the formula for the area of a rectangle; calculating the perimeter and area of shapes made from rectangles (pages 234–7).

- Evens

  The sum of four even numbers is divisible by 4.

  When is this statement true? When is it false?

  Related objectives:
  Solve mathematical problems in a range of contexts.
  Present and interpret solutions in the context of the original problem; explain and justify methods and conclusions, orally and in writing.

  Link to multiples and factors (pages 52–5).

- Paraffin jugs

  A hardware shop has only a 5 gallon jug and a 3 gallon jug to measure out paraffin for customers.
  How can the shop assistant measure 1 gallon without wasting any paraffin?

  What if the assistant has a 7 gallon jug and a 4 gallon jug?

  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to rapid recall of addition and subtraction facts (pages 88–9).

- Classifying quadrilaterals

  Copy this table on to a large piece of paper.

		Number of pairs of parallel sides		
		0	1	2
Number of pairs of equal sides	0			
	1			
	2			

  Draw and name quadrilaterals in the appropriate spaces.
  Will any of the spaces remain empty? If so, explain why.

  Related objectives:
  Identify the necessary information; represent problems mathematically making correct use of symbols, words, diagrams, tables or graphs.

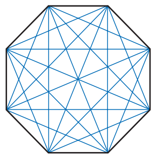
  Link to identifying and using angle, side and symmetry properties of triangles and quadrilaterals (pages 184–9).

See Y456 examples (pages 78–9).

As outcomes, Year 8 pupils should, for example:

- Diagonals
  Prove that the number of diagonals d in a regular polygon with n sides is

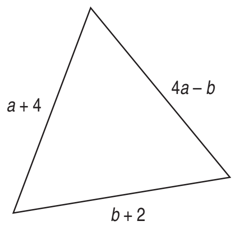
  $d = \frac{1}{2}n(n - 3)$



  Related objectives:
  Identify the necessary information; represent problems mathematically making correct use of symbols, words, diagrams, tables or graphs.

  Link to expressing simple functions in symbols (pages 112–13); using linear expressions, justifying the form by relating to the context (pages 154–7).

- Perimeter of a triangle
  The diagram shows the lengths of the sides of a triangle in centimetres.
  The triangle is equilateral.
  Find its perimeter.



  Related objectives:
  Solve mathematical problems in a range of contexts.

  Link to finding the perimeter of a triangle (pages 234–7); solving linear equations with integer coefficients (pages 122–5); substituting integers into simple formulae (pages 138–43).

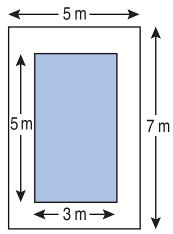
- True or false?
  Use logical argument to establish whether these statements are true or false.
  a. An isosceles triangle is made up of two identical right-angled triangles.
  b. A rhombus is a parallelogram but a parallelogram is not a rhombus.

  Related objectives:
  Present problems and solutions in algebraic, geometric or graphical form, using correct notation and appropriate diagrams.

  Link to explaining and justifying inferences, deductions and conclusions using mathematical reasoning (pages 184–9).

As outcomes, Year 9 pupils should, for example:

- Garden path
  A metre wide garden path surrounds a rectangular lawn.



  The area of the path can be found by subtracting the area of the lawn from the whole area,
  i.e. $5 \times 7 - 3 \times 5 = 20$.
  So the area of the path is 20 m^2.

  However, $5 + 7 + 3 + 5 = 20$.

  Prove that the sum of the inner and outer width and length will always give the numerical value of the area.

  Related objectives:
  Solve increasingly demanding problems; explore connections in mathematics across a range of contexts; generate fuller solutions.
  Represent problems and synthesise information in algebraic, geometric or graphical form; move from one form to another to gain a different perspective on the problem.

  Link to taking out single-term common factors; squaring a linear expression, expanding the product of two linear expressions, establishing identities (pages 116–21);
  deriving and using a formula and changing its subject, extending to more complex formulae (pages 140–3).

- Prove it
  Prove these statements:
  a. Any three-digit whole number is divisible by 9 if the sum of the digits is divisible by 9.
  b. The difference between a two-digit number and its reverse is always a multiple of 9.

  Related objectives:
  Represent problems and synthesise information in algebraic, geometric or graphical form.

  Link to relevant content.

- Hexagons
  A regular hexagon has perimeter of 60 cm.
  Calculate its area.
  Find a method to calculate the area of any regular hexagon given its perimeter.
  What about other regular polygons?

  Related objectives:
  Solve increasingly demanding problems; explore connections in mathematics across a range of contexts; generate fuller solutions.

  Link to using mathematical reasoning and applying Pythagoras' theorem (pages 184–9); using sine, cosine, tangent (pages 242–7).

Pupils should be taught to:

Understand and use decimal notation and place value; multiply and divide integers and decimals by powers of 10

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
place value, zero place holder, tenth, hundredth, thousandth…
equivalent, equivalence…

Understand and use decimal notation and place value. Read and write any number from 0.001 to 1 000000, knowing what each digit represents. For example, know that:

- In 5.239 the digit 9 represents nine thousandths, which is written as 0.009.
- The number 5.239 in words is 'five point two three nine', not 'five point two hundred and thirty-nine'.
- The fraction $5^{239}/_{1000}$ is read as 'five and two hundred and thirty-nine thousandths'.

Know the significance of 0 in 0.35, 3.05, 3.50, and so on.

Know that decimals used in context may be spoken in different ways. For example:
- 1.56 is spoken in mathematics as 'one point five six'.
- £1.56 is spoken as 'one pound fifty-six'.
- £1.06 is spoken as 'one pound and six pence'.
- £0.50 is spoken as 'fifty pence'.
- 1.56 km is sometimes spoken as 'one kilometre, five hundred and sixty metres'.
- 3.5 hours can be spoken as 'three and a half hours' or 'three hours and thirty minutes'.

Answer questions such as:
- Write in figures:
  four hundred and three thousand, and seventeen.
- Write in words: 4.236, 0.5, 35.08, …
- Write as a decimal the fraction
  six, and two hundred and forty-three thousandths.
- Make the largest and smallest number you can using:
  the digits 2, 0, 3, 4;
  the digits 2, 0, 3, 4, and a decimal point.

Add or subtract 0.1 and 0.01 to or from any number.

Count forwards or backwards from any number. For example:
- Count on in 0.1s from 4.5.
- Count back from 23.5 in 0.1s.
- Count on in 0.01s from 4.05.

Answer questions such as:

- What is 0.1 less than 2.0? What is 0.01 more than 2.09?

- What needs to be added or subtracted to change:
  27.48 to 37.48, 27.48 to 27.38, 27.48 to 26.38?
  5.032 to 5.037, 5.032 to 5.302?

See Y456 examples (pages 2–5, 28–9).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to: billion, power, index…

Read and write positive integer powers of 10.

Know that:

1 hundred is $10 \times 10 = 10^2$
1 thousand is $10 \times 10 \times 10 = 10^3$
10 thousand is $10 \times 10 \times 10 \times 10 = 10^4$, etc.
1 million is 10^6
1 billion is 10^9, one thousand millions
(In the past, 1 billion was 10^{12}, one million millions, in the UK.)

Recognise that successive powers of 10 (i.e. 10, 10^2, 10^3, …) underpin decimal (base 10) notation.

Read numbers in standard form, e.g. read 7.2×10^3 as 'seven point two times ten to the power three'.

Link to using index notation (pages 56–9).

Add or subtract 0.001 to or from any number.

Answer questions such as:

- What is 0.001 more than 3.009?
  What is 0.001 more than 3.299?
  What is 0.002 less than 5?
  What is 0.005 less than 10?

- What needs to be added or subtracted to change:
  4.257 to 4.277? 6.132 to 6.139?

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to: standard (index) form… exponent…

Extend knowledge of integer powers of 10.

Know that:

$10^0 = 1$ $10^{-1} = 1/10^1 = \frac{1}{10}$
$10^1 = 10$ $10^{-2} = 1/10^2 = \frac{1}{100}$

Know the prefixes associated with powers of 10. Relate to commonly used units. For example:

10^9	giga	10^{-2}	centi
10^6	mega	10^{-3}	milli
10^3	kilo	10^{-6}	micro
		10^{-9}	nano
		10^{-12}	pico

Know the term standard (index) form and read numbers such as 7.2×10^{-3}.

Link to using index notation (pages 56–9) and writing numbers in standard form (pages 38–9).

Know that commonly used units in science, other subjects and everyday life are:

kilogram (kg) – SI unit	**metre (m)** – SI unit	
gram (g)	kilometre (km)	litre (l)
milligram (mg)	millimetre (mm)	millilitre (ml)

Pupils should be taught to:

Understand and use decimal notation and place value; multiply and divide integers and decimals by powers of 10 (continued)

As outcomes, Year 7 pupils should, for example:

Multiply and divide numbers by 10, 100 and 1000.

Investigate, describe the effects of, and explain multiplying and dividing a number by 10, 100, 1000, e.g. using a place value board, **calculator** or **spreadsheet**.

In particular, recognise that:

- Multiplying a positive number by 10, 100, 1000… has the effect of increasing the value of that number.
- Dividing a positive number by 10, 100, 1000… has the effect of decreasing the value of that number.
- When a number is multiplied by 10, the digits move one place to the left:

```
    3   4   .   1   2       34.12 multiplied by 10  =  341.2
3   4   1   .   2
```

- When a number is divided by 10, the digits move one place to the right:

```
3   4   .   1           34.1 divided by 10  =  3.41
    3   .   4   1
```

Complete statements such as:

4 × 10 = □	4 × □ = 400
4 ÷ 10 = □	4 ÷ □ = 0.04
0.4 × 10 = □	0.4 × □ = 400
0.4 ÷ 10 = □	0.4 ÷ □ = 0.004
□ ÷ 100 = 0.04	□ ÷ 10 = 40
□ × 1000 = 40 000	□ × 10 = 400

See Y456 examples (pages 6–7).

Link to converting mm to cm and m, cm to m, m to km… (pages 228–9).

As outcomes, Year 8 pupils should, for example:

Multiply and divide numbers by 0.1 and 0.01.

Investigate, describe the effects of, and explain multiplying and dividing a number by 0.1 and 0.01, e.g. using a **calculator** or **spreadsheet**.

In particular, recognise how numbers are increased or decreased by these operations.

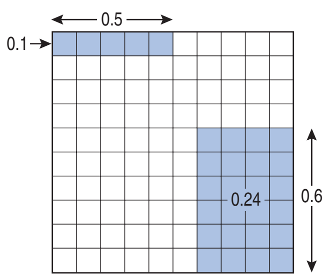
0.1 is equivalent to $\frac{1}{10}$ and 0.01 is equivalent to $\frac{1}{100}$, so:

- Multiplying by 0.1 has the same effect as multiplying by $\frac{1}{10}$ or dividing by 10. For example, 3×0.1 has the same value as $3 \times \frac{1}{10}$, which has the same value as $3 \div 10 = 0.3$, and 0.3×0.1 has the same value as $\frac{3}{10} \times \frac{1}{10} = \frac{3}{100} = 0.03$.
- Multiplying by 0.01 has the same effect as multiplying by $\frac{1}{100}$ or dividing by 100. For example, 3×0.01 has the same value as $3 \times \frac{1}{100}$, which has the same value as $3 \div 100 = 0.03$, and 0.3×0.01 has the same value as $\frac{3}{10} \times \frac{1}{100} = \frac{3}{1000} = 0.003$.
- Dividing by 0.1 has the same effect as dividing by $\frac{1}{10}$ or multiplying by 10. For example, $3 \div 0.1$ has the same value as $3 \div \frac{1}{10}$.
  (How many tenths in three? $3 \times 10 = 30$)
  $0.3 \div 0.1$ has the same value as $\frac{3}{10} \div \frac{1}{10}$.
  (How many tenths in three tenths? $0.3 \times 10 = 3$)
- Dividing by 0.01 has the same effect as dividing by $\frac{1}{100}$ or multiplying by 100. For example, $3 \div 0.01$ has the same value as $3 \div \frac{1}{100}$.
  (How many hundredths in three? $3 \times 100 = 300$)
  $0.3 \div 0.01$ has the same value as $\frac{3}{10} \div \frac{1}{100}$.
  (How many hundredths in three tenths? $0.3 \times 100 = 30$)

Complete statements such as:

$0.5 \times 0.1 = \square$	$0.8 \times \square = 0.08$
$0.7 \div 0.1 = \square$	$0.6 \div \square = 6$

Understand a diagrammatic explanation to show, for example, that $0.1 \times 0.5 = 0.05$, or $0.24 \div 0.6 = 0.4$.



Discuss the effects of multiplying and dividing by a number less than 1.

- Does division always make a number smaller?
- Does multiplication always make a number larger?

As outcomes, Year 9 pupils should, for example:

Multiply and divide by any integer power of 10.

For example:

- Calculate:

7.34×100	$37.4 \div 100$
46×1000	$3.7 \div 1000$
$8042 \times 10\,000$	$4982 \div 10000$
9.3×0.1	$0.27 \div 0.1$
0.63×0.01	$5.96 \div 0.01$

Link to converting mm^2 to cm^2, cm^2 to m^2, mm^3 to cm^3 and cm^3 to m^3 (pages 228–9).

Begin to write numbers in standard form, expressing them as

$A \times 10^n$ where $1 \le A < 10$, and n is an integer.

For example:

$734.6 = 7.346 \times 10^2$
$0.0063 = 6.3 \times 10^{-3}$

Know how to use the 'EXP' key on a **calculator** to convert from index form.

Answer questions such as:

- Complete these. The first is done for you.
  $3 \times 10^n = 300 \times 10^{n-2}$
  $0.3 \times 10^n = 30\,000 \times \square$
  $0.3 \times 10^n = 0.0003 \times \square$
  $3 \div 10^n = 0.003 \times \square$
  $0.3 \div 10^n = 300 \times \square$
  $0.003 \div 10^n = 3 \times \square$
- Put these numbers in ascending order:
  2×10^{-2}, 3×10^{-1}, 2.5×10^{-3}, 2.9×10^{-2}, 3.2×10^{-1}
- Write these numbers in standard form:
  a. The population of the UK is 57 million.
  b. The dwarf pigmy goby fish weighs 0.000 14oz.
  c. The shortest millipede in the world measures 0.082 inches.
  d. After the Sun, the nearest star is 24800000000000 miles away.
- The probability of dying before the age of 40 is 1 in 850, or 0.00118, or 1.8×10^{-3}.

  These are the risks of dying from particular causes:

smoking 10 cigarettes a day	1 in 200
road accident	1 in 8000
accident at home	1 in 260 000
railway accident	1 in 500000

  Write each of these as a probability in standard form.

Link to writing numbers in standard form in science and geography.

Pupils should be taught to:

Compare and order decimals

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
decimal number, decimal fraction, less than, greater than, between, order, compare, digit, most/least significant digit…
and use accurately these symbols: =, ≠, <, >, ≤, ≥.

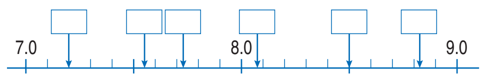
Know that to **order decimals**, digits in the same position must be compared, working from the left, beginning with the first non-zero digit. In these examples, the order is determined by:

- 0.325 < 0.345 — the second decimal place;
- 3.18 km > 3.172km — the second decimal place;
- 0.42 < 0.54 — the first decimal place;
- 5.4 < 5.6 < 5.65 — the first decimal place initially, then the second decimal place.

Know that when comparing measures it is necessary to convert all measures into the same units. For example:

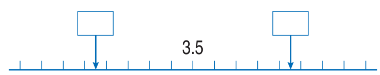
- Order these measurements, starting with the smallest:
  5 kg, 500g, 0.55kg
  45cm, 1.23m, 0.96m
  £3.67, £3.71, 39p

Identify and estimate decimal fractions on a number line and find a number between two others by looking at the next decimal place. For example:



- Find the number that is half way between:
  3 and 4, 0.3 and 0.4, $^{-}3$ and 4, $^{-}4$ and 3.

- 3.5 lies half way between two other numbers.
  What could they be?



- Use a **computer simulation** to zoom in and out of a number line to compare and order decimals to at least 2 d.p.

- Use a **graphical calculator** to generate ten random numbers lying between 0 and 1, with a maximum of 2 d.p.
  Arrange the numbers in order. For example, enter:

  [Int] [(] [Ran#] [×] [1] [0] [0] [)] [÷] [1] [0] [0]

  then keep pressing the [EXE] button.

Use accurately the symbols <, >, ≤, ≥. For example:

- Place > or < between these:
  12.45 ☐ 12.54 — $^{-}6$ °C ☐ $^{-}7$°C
  3.424 ☐ 3.42 — 6.75 litres ☐ 675 millilitres

- Given that 31.6 ≤ x ≤ 31.8, give possible values for x:
  a. if x has 1 d.p. — b. if x has two decimal places

…

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
ascending, descending…

Know that to **order decimals**, digits in the same position must be compared, working from the left, beginning with the first non-zero digit.
In these examples, the order is determined by:

- 0.024 37 < 0.02452 — the fourth decimal place;
- 3.1895 > 3.1825 — the third decimal place;
- 23.451 < 23.54 — the first decimal place;
- 5.465 < 5.614 < 5.65 — the first decimal place initially, then the second.

Extend to negative numbers, e.g. $^{-}0.0237 > {}^{-}0.0241$.

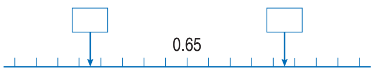
Know how to order data collected by measuring, and group the data in intervals. For example:

- When collecting data on pupils' heights, construct a frequency table using groups such as:
  $1.50 \le h < 1.55$ $\quad$ $1.55 \le h < 1.60$, etc.

Identify decimal fractions on a number line and find a number between two others by looking at the next decimal place. For example:

- Find the number that is half way between:
  0.03 and 0.04 $\quad$ $^{-}0.3$ and 0.4.

- 0.65 lies half way between two other numbers.
  What could they be?



- Use a **computer simulation** to zoom in and out of a number line to compare decimals to at least 3 d.p.

Link to graphs (pages 164–77), e.g. using a graphical calculator to zoom in on a graph as the axes change.

Use accurately the symbols <, >, ≤, ≥. For example:

- Place > or < between:
  0.503 ☐ 0.53 $\quad$ 3.2 metres ☐ 320 millimetres

- Given that $31.62 \le z \le 31.83$, discuss possible values for z. Understand that there is an infinite number of possible solutions.

As outcomes, Year 9 pupils should, for example:

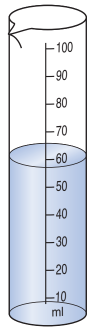
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Round numbers, including to a given number of decimal places

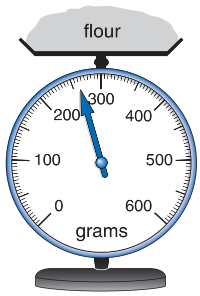
Use, read and write, spelling correctly:
round, nearest, to one decimal place (1 d.p.)… approximately…

Round positive whole numbers to the nearest 10, 100 or 1000, in mathematics and subjects such as science, design and technology, geography… For example:

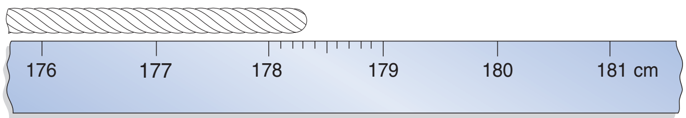
- What is the volume of the liquid in the measuring cylinder to the nearest 10 ml?



- What is the mass of the flour to the nearest 100 g?
  Estimate the mass of the flour to the nearest 10g.



- How long is the rope to:
  a. the nearest 10 cm?
  b. the nearest 100cm?
  c. the nearest cm?
  d. the nearest mm?



- How many people visited the Dome to the nearest 100?
  Was the headline correct?

The Daily Record 10 Feb

Nearly 20 thousand people visit Dome

15,437 people visited the Dome yesterday.

In other subjects, round whole numbers to the nearest 10, 100 or 1000 in order to classify them or put them in order. For example:

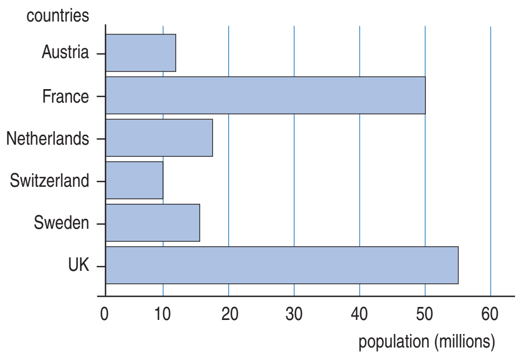
- In geography, round and then place in order:
  populations of towns, heights of mountains, weather data…
- In science, round and then place in order:
  the proportion of lead in the air at different places, the diameters of the planets…
- In design and technology, round and then place in order:
  the grams of fat in different foods…

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
recurring decimal…

Round positive whole numbers to a given power of 10, in mathematics and in other subjects.
For example:

- This chart shows the estimated population of six countries. Write in figures the approximate population of each country.



- On the chart above, Sweden is recorded as having an estimated population of 15 million. What is the highest/lowest population that it could actually have?

- There are 1 264317 people out of work.
  Politician A says: 'We have just over 1 million people out of work.'
  Politician B says: 'We have nearly one and a half million people out of work.'
  Who is more accurate, and why?

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
significant figures, upper and lower bounds…
Read and write the 'approximately equal to' sign (≈).

Use rounding to make estimates.

For example:

- The population of the world is about 5300 million.

  The approximate populations of the four largest cities are:

Mexico City	21.5 million
Sao Paulo	19.9 million
Tokyo	19.5 million
New York	15.7 million

  The tenth largest city is Rio de Janeiro with a population of 11.9 million.

  Estimate the percentage of the world's population which lives in the ten largest cities.

- A heavy metal in water kills fish when it reaches levels of more than 4 parts per million.
  A lake contains 4.7 megalitres of water.
  How much heavy metal can be in the water for the fish to be safe, if 1 litre of the heavy metal has a mass of 2.4 kg?

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Round numbers, including to a given number of decimal places (continued)

Round positive whole numbers and decimals.
Know that if a measurement is half way between two numbers it is normally rounded up to the next number. Recognise that in some practical situations, such as a division problem, this may not be appropriate. For example:

- 124 children want to go on a school trip. If each coach holds 49 people, how many coaches are needed?
- I have 52 drawing pins. If each poster for my bedroom needs 6 pins, how many posters can I put up?
- A pupil in technology needs to cut a 1 metre length of wood into three pieces. How long should each piece be?

Link to understanding division (pages 82–5).

Round decimals to the nearest whole number or to one decimal place.

When rounding a decimal to a whole number, know that:
- if there are 5 or more tenths, then the number is rounded up to the next whole number; otherwise, the whole number is left unchanged;
- decimals with more than one decimal place are not first rounded to one decimal place, e.g. 7.48 rounds to 7, not to 7.5 which then rounds to 8.

When rounding a decimal such as 3.96 to one decimal place, know that the answer is 4.0, not 4, because the zero in the first decimal place is significant.

For example:
- 4.48 rounded to the nearest whole number is 4.
- 4.58 rounded to the nearest whole number is 5, and rounded to one decimal place is 4.6.
- 4.97 rounded to the nearest whole number is 5.
- 4.97 rounded to one decimal place is 5.0.

Answer questions such as:

- Round 5.28:
  a. to the nearest whole number;
  b. to one decimal place.

- Here are the winning heights and distances for some women's field events in an international competition. Round each height or distance:
  a. to the nearest whole number;
  b. to one decimal place.

Women's events	
High jump	2.09 metres
Long jump	7.48 metres
Shot-put	21.95 metres
Discus throw	76.80 metres
Javelin throw	80.00 metres

See Y456 examples (pages 12–13, 30–1, 56–7).

As outcomes, Year 8 pupils should, for example:

Recognise recurring decimals.

Recurring decimals contain an infinitely repeating block of one or more decimal digits.
For example:

- $\frac{1}{6}$ = 0.16666… is written as $0.1\dot{6}$
- $\frac{2}{11}$ = 0.181818… is written as $0.\dot{1}\dot{8}$

Fractions with denominators containing prime factors other than 2 or 5 will recur if written in decimal form.

Round decimals to the nearest whole number or to one or two decimal places.

For example, know that:

- 3.7452 rounded to the nearest whole number is 4, to one decimal place is 3.7, and to two decimal places is 3.75.
- 2.199 rounded to the nearest whole number is 2, to one decimal place is 2.2, and to two decimal places is 2.20.
- 6.998 rounded to two decimal places is 7.00.

When substituting numbers into expressions and formulae, know that rounding should not be done until the final answer has been computed.

Answer questions such as:

- Round 12.3599 to one decimal place
- Use a **calculator** to do these calculations.
  Write the answers to two decimal places.
  2 ÷ 3   3 ÷ 16   11 ÷ 9   9 ÷ 11   14 ÷ 17

Round decimals in context, selecting an appropriate number of decimal places to use when, for example:

- using decimal measurements for work on perimeter, area and volume;
- collecting measurements to use as data for statistics;
- calculating summary statistics, such as the mean;
- investigating recurring decimals;
- dividing;
- carrying out science experiments;
- measuring in design and technology or geography…

As outcomes, Year 9 pupils should, for example:

Round decimals to the nearest whole number or to one, two and three decimal places.

For example, know that:

- 3.0599 rounded to the nearest whole number is 3, rounded to 1 d.p. is 3.1, to 2 d.p. is 3.06, and to 3 d.p. is 3.060.
- 9.953 rounded to the nearest whole number is 10, to 1 d.p. is 10.0, and to 2 d.p. is 9.95.
- $\frac{22}{7}$ is an approximation to π and can be given as 3.14 to 2 d.p. or 3.143 correct to 3 d.p.

Know that rounding should not be done until a final result has been computed.

Answer questions such as:

- Use a **calculator** to evaluate $\frac{1}{850}$ correct to one decimal place.

Round decimals in context. Select an appropriate number of decimal places to use, knowing at which stage to round when, for example:

- approximating π in circle measurements and calculations;
- making measurements in mathematics and other subjects;
- when presenting results of calculations in geometrical and statistical contexts;
- when substituting decimals into expressions and formulae.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Round numbers, including to a given number of decimal places or significant figures (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Understand upper and lower bounds. For example:

- For **discrete data** such as:

  > The population p of Sweden to the nearest million is 15 million.

  know that the least population could be 14 500000 and the greatest population could be 15499999; understand that this can be written as:

  $$14500000 \leq p < 15500000$$

- For **continuous data** such as measurements of distance:

  > The distance d km from Exeter to Plymouth is 62 km to the nearest km.

  know that the shortest possible distance is 61.5km and the longest possible distance is 62.5km, which can be written as:

  $$61.5 \leq d < 62.5$$

Round numbers to a given number of significant figures. Know, for example, that:

- 5.78 is 5.8 to two significant figures (2 s.f.).
- 34.743 is 35 to 2 s.f. and 34.7 to 3 s.f.
- 5646 is 6000 to 1 s.f., 5600 to 2 s.f. and 5650 to 3 s.f.
- 0.004 36 is 0.004 to 1 s.f. and 0.0044 to 2 s.f.

Know when to insert zeros as place holders to indicate the degree of significance of the number. For example, 1.4007 is 1.40 to 3 s.f.

Use numbers to a given number of significant figures to work out an approximate answer. For example:

- The area of a circle with radius 7 cm is approximately 3×50 cm^2.
  Compare this answer with the approximations $^{22}/_7 \times 7 \times 7$ cm^2 and $3.14 \times 7 \times 7$ cm^2,
  and with $\pi \times 7 \times 7$ cm^2 calculated using the π key on a calculator.

Give answers to calculations to an appropriate number of significant figures. For example:

- $\dfrac{65 + 78}{41 \times 56} \approx 0.0623$ to 3 s.f.
- $5.84 + \dfrac{3.26 + 4.17}{1.23} \approx 12$ to 2 s.f.

Pupils should be taught to:

Order, add, subtract, multiply and divide positive and negative numbers

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
integer, positive, negative, plus, minus…
and know that $^{-}6$ is read as 'negative six'.

Order integers and position them on a number line. For example:

- Put a > or < sign between these pairs of temperatures:
  $^{-}6$ °C ☐ 4°C   6°C ☐ $^{-}4$°C $^{-}6$°C ☐ $^{-}4$°C $^{-}4$°C ☐ $^{-}6$°C
- On a number line, mark numbers half way between two given negative numbers, or between a given positive number and a given negative number.
- Use a **graphical calculator** to generate ten random numbers lying between $^{-}20$ and $^{+}20$, then arrange them in order. For example, enter:

  [Int] [(] [Ran#] [×] [4] [0] [)] [−] [2] [0]

  then keep pressing the [EXE] button.

Link to plotting coordinates in all four quadrants (pages 218–19).

Begin to add and subtract integers.

Extend patterns such as:

$2 + 1 = 3$	$^{-}3 - 1 = {}^{-}4$
$2 + 0 = 2$	$^{-}3 - 0 = {}^{-}3$
$2 + {}^{-}1 = 1$	$^{-}3 - {}^{-}1 = {}^{-}2$
$2 + {}^{-}2 = 0$	$^{-}3 - {}^{-}2 = {}^{-}1$
$2 + {}^{-}3 = {}^{-}1$	$^{-}3 - {}^{-}3 = 0$

Use negative number cards to help answer questions such as:

$^{-}3 + {}^{-}5 =$ ☐	$^{-}13 + {}^{-}25 =$ ☐
$^{-}146 + {}^{-}659 =$ ☐	$^{-}99 + {}^{-}99 =$ ☐
$^{-}9 - {}^{-}4 =$ ☐	$^{-}43 - {}^{-}21 =$ ☐
$^{-}537 - {}^{-}125 =$ ☐	$^{-}99 - {}^{-}99 =$ ☐

Answer open-ended questions such as:

- The answer to a question was $^{-}8$.
  What was the question?
- The result of subtracting one integer from another is $^{-}2$.
  What could the two integers be?
- The temperature is below freezing point.
  It falls by 10 degrees, then rises by 7 degrees.
  What could the temperature be now?

Solve simple puzzles or problems involving addition and subtraction of positive and negative numbers, such as:

- Complete this magic square.

$^{-}5$	2	$^{-}6$
	$^{-}8$	$^{-}1$

Link to substituting positive and negative numbers in expressions and formulae (pages 138–41).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

Order positive and negative decimals.

See ordering decimals (pages 40–1).

Add and subtract integers.

Understand that 1 add $^{-}1$ is zero, and use this to calculate, for example:

$1 + 1 + {^{-}1} + 1 + {^{-}1} = \square$ $\quad$ ${^{-}1} + 1 = \square$

$3 + {^{-}3} = \square$ $\quad$ ${^{-}29} + 29 = \square$
$827 + {^{-}827} = \square$ $\quad$ ${^{-}3} + \square = 0$

$62 + {^{-}51} = \square$ $\quad$ ${^{-}87} + 90 = \square$
$40 + {^{-}30} + 20 + {^{-}10} = \square$ $\quad$ ${^{-}3} + \square = 7$

Recognise that:

- $0 - {^{-}1}$ has the same value as $(1 + {^{-}1}) - {^{-}1} = 1$;
- $0 - 1$ has the same value as $(1 + {^{-}1}) - 1 = {^{-}1}$.

Use this to calculate, for example:

$5 - {^{-}3} = \square$ $\quad$ ${^{-}4} - {^{-}5} = \square$ $\quad$ ${^{-}2} - \square = 7$

Use the sign change key on a **calculator** to work out:

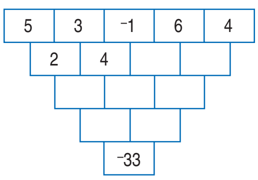
$134 - {^{-}965}$ $\quad$ ${^{-}437} + 356$ $\quad$ $538 - {^{-}702}$
$468 + {^{-}727}$ $\quad$ ${^{-}815} + {^{-}768}$ $\quad$ ${^{-}646} - {^{-}573}$

Solve puzzles such as:

- Complete this table.

a	19	6	7	3	$^{-}4$		$^{-}8$
b	12	14	$^{-}4$		$^{-}6$	$^{-}5$	
$a - b$				8		2	0

- In this 'wall', subtract the right-hand from the left-hand number to find the number directly below. Fill in the empty boxes.



Link to substituting positive and negative numbers in expressions and formulae (pages 138–41).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Order, add, subtract, multiply and divide positive and negative numbers (continued)

Use positive and negative numbers in context.

For example, find:

- the final position of an object after moves forwards and backwards along a line;
- a total bank balance after money is paid in and taken out;
- the total marks in a test of 10 questions, with $^{+}2$ marks for a correct answer and $^{-}1$ mark for an incorrect answer;
- the total of scores above and below par in a round of golf;
- the mean of a set of temperatures above and below zero…

Know how to, for example:

- find the distance between two floors using a lift, including above and below ground level;
- calculate game scores which include positive and negative points;
- identify measurements above and below sea-level, using contour lines on maps;
- interpret world weather charts to find differences in temperatures around the globe;
- identify the level of accuracy in measurements, e.g. 20 cm ± 0.5cm…

Link to work in other subjects.

As outcomes, Year 8 pupils should, for example:

Multiply and divide positive and negative numbers.

Link known multiplication tables to negative number multiplication tables. For example:

- $^{-}2 \times 1 = {}^{-}2$, $^{-}2 \times 2 = {}^{-}4$, $^{-}2 \times 3 = {}^{-}6$
  and so on …

- Write tables, continuing the pattern:

$2 \times 2 = 4$	$2 \times {}^{-}2 = {}^{-}4$
$1 \times 2 = 2$	$1 \times {}^{-}2 = {}^{-}2$
$0 \times 2 = 0$	$0 \times {}^{-}2 = 0$
$^{-}1 \times 2 = {}^{-}2$	$^{-}1 \times {}^{-}2 = 2$
$^{-}2 \times 2 = {}^{-}4$	$^{-}2 \times {}^{-}2 = 4$
$^{-}3 \times 2 = {}^{-}6$	$^{-}3 \times {}^{-}2 = 6$

Complete a multiplication table. Shade positive and negative numbers, and zero, using different colours.

Look for patterns.

×	⁻3	⁻2	⁻1	0	1	2	3
3	⁻9	⁻6	⁻3	0	3	6	9
2	⁻6	⁻4	⁻2	0	2	4	6
1	⁻3	⁻2	⁻1	0	1	2	3
0	0	0	0	0	0	0	0
⁻1	3	2	1	0	⁻1	⁻2	⁻3
⁻2	6	4	2	0	⁻2	⁻4	⁻6
⁻3	9	6	3	0	⁻3	⁻6	⁻9

Recognise that division by a negative number is the inverse of multiplication by a negative number. Use this, and the negative number multiplication tables, to show, for example, that $^{-}4 \div {}^{-}2 = 2$, and relate this to the question 'How many $^{-}2$s in $^{-}4$?'

For a fact such as $^{-}3 \times 2 = {}^{-}6$, write three other facts, i.e. $2 \times {}^{-}3 = {}^{-}6$, $^{-}6 \div 2 = {}^{-}3$, $^{-}6 \div 3 = {}^{-}2$.

Answer questions such as:

- How many negative twos make negative four? (Two.)
- The answer to a question was $^{-}24$. What was the question?

Use the sign change key on a **calculator** to work out:

$48 \times {}^{-}53$	$^{-}74 \times 3$	$9.02 \div {}^{-}22$
$68 \times {}^{-}49$	$^{-}8 \times {}^{-}73.7$	$^{-}6450 \div {}^{-}15$

Solve puzzles such as:

- Complete this multiplication grid. Find two ways to do it.

×		4	⁻9	
		⁻8	18	
⁻3		⁻12		
	35			⁻14
				12

Extend to the distributive law. For example:

$$^{-}1 \times (3 + 4) = {}^{-}1 \times 7 = {}^{-}7$$
$$^{-}1 \times (3 + 4) = ({}^{-}1 \times 3) + ({}^{-}1 \times 4) = {}^{-}3 + {}^{-}4 = {}^{-}7$$

Link to substituting positive and negative numbers in expressions and formulae (page 138–41).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Recognise and use multiples, factors and primes; use tests of divisibility

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
multiple, lowest common multiple (LCM), factor, common factor, highest common factor (HCF), divisor, divisible, divisibility, prime, prime factor, factorise…

Know that a prime number has two and only two distinct factors (and hence that 1 is not a prime number).

Know the prime numbers up to 30 and test whether two-digit numbers are prime by using simple tests of divisibility, such as:

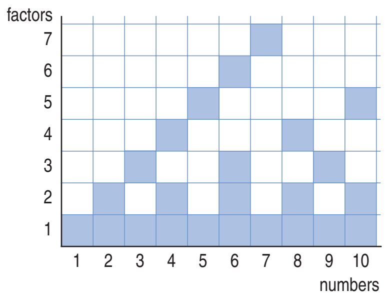
- 2 the last digit is 0, 2, 4, 6 or 8;
- 3 the sum of the digits is divisible by 3;
- 4 the last two digits are divisible by 4;
- 5 the last digit is 0 or 5;
- 6 it is divisible by both 2 and 3;
- 8 half of it is divisible by 4;
- 9 the sum of the digits is divisible by 9.

Explore links between factors, primes and multiples.
For example:

- Find the primes to 100 by using the sieve of Eratosthenes. On a hundred square, colour in 1, then the multiples of 2 that are greater than 2 in one colour, the multiples of 3 that are greater than 3 in another colour… so that the remaining uncoloured numbers are the primes.

Find the factors of a number.

- Make a 'factor finder'.



- Find the factors of a number by checking for divisibility by primes. For example, to find the factors of 123, check mentally or otherwise if the number divides by 2, then 3, 5, 7, 11…

- Find all the pairs of factors of non-prime numbers. For example:
  the pairs of factors of 51 are 1 × 51 and 3 × 17;
  the pairs of factors of 56 are 1 × 56, 2 × 28, 4 × 14, 7 × 8.

Use factors when appropriate to calculate mentally, as in:

$$35 \times 12 = 35 \times 2 \times 6 = 70 \times 6 = 420$$

$$\frac{144}{36} = \frac{12 \times 12}{3 \times 12} = \frac{12}{3} = 4$$

Link to cancelling fractions (pages 60–3).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
prime factor decomposition…

Apply tests of divisibility for 12, 15, 18… by applying two simpler tests. For example, for:

- 15 the number is divisible by 3 and divisible by 5;
- 18 the number is even and divisible by 9.

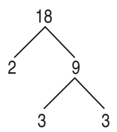
Use a **calculator** to explore divisibility. For example:

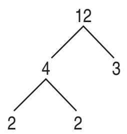
- Is 2003 a prime number?

```
2003/7
                286.1428571
2003/11
                182.0909091
2003/13
                154.0769231
```

Find the prime factor decomposition of a number.

Use factor trees to find prime factors and write non-prime numbers as the products of prime factors. For example, $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$.





$18 = 2 \times 3 \times 3 = 2 \times 3^2$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

Divide by prime numbers, in ascending order, to find all the prime factors of a non-prime number. Write the number as a product of prime factors.

2	24
2	12
2	6
3	3
	1

2	180
2	90
3	45
3	15
5	5
	1

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$

Use factors when appropriate to calculate, as in:

$64 \times 75 = 64 \times 25 \times 3 = 1600 \times 3 = 4800$

$\sqrt{576} = \sqrt{(3 \times 3 \times 8 \times 8)} = 3 \times 8 = 24$

Link to cancelling fractions (pages 60–3).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Recognise and use multiples, factors and primes; use tests of divisibility (continued)

Find the lowest common multiple (LCM) of two numbers, such as: 6 and 8; 25 and 30.

6 times table: 6 12 18 24 30…

8 times table: 8 16 24 32…

The lowest common multiple of 6 and 8 is 24.

Find the highest common factor (HCF) of two numbers, such as: 18 and 24; 40 and 65.

The factors of 18 are 1 2 3 6 9 18

The factors of 24 are 1 2 3 4 6 8 12 24

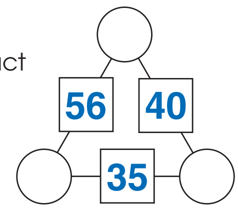
1, 2, 3 and 6 are common factors of 18 and 24, so 6 is the highest common factor of 18 and 24.

Link to cancelling fractions (pages 60–3).

See Y456 examples (pages 18–21).

Investigate problems such as:

- Which numbers less than 100 have exactly three factors?
- What number up to 100 has the most factors?
- Find some prime numbers which, when their digits are reversed, are also prime.
- There are 10 two-digit prime numbers that can be written as the sum of two square numbers. What are they?
- Write a number in each circle so that the number in each square is the product of the two numbers on either side of it.

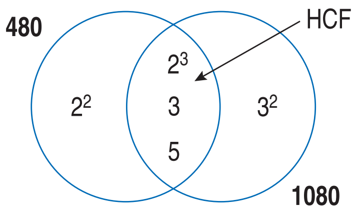


As outcomes, Year 8 pupils should, for example:

Use prime factors to find the highest common factor and lowest common multiple of a set of numbers.
For example:

Find the HCF and LCM of 480 and 1080.

480	$= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$
	$= 2^5 \times 3 \times 5$
1080	$= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5$
	$= 2^3 \times 3^3 \times 5$
HCF	$= 2^3 \times 3 \times 5$
LCM	$= 2^5 \times 3^3 \times 5$



Link to cancelling fractions (pages 60–3), and adding and subtracting fractions (pages 66–7).

Investigate problems such as:

- Show that 60 has exactly 12 factors.
  Find three more numbers less than 100 with exactly 12 factors.

- The sum of the digits of 715 is 13, and 13 is a factor of 715. What other three-digit numbers have this property?

As outcomes, Year 9 pupils should, for example:

Use prime factor decomposition to find the lowest common multiple of denominators of fractions in order to add or subtract them efficiently.
For example:

- $\frac{31}{56} + \frac{29}{70} = \frac{155 + 116}{280}$
  because $56 = 2^3 \times 7$ and $70 = 2 \times 5 \times 7$ so
  LCM $= 2^3 \times 5 \times 7 = 280$.

- $\frac{17}{28} - \frac{12}{38} = \frac{323 - 168}{532}$
  because $28 = 2^2 \times 7$ and $38 = 2 \times 19$ so
  LCM $= 2^2 \times 7 \times 19 = 532$.

Link to adding and subtracting fractions (pages 66–7).

Use prime factor decomposition to find the highest common factor in order to cancel fractions.

Link to cancelling fractions (pages 60–3), and multiplying and dividing fractions (pages 68–9).

Investigate problems such as:

- Take any two-digit number. Reverse the digits. Subtract the smaller number from the larger. Prove that the difference is always divisible by 9.
  Let the number be $10t + u$.
  Reversing the digits gives $10u + t$.
  The difference is
  $10t + u - 10u - t = 9t - 9u = 9(t - u)$
  showing that 9 is always a factor.

- Prove that a two-digit number in which the tens digit equals the units digit is always divisible by 11.
  The number is of the form $10t + t = 11t$.

- Prove that a three-digit number in which the sum of the hundreds digit and the units digit equals the tens digit is always divisible by 11.
  The number is of the form
  $100h + 10t + (t - h) = 99h + 11t = 11(9h + t)$

Find the common factors of algebraic expressions.
For example:

- $2x^2yz$ and $3wxy$ have a common factor xy.

- Find the HCF and LCM of a^5b^4 and a^4b^4.
  HCF $= a^4b^4$ LCM $= a^5b^4$

- $(x - 1)(2x + 3)^3$ and $(x - 1)^2(2x - 3)$ have the common factor $(x - 1)$.

Link to finding common factors in algebra (pages 116–17).

Pupils should be taught to:

Recognise squares and cubes, and the corresponding roots; use index notation and simple instances of the index laws

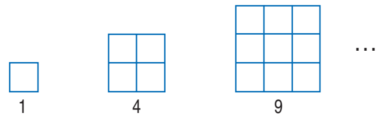
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
property, consecutive, classify…
square number, squared, square root… triangular number…
the notation 6^2 as six squared and the square root sign √.

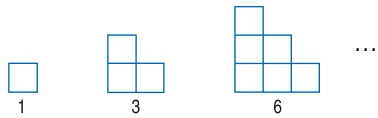
Use index notation to write squares such as 2^2, 3^2, 4^2, …

Recognise:

- squares of numbers 1 to 12 and the corresponding roots;

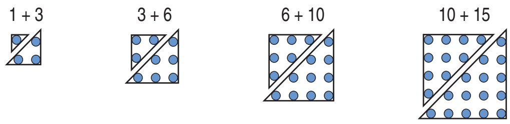


- triangular numbers: 1, 3, 6, 10, 15, …



Work out the values of squares such as 15^2, 21^2.

Investigate the relationship between square numbers and triangular numbers, using interlocking cubes or pegboard.



Link to generating sequences from practical contexts (pages 146–7).

Square roots

Find square roots of multiples of 100 and 10 000 by factorising. For example, find:

- $\sqrt{900} = \sqrt{(9 \times 100)}$
  $= \sqrt{9} \times \sqrt{100} = 3 \times 10 = 30$

- $\sqrt{160\,000} = \sqrt{(16 \times 100 \times 100)}$
  $= \sqrt{16} \times \sqrt{100} \times \sqrt{100} = 4 \times 10 \times 10 = 400$

Use a **calculator**, including the square root key, to find square roots, rounding as appropriate.

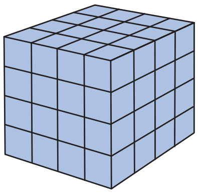
Recognise that squaring and finding the square root are the inverse of each other.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
cube number, cubed, cube root… power…
the notation 6^3 as six cubed, and 6^4 as six to the power 4… and the cube root sign $\sqrt[3]{}$.

Use index notation to write cubes and small positive integer powers of 10.

Know cubes of 1, 2, 3, 4, 5 and 10 and the corresponding roots.



$4^3 = 64$

Work out the values of cubes, such as 6^3, $^{-}9^3$, $(0.1)^3$.

Know that $100 = 10 \times 10 = 10^2$, and that successive powers of 10 (10, 10^2, 10^3, …) underpin decimal (base 10) notation. For example:

1 thousand is 10^3;
10 thousand is 10^4;
1 million is 10^6;
1 billion is 10^9 (one thousand millions).

Link to place value (pages 36–7), prime factor decomposition of a number and tree diagrams (pages 52–3), and generating sequences from practical contexts (pages 146–7).

Squares, cubes and square roots

Know that a positive integer has two square roots, one positive and one negative; by convention the square root sign √ denotes the positive square root.

Find square roots by factorising, for example:
$\sqrt{196} = \sqrt{(4 \times 49)} = 2 \times 7 = 14$

Find an upper and lower bound for a square root by comparing with the roots of two consecutive square numbers:
$\sqrt{4} < \sqrt{7} < \sqrt{9}$ so $2 < \sqrt{7} < 3$

Use a **calculator** to find cubes, squares and estimate square roots, including using the square root key. For example:

- Find the square root of 12.

  3^2 = 9 (3 to the power 2)
  4^2 = 16
  so the square root of 12 lies between 3 and 4.

```
3^2
4^2                  9
3.5^2               16
3.4^2            12.25
```

  Try 3.5, and so on.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
index, indices, index notation, index law…

Use index notation for small integer powers.
For example:

- $19^3 = 6859$ $6^5 = 7776$ $14^4 = 38\,416$

Know that $x^0 = 1$, for all values of x.

Know that:
$10^{-1} = 1/10 = 0.1$ $10^{-2} = 1/100 = 0.01$

Know how to use the x^y key on a **calculator** to calculate powers.

Recognise applications of indices in biology, where cells and organisms grow by doubling, giving rise to the powers of 2.

Link to writing numbers in standard form (pages 38–9).

Square roots and cube roots

Know that:
- $\sqrt{a} + \sqrt{b} \neq \sqrt{(a + b)}$

Know that:
- there are two square roots of a positive integer, one positive and one negative, written as ±√;
- the cube root of a positive number is positive and the cube root of a negative number is negative.

Use **ICT** to estimate square roots or cube roots to the required number of decimal places. For example:

- Estimate the solution of $x^2 = 70$.

  The positive value of x lies between 8 and 9, since $8^2 = 64$ and $9^2 = 81$.
  Try numbers from 8.1 to 8.9 to find a first approximation lying between 8.3 and 8.4.
  Next try numbers from 8.30 to 8.40.

Link to using trial and improvement and ICT to find approximate solutions to equations (pages 132–5).

Pupils should be taught to:

Recognise squares and cubes, and the corresponding roots; use index notation and simple instances of the index laws (continued)

As outcomes, Year 7 pupils should, for example:

Investigate problems such as:

- Without using a calculator, find a number that when multiplied by itself gives 2304.
- Describe the pattern formed by the last digits of square numbers. Do any numbers not appear as the last digits? Could 413 be a square number? Or 517?
- Can every square number up to 12 × 12 be expressed as the sum of two prime numbers?
- Some triangular numbers are equal to the sum of two other triangular numbers. Find some examples.

See Y456 examples (pages 20–1).

As outcomes, Year 8 pupils should, for example:

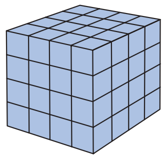
Investigate problems such as:

- Using a **calculator**, find two consecutive numbers with a product of 7482.
- Some numbers are equal to the sum of two squares: for example, $34 = 3^2 + 5^2$.
  Which numbers less than 100 are equal to the sum of two squares?
  Which can be expressed as the sum of two squares in at least two different ways?
- What are the 20 whole numbers up to 30 that can be written as the difference of two squares?
- Find the smallest number that can be expressed as the sum of two cubes in two different ways.
- What are the three smallest numbers that are both triangular and square?

As outcomes, Year 9 pupils should, for example:

Investigate problems such as:

- Estimate the cube root of 20.
- The outside of a cube made from smaller cubes is painted blue. How many small cubes have 0, 1, 2 or 3 faces painted blue? Investigate.



- Three integers, each less than 100, fit the equation $a^2 + b^2 = c^2$.
  What could the integers be?

Link to Pythagoras' theorem (pages 186–9); graphs of quadratic and cubic functions (pages 170–1).

Use simple instances of the index laws and start to multiply and divide numbers in index form.

Recognise that:

- indices are added when multiplying, e.g.
  $4^3 \times 4^2 = (4 \times 4 \times 4) \times (4 \times 4)$
  $= 4 \times 4 \times 4 \times 4 \times 4$
  $= 4^5 = 4^{(3+2)}$
- indices are subtracted when dividing, e.g.
  $4^5 \div 4^2 = (4 \times 4 \times 4 \times 4 \times 4) \div (4 \times 4)$
  $= 4 \times 4 \times 4$
  $= 4^3 = 4^{(5-2)}$
- $4^2 \div 4^5 = 4^{(2-5)} = 4^{-3}$
- $7^5 \div 7^5 = 7^0 = 1$

Generalise to algebra. Apply simple instances of the index laws (small integral powers), as in:

- $n^2 \times n^3 = n^{2+3} = n^5$
- $p^3 \div p^2 = p^{3-2} = p$

Know and use the general forms of the index laws for multiplication and division of integer powers.

$p^a \times p^b = p^{a+b}$, $p^a \div p^b = p^{a-b}$, $(p^a)^b = p^{ab}$

Begin to extend understanding of index notation to negative and fractional powers; recognise that the index laws can be applied to these as well.

2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3	2^4
$\frac{1}{2^4} = \frac{1}{16}$	$\frac{1}{2^3} = \frac{1}{8}$	$\frac{1}{2^2} = \frac{1}{4}$	$\frac{1}{2^1} = \frac{1}{2}$	1	2	4	8	16

Know the notation $5^{1/2} = \sqrt{5}$ and $5^{1/3} = \sqrt[3]{5}$.

Extend to simple surds (unresolved roots):

- $\sqrt{3} \times \sqrt{3} = 3$
- $\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}$
- $\sqrt{32} = \sqrt{(2 \times 16)} = \sqrt{2} \times \sqrt{16} = 4\sqrt{2}$

- Can a square have an exact area of 32 cm²?
  If so, what is its exact perimeter?

Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals

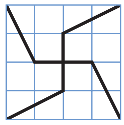
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
numerator, denominator, mixed number, proper fraction, improper fraction… decimal fraction, percentage… equivalent, cancel, simplify, convert…
lowest terms, simplest form…

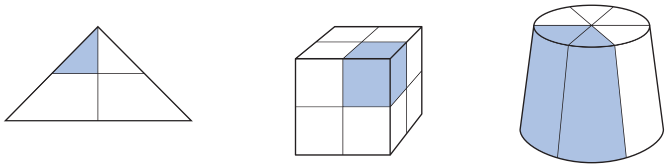
Understand a fraction as part of a whole.

Use fraction notation to describe a proportion of a shape.
For example:

- Find different ways of dividing a 4 by 4 grid of squares into quarters using straight lines.



- Estimate the fraction of each shape that is shaded.



- Shade one half of this shape.

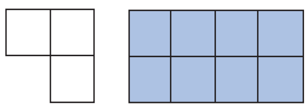
- Watch a **computer simulation** of a square being sectioned into fractional parts.
  Shade a fraction of a 6 by 6 grid of squares, e.g. one third.
  Convince a partner that exactly one third is shaded.

Relate fractions to division. Know that 4 ÷ 8 is another way of writing $\frac{4}{8}$, which is the same as $\frac{1}{2}$.

Express a smaller number as a fraction of a larger one.
For example:

- What fraction of:
  - 1 metre is 35 centimetres?
  - 1 kilogram is 24 grams?
  - 1 hour is 33 minutes?
  - 1 yard is 1 foot?
- What fraction of a turn does the minute hand turn through between:
  - 7:15 p.m. and 7:35 p.m.?
  - 3:05 p.m. and 6:50 p.m.?

- What fraction of a turn takes you from facing north to facing south-west?
- What fraction of a turn is 90°, 36°, 120°, 450°?
- What fraction of the big shape is the small one? ($\frac{3}{8}$)



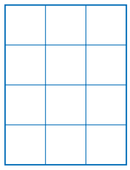
Know the meaning of numerator and denominator.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
terminating decimal, recurring decimal…
unit fraction…

Use fraction notation to describe a proportion of a shape. For example:

- Draw a 3 by 4 rectangle.

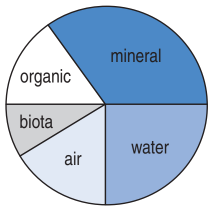


  Divide it into four parts that are $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{12}$ of the whole rectangle. Parts must not overlap.

  Now draw a 4 by 5 rectangle. Divide it into parts. Each part must be a unit fraction of the whole rectangle, i.e. with numerator 1.

  Try a 5 by 6 rectangle. And a 3 by 7 rectangle?

- The pie chart shows the proportions of components in soil.



  Estimate the fraction of the soil that is:
  a. water; b. air.

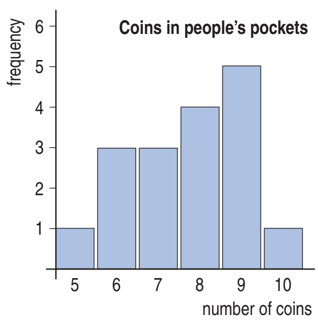
Relate fractions to division. Know that 43 ÷ 7 is another way of writing $\frac{43}{7}$, which is the same as $6\frac{1}{7}$.

Express a number as a fraction (in its lowest terms) of another. For example:

- What fraction of 180 is 120?

- What fraction of:
  1 foot is 3 inches?
  1 year is February?

- The bar chart shows the numbers of coins in people's pockets.
  What fraction of the total number of people had 7 coins in their pockets?

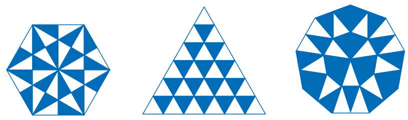


Link to enlargement and scale factor (pages 212–15).

As outcomes, Year 9 pupils should, for example:

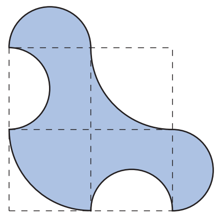
Use fraction notation to describe a proportion of a shape. For example:

- Estimate the fraction of each shape that is shaded.



- The curves of this shape are semicircles or quarter circles.

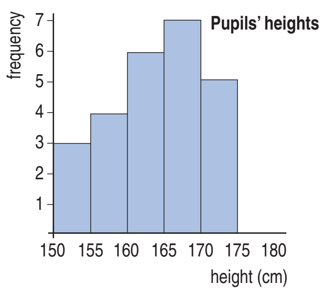
  Express the shaded shape as a fraction of the large dashed square.



Express a number as a fraction (in its lowest terms) of another. For example:

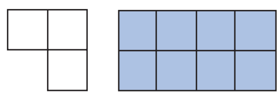
- What fraction of 120 is 180? ($\frac{3}{2}$ or $1\frac{1}{2}$)

- This frequency diagram shows the heights of a class of girls, classified in intervals $150 \le h < 155$, etc.



  What fraction of the girls are between 150 cm and 160cm tall?

- What fraction of the small shape is the large one? ($\frac{8}{3}$ or $2\frac{2}{3}$)



Link to enlargement and scale factor (pages 212–15).

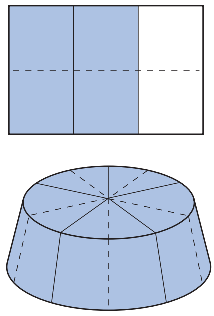
Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals (continued)

As outcomes, Year 7 pupils should, for example:

Simplify fractions by cancellation and recognise equivalent fractions.

Understand how equivalent fractions can be shown in diagrammatic form, with shapes sectioned into equal parts.



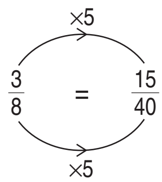
$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{5}{5} = 1$$

$$\frac{10}{10} = 1$$

Find equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

For example, recognise that because
$3 \times 5 = 15$
$8 \times 5 = 40$
it follows that $\frac{3}{8}$ is equivalent to $\frac{15}{40}$.



Know that if the numerator and the denominator have no common factors, the fraction is expressed in its lowest terms.

Answer questions such as:

- Cancel these fractions to their simplest form by looking for highest common factors:
  $\frac{9}{15}$ $\frac{12}{18}$ $\frac{42}{56}$
- Find two other fractions equivalent to $\frac{4}{5}$.
- Show that $\frac{12}{18}$ is equivalent to $\frac{6}{9}$ or $\frac{4}{6}$ or $\frac{2}{3}$.
- Find the unknown numerator or denominator in:
  $\frac{1}{4} = \frac{\square}{48}$ $\frac{7}{12} = \frac{35}{\square}$ $\frac{36}{24} = \frac{\square}{16}$

Link to finding the highest common factor (pages 54–5).

Continue to convert improper fractions to mixed numbers and vice versa: for example, change $\frac{34}{8}$ to $4\frac{1}{4}$, and $5\frac{7}{12}$ to $\frac{67}{12}$.

Answer questions such as:
- Convert $\frac{36}{5}$ to a mixed number.
- Which fraction is greater, $4\frac{4}{7}$ or $\frac{29}{7}$?
- How many fifths are there in $7\frac{1}{5}$?
- The fraction $\frac{7}{14}$ has three digits, 7, 1 and 4. It is equal to $\frac{1}{2}$. Find all the three-digit fractions that are equal to $\frac{1}{2}$. Explain how you know you have found them all.
- Find all the three-digit fractions that are equal to $\frac{1}{3}$. And $\frac{1}{4}$…
- There is only one three-digit fraction that is equal to $1\frac{1}{2}$. What is it?
- Find all the three-digit fractions that are equal to $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$…

See Y456 examples (pages 22–3).

As outcomes, Year 8 pupils should, for example:	As outcomes, Year 9 pupils should, for example:

Understand the equivalence of algebraic fractions.

For example:

$$\frac{ab}{ac} \equiv \frac{b}{c}$$

$$\frac{ab^2}{abc} = \frac{\overset{1}{\cancel{a}} \times \overset{1}{\cancel{b}} \times b}{\underset{1}{\cancel{a}} \times \underset{1}{\cancel{b}} \times c} = \frac{b}{c}$$

$$\frac{3ab}{6bc} = \frac{a}{2c}$$

Simplify algebraic fractions by finding common factors. For example:

- Simplify $\frac{3a + 2ab}{4a^2}$

Recognise when cancelling is inappropriate. For example, recognise that:

- $\frac{a + b}{b}$ is not equivalent to a + 1;
- $\frac{a + b}{b}$ is not equivalent to a;
- $\frac{ab - 1}{b}$ is not equivalent to a – 1.

Link to adding algebraic fractions (pages 118–19).

Pupils should be taught to:

Use fraction notation; recognise and use the equivalence of fractions and decimals (continued)

As outcomes, Year 7 pupils should, for example:

Convert terminating decimals to fractions.
Recognise that each terminating decimal is a fraction: for example, $0.27 = \frac{27}{100}$.

Convert decimals (up to two decimal places) to fractions. For example:

- Convert 0.4 to $\frac{4}{10}$ and then cancel to $\frac{2}{5}$.
- Convert 0.32 to $\frac{32}{100}$ and then cancel to $\frac{8}{25}$.
- Convert 3.25 to $3\frac{25}{100} = 3\frac{1}{4}$.

Link to place value (pages 36–9).

Convert fractions to decimals.
Convert a fraction to a decimal by using a known equivalent fraction. For example:

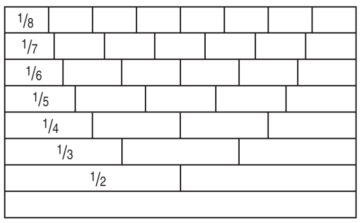
- $\frac{2}{8} = \frac{1}{4} = 0.25$
- $\frac{3}{5} = \frac{6}{10} = 0.6$
- $\frac{3}{20} = \frac{15}{100} = 0.15$

Convert a fraction to a decimal by using a known equivalent decimal. For example:

- Because $\frac{1}{5} = 0.2$
  $\frac{3}{5} = 0.2 \times 3 = 0.6$

See Y456 examples (pages 30–1).

Compare two or more simple fractions.
Deduce from a model or diagram that $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{5} > \ldots$ and that, for example, $\frac{2}{3} < \frac{3}{4}$.



Answer questions such as:

- Insert a > or < symbol between each pair of fractions:
  $\frac{1}{2}$ □ $\frac{7}{10}$ $\frac{3}{8}$ □ $\frac{1}{2}$ $\frac{1}{2}$ □ $\frac{2}{3}$ $\frac{7}{15}$ □ $\frac{1}{2}$
- Write these fractions in order, smallest first:
  $\frac{3}{4}$, $\frac{2}{3}$ and $\frac{5}{6}$;
  $2\frac{1}{10}$, $1\frac{3}{10}$, $2\frac{1}{2}$, $1\frac{1}{5}$, $1\frac{3}{4}$.
- Which of $\frac{5}{6}$ or $\frac{6}{5}$ is nearer to 1?
  Explain your reasoning.

See Y456 examples (pages 22–3).

As outcomes, Year 8 pupils should, for example:

Convert decimals to fractions.
Continue to recognise that each terminating decimal is a fraction. For example, 0.237 = $\frac{237}{1000}$.

Recognise that a recurring decimal is a fraction.

Convert decimals (up to three decimal places) to fractions. For example:
- Convert 0.625 to $\frac{625}{1000}$ and then cancel to $\frac{5}{8}$.

Link to percentages (pages 70–1).

Convert fractions to decimals.
Use division to convert a fraction to a decimal, without and with a calculator. For example:
- Use short division to work out that:
  $\frac{1}{5}$ = 0.2 $\frac{3}{8}$ = 0.375 $\frac{27}{8}$ = … $\frac{3}{7}$ = …
- Use a **calculator** to work out that $\frac{7}{53}$ = …

Investigate fractions such as $\frac{1}{3}$, $\frac{1}{6}$, $\frac{2}{3}$, $\frac{1}{9}$, $\frac{1}{11}$, … converted to decimals. For example:
- Predict what answers you will get when you use a **calculator** to divide:
  3 by 3, 4 by 3, 5 by 3, 6 by 3, and so on.

Order fractions.
Compare and order fractions by converting them to fractions with a common denominator or by converting them to decimals. For example, find the larger of $\frac{7}{8}$ and $\frac{4}{5}$:
- using common denominators:
  $\frac{7}{8}$ is $\frac{35}{40}$, $\frac{4}{5}$ is $\frac{32}{40}$, so $\frac{7}{8}$ is larger.
- using decimals:
  $\frac{7}{8}$ is 0.875, $\frac{4}{5}$ is 0.8, so $\frac{7}{8}$ is larger.

Use equivalent fractions or decimals to position fractions on a number line. For example:
- Mark fractions such as $\frac{2}{5}$, $\frac{6}{20}$, $\frac{3}{15}$, $\frac{18}{12}$ on a number line graduated in tenths, then on a line graduated in hundredths.

Answer questions such as:
- Which is greater, 0.23 or $\frac{3}{16}$?
- Which fraction is exactly half way between $\frac{3}{5}$ and $\frac{5}{7}$?

As outcomes, Year 9 pupils should, for example:

Know that a recurring decimal is an exact fraction.
Know and use simple conversions for recurring decimals to fractions. For example:
- 0.333 333… = $\frac{1}{3}$ (= $\frac{3}{9}$)
- 0.666666… = $\frac{2}{3}$
- 0.111111… = $\frac{1}{9}$
- 0.999999… = $\frac{9}{9}$ = 1

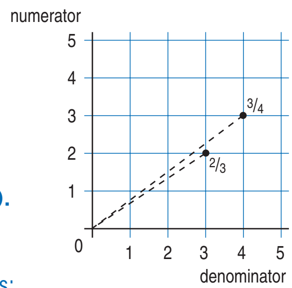
Convert recurring decimals to fractions in simple cases, using an algebraic method. For example:
- z = 0.333 333… (1)
  $10z$ = 3.333 333… (2)
  Subtracting (1) from (2) gives:
  $9z = 3$
  $z = \frac{1}{3}$
- Comment on:
  z = 0.999 999…
  $10z$ = 9.999 999…
  $9z = 9$
  $z = 1$

Order fractions.
Answer questions such as:
- The numbers $\frac{1}{2}$, a, b, $\frac{3}{4}$ are in increasing order of size. The differences between successive numbers are all the same. What is the value of b?
- z is a decimal with one decimal place. Write a list of its possible values, if both these conditions are satisfied:
  $\frac{1}{3} < z < \frac{2}{3}$ $\frac{1}{6} < z < \frac{5}{6}$

Link to inequalities (pages 112–13).

Order fractions by graphing them. Compare gradients.

Link to gradients (page 167–9).



Investigate sequences such as:

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots, \frac{n}{n+1}$$

Investigate what happens as the sequence continues and n tends towards infinity.
Convert the fractions to decimals or draw a graph of the decimal against the term number.

Pupils should be taught to:

Calculate fractions of quantities; add, subtract, multiply and divide fractions

As outcomes, Year 7 pupils should, for example:

Add and subtract simple fractions.

Know addition facts for simple fractions, such as:

- $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
- $\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$
- $\frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

and derive other totals from these results, such as:

- $\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ (knowing that $\frac{1}{4} = \frac{2}{8}$)
- $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$ (knowing that $\frac{4}{8} = \frac{1}{2}$)

Add and subtract simple fractions with the same denominator. For example:

- $\frac{3}{8} + \frac{5}{8}$ $\frac{3}{5} + \frac{4}{5} + \frac{1}{5}$ $\frac{7}{10} + \frac{3}{10} + \frac{5}{10} + \frac{8}{10}$
- $\frac{6}{7} - \frac{4}{7}$ $\frac{9}{10} + \frac{4}{10} - \frac{3}{10}$

Calculate fractions of numbers, quantities or measurements.

Know that, for example:

- $\frac{1}{5}$ of 35 has the same value as $35 \div 5 = 7$;
- $\frac{2}{3}$ of 15 has the same value as $15 \div 3 \times 2 = 10$;
- 0.5 of 18 has the same value as $\frac{1}{2}$ of 18 = 9.

Use mental methods to answer short questions with whole-number answers, such as:

- Find: one fifth of 40; two thirds of 150 g.
- Find: $\frac{1}{3}$ of 24; $\frac{3}{8}$ of 160; $\frac{9}{10}$ of 1 metre.
- Find: 0.5 of 50; 0.75 of 56; 1.25 of 40.

Use informal written methods to answer questions such as:

- If I make one fifth of a turn, how many degrees do I turn?
- Calculate: $\frac{7}{10}$ of £420; $\frac{6}{5}$ of 35;
  $\frac{3}{7}$ of 210; $1\frac{1}{4}$ of 2.4.

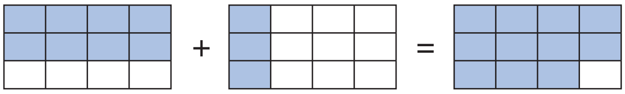
See Y456 examples (pages 24–5).

Link to multiplying fractions (pages 68–9).

As outcomes, Year 8 pupils should, for example:

Add and subtract fractions.

Use diagrams to illustrate adding and subtracting fractions, showing equivalence.



Know that fractions can only be added and subtracted if they have the same denominator.
Use, for example, a single bar to avoid the problem of adding denominators:

$$\frac{3}{5} + \frac{5}{10} = \frac{6+5}{10} = \frac{11}{10} = 1\frac{1}{10}$$

Answer questions such as:

- Add/subtract these fractions:
  $\frac{1}{4} + \frac{5}{12}$ $\frac{3}{5} + \frac{3}{4}$ $\frac{5}{6} - \frac{3}{4}$
- Ancient Egyptian fractions were written with 1 as the numerator (unit fractions).
  Express these fractions as sums of unit fractions:
  $\frac{5}{8}$, $\frac{11}{12}$, $\frac{7}{10}$, $\frac{7}{12}$, $\frac{9}{20}$
- This fraction sum is made from four different digits, 1, 2, 4 and 8. The fraction sum is 1.

  $\frac{1}{2} + \frac{4}{8}$

  Find other fraction sums made from four different digits and with a fraction sum of 1.

Calculate fractions of numbers, quantities or measurements.

Develop written methods to answer short questions with fraction answers, such as:

- Find: three fifths of 17;
  two thirds of 140 g;
  $\frac{6}{25}$ of 34.

Link to multiplying and dividing fractions **(pages 68–9).**

As outcomes, Year 9 pupils should, for example:

Add and subtract fractions.

Add and subtract more complex fractions.
For example:

- $\frac{11}{18} + \frac{7}{24} = \frac{44+21}{72} = \frac{65}{72}$
- A photograph is $6\frac{1}{4}$ inches tall and $8\frac{5}{8}$ inches wide. Calculate its perimeter.
- Investigate $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$
  and similar series.
- Begin to add and subtract algebraic fractions (pages 118–19), linking to number examples.

Link to finding lowest common multiples **(pages 54–5).**

Calculate fractions of numbers, quantities or measurements.

Understand the multiplicative nature of fractions as operators. For example:

- Which is the greater: $\frac{3}{4}$ of 24 or $\frac{2}{3}$ of 21?
  34% of 75 or $\frac{5}{7}$ of 85?
  5 out of 16 or 8 out of 25?
- In a survey of 24 pupils, $\frac{1}{3}$ liked football best, $\frac{1}{4}$ liked basketball, $\frac{3}{8}$ liked athletics. The rest liked swimming. How many liked swimming?
- Brian used $\frac{1}{3}$ of a 750 g bag of flour to make scones. Claire used $\frac{2}{5}$ of the flour that remained to make a cake.
  How many grams of flour were left in the bag?
- In a bag of 20 coloured beads, $\frac{2}{5}$ are red, $\frac{1}{4}$ are blue, $\frac{1}{10}$ are yellow and 3 are green. The rest are black. What fraction are black?

Link to mutually exclusive events in probability **(pages 278–81).**

Pupils should be taught to:

Calculate fractions of quantities; add, subtract, multiply and divide fractions (continued)

As outcomes, Year 7 pupils should, for example:

Multiply a fraction by an integer or an integer by a fraction.

Know that $\frac{1}{4}$ of 12, $\frac{1}{4} \times 12$, $12 \times \frac{1}{4}$ and $12 \div 4$ are all equivalent.

Multiply a simple fraction by an integer. For example:

$\frac{1}{5} \times 3 = \frac{3}{5}$ $\quad$ $\frac{2}{5} \times 4 = \frac{8}{5}$

Simplify the product of a simple fraction and an integer.
For example:

$\frac{1}{5} \times 15 = 3$

$\frac{2}{5} \times 15 = 2 \times \frac{1}{5} \times 15 = 2 \times 3 = 6$

$12 \times \frac{5}{6} = \frac{5}{6} \times 12 = 5 \times \frac{1}{6} \times 12 = 5 \times 2 = 10$

Answer questions such as:

- Find: $\frac{1}{9} \times 63$ $\quad$ $\frac{7}{9} \times 90$ $\quad$ $1\frac{1}{4} \times 10$
- Find: 0.25×24 $\quad$ 0.2×50 $\quad$ 3.3×40

As outcomes, Year 8 pupils should, for example:

Multiply an integer by a fraction.

Know that $\frac{2}{3}$ of 12, $\frac{2}{3} \times 12$ and $12 \times \frac{2}{3}$ are all equivalent.

Connect ordinary multiplication tables with patterns in fraction multiplication tables:

$\frac{1}{5} \times 1 = \frac{1}{5}$	$\frac{2}{5} \times 1 = \frac{2}{5}$	$\frac{3}{5} \times 1 = \frac{3}{5}$
$\frac{1}{5} \times 2 = \frac{2}{5}$	$\frac{2}{5} \times 2 = \frac{4}{5}$	$\frac{3}{5} \times 2 = \frac{6}{5}$
$\frac{1}{5} \times 3 = \frac{3}{5}$	$\frac{2}{5} \times 3 = \frac{6}{5}$	$\frac{3}{5} \times 3 = \frac{9}{5}$
$\frac{1}{5} \times 4 = \frac{4}{5}$	$\frac{2}{5} \times 4 = \frac{8}{5}$	$\frac{3}{5} \times 4 = \frac{12}{5}$

Think of multiplication by $\frac{1}{8}$ as division by 8, so $6 \times \frac{1}{8} = 6 \div 8$, and $6 \times \frac{3}{8} = 6 \times 3 \times \frac{1}{8} = 18 \div 8$.

Use cancellation to simplify the product of a fraction and an integer. For example:

$$\frac{7}{24} \times \frac{15}{1} = \frac{7}{\underset{8}{\cancel{24}}} \times \frac{\overset{5}{\cancel{15}}}{1} = \frac{35}{8}$$

Answer questions such as:

- Find: $\frac{3}{12} \times 30$ $\frac{5}{9} \times 24$ $2\frac{1}{8} \times 10$

Understand that when multiplying a positive number by a fraction less than one, the result will be a smaller number. For example:
$24 \times \frac{1}{4} = 6$

Divide an integer by a fraction.

Know that a statement such as $24 \div \frac{1}{4}$ can be interpreted as:

- How many quarters are there in 24?
  $24 = \square \times \frac{1}{4}$ or $24 = \frac{1}{4} \times \square$.

For example:

- Look at one whole circle (or rectangle, prism…). How many sevenths can you see? (Seven.)
- Look at 1. How many fifths can you see? (Five.)
- Look at 4. How many fifths can you see? (Twenty.)
- Look at 4. How many two fifths can you see? (Ten.)

Use patterns. For example:

$60 \times \frac{1}{6} = 10$	and	$10 \div \frac{1}{6} = 60$
$30 \times \frac{2}{6} = 10$	and	$10 \div \frac{2}{6} = 30$
$20 \times \frac{3}{6} = 10$	and	$10 \div \frac{3}{6} = 20$
$15 \times \frac{4}{6} = 10$	and	$10 \div \frac{4}{6} = 15$
$12 \times \frac{5}{6} = 10$	and	$10 \div \frac{5}{6} = 12$

Understand that when dividing a positive number by a fraction less than one, the result will be a larger number. For example:
$24 \div \frac{1}{4} = 96$

As outcomes, Year 9 pupils should, for example:

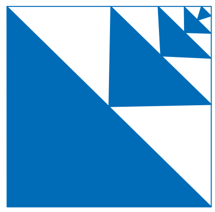
Multiply a fraction by a fraction.

Multiply fractions, using cancelling to simplify:

$$\frac{3}{4} \times \frac{2}{9} = \frac{\overset{1}{\cancel{3}} \times \overset{1}{\cancel{2}}}{\underset{2}{\cancel{4}} \times \underset{3}{\cancel{9}}} = \frac{1}{6}$$

For example:

- Calculate:
  a. $\frac{3}{5} \times \frac{20}{33} \times \frac{22}{14}$
  b. $\frac{22}{7} \times 14 \times 14$
  c. $4\frac{2}{3} \times 1\frac{3}{4}$
  d. $\frac{1}{2}(2 - \frac{1}{4})$
  e. $(2\frac{1}{2})^3$
- A photograph is $6\frac{1}{4}$ inches tall and $8\frac{5}{8}$ inches wide. Calculate its area.
- Imagine a square with sides of 1 metre. The area of the largest shaded triangle is $\frac{1}{2}\,m^2$.

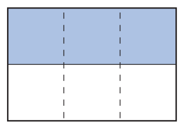


  a. Write the areas of the next two largest shaded triangles.
  b. Use the diagram to help you find the sum of the infinite series:
  $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \ldots$
  Explain how you arrived at your solution.

Divide a fraction by a fraction.

Use the inverse rule to divide fractions, first converting mixed numbers to improper fractions.
For example:

- Look at one half of a shape.



  How many sixths of the shape can you see? (Six.)
  So, how many sixths in one half? (Three.)
  So $\frac{1}{2} \div \frac{1}{6} = \frac{1}{2} \times \frac{6}{1} = \frac{6}{2} = 3$
- $\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} \times \frac{7}{4} = \frac{14}{12}$ or $\frac{7}{6}$
- $2\frac{1}{3} \div \frac{4}{5} = \frac{7}{3} \times \frac{5}{4} = \frac{35}{12}$ or $2\frac{11}{12}$

Answer questions such as:

- Calculate: $(1 - \frac{1}{3})/(1 - \frac{5}{8})$
- The area of a circle is $154\,cm^2$.
  Taking π as $\frac{22}{7}$, find the radius of the circle.

Link to multiplying and dividing algebraic fractions (pages 118–19).

Pupils should be taught to:

Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems

As outcomes, Year 7 pupils should, for example:

Understand percentage as the number of parts in every 100, and express a percentage as an equivalent fraction or decimal. For example:

Convert percentages to fractions by writing them as the number of parts per 100, then cancelling. For example:

- 60% is equivalent to $\frac{60}{100} = \frac{3}{5}$;
- 150% is equivalent to $\frac{150}{100} = \frac{3}{2} = 1\frac{1}{2}$.

Convert percentages to decimals by writing them as the number of parts per 100, then using knowledge of place value to write the fraction as a decimal. For example:

- 135% is equivalent to $135 \div 100 = 1.35$.

Recognise the equivalence of fractions, decimals and percentages.

Know decimal and percentage equivalents of simple fractions.

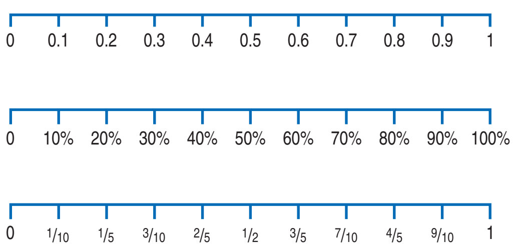
For example, know that $1 \equiv 100\%$. Use this to show that:

- $\frac{1}{10}$ = 0.1 which is equivalent to 10%;
- $\frac{1}{100}$ = 0.01 which is equivalent to 1%;
- $\frac{1}{8}$ = 0.125 which is equivalent to $12\frac{1}{2}\%$;
- $1\frac{3}{4}$ = 1.75 which is equivalent to 175%;
- $\frac{1}{3}$ = 0.333… which is equivalent to $33\frac{1}{3}\%$.

Express simple fractions and decimals as equivalent percentages by using equivalent fractions. For example:

- $\frac{3}{5} = \frac{60}{100}$ which is equivalent to 60%;
- $\frac{7}{20} = \frac{35}{100}$ which is equivalent to 35%;
- $2\frac{3}{4} = \frac{275}{100}$ which is equivalent to 275%;
- $0.48 = \frac{48}{100}$ which is equivalent to 48%;
- $0.3 = \frac{30}{100}$ which is equivalent to 30%.

Use number lines to demonstrate equivalence.



See Y456 examples (pages 32–3).

Link the equivalence of fractions, decimals and percentages to the probability scale (pages 278–9), and to the interpretation of data in pie charts and bar charts (pages 268–71).

As outcomes, Year 8 pupils should, for example:

Understand percentage as the operator 'so many hundredths of'.
For example, know that 15% means 15 parts per hundred, so 15% of Z means $^{15}/_{100} \times Z$.

Convert fraction and decimal operators to percentage operators by multiplying by 100.
For example:

- 0.45 — $0.45 \times 100\% = 45\%$
- $^{7}/_{12}$ — $(7 \div 12) \times 100\% = 58.3\%$ (to 1 d.p.)

Link the equivalence of fractions, decimals and percentages to the probability scale (pages 278–9), and to the interpretation of data in pie charts and bar charts (pages 268–71).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems (continued)

As outcomes, Year 7 pupils should, for example:

Calculate percentages of numbers, quantities and measurements.

Know that 10% is equivalent to $\frac{1}{10}$ = 0.1, and 5% is half of 10%.

Use **mental methods**. For example, find:

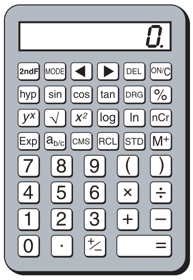
- 10% of £20 by dividing by 10;
- 10% of 37 g by dividing by 10;
- 5% of £5 by finding 10% and then halving;
- 100% of 5 litres by knowing that 100% represents the whole;
- 15% of 40 by finding 10% then 5% and adding the results together.

Use **informal written methods**. For example, find:

- 11% of £2800 by calculating 10% and 1% as jottings, and adding the results together;
- 70% of 130 g by calculating 10% and multiplying this by 7 as jottings; or by calculating 50% and 20% as jottings and adding the results.

Use a **calculator**, without the % key, to work out percentages of numbers and measures. For example:

- What is 24% of 34?
- Find 14.5% of 56 litres.



Know that there is more than one way to find a percentage using a calculator. For example, to find 12% of 45:

Convert a percentage calculation to an equivalent decimal calculation.

12% of 45 0.12 × 45

[.] [1] [2] [×] [4] [5] [=]

Convert a percentage calculation to an equivalent fraction calculation.

12% of 45 $\frac{12}{100}$ × 45

[1] [2] [÷] [1] [0] [0] [×] [4] [5] [=]

Recognise that this method is less efficient than the first.

Understand a calculator display when finding percentages in the context of money. For example:

- Interpret 15% of £48, displayed by most calculators as 7.2, as £7.20.

See Y456 examples (pages 32–3).

As outcomes, Year 8 pupils should, for example:

Calculate percentages of numbers, quantities and measurements.

Continue to use **mental methods**. For example, find:

- 65% of 40 by finding 50%, then 10% then 5% and adding the results together.
- 35% of 70 ml by finding 10%, trebling the result and then adding 5%;
- 125% of £240 by finding 25% then adding this to 240.

Use **written methods**. For example:

Use an equivalent fraction, as in:

- 13% of 48 $\quad {}^{13}/_{100} \times 48 = {}^{624}/_{100} = 6.24$

Use an equivalent decimal, as in:

- 13% of 48 $\quad 0.13 \times 48 = 6.24$

Use a unitary method, as in:

- 13% of 48 $\quad$ 1% of 48 $= 0.48$
  13% of 48 $= 0.48 \times 13 = 6.24$

Use a **calculator**, without the % key, to work out percentages of numbers and measures.

Use an equivalent decimal calculation.

12% of 45 $\quad 0.12 \times 45$

[.] [1] [2] [×] [4] [5] [=]

Use a unitary method; that is, find 1% first.

12% of 45 $\quad$ 1% is 0.45, so 12% is 0.45×12

[.] [4] [5] [×] [1] [2] [=]

Recognise that these methods are equally efficient.

Extend understanding of the display on the calculator when using percentages of money.
For example:

- Interpret the answer to 33⅓% of £27, displayed by

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems (continued)

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
change, total, value, amount, sale price, discount, decrease, increase, exchange rate, currency, convert…

Use the equivalence of fractions, decimals and percentages to compare two or more simple proportions and to solve simple problems.

Discuss percentages in everyday contexts. For example:

- Identify the percentage of wool, cotton, polyester… in clothes by examining labels.
- Work out what percentage of the pupils in the class are boys, girls, aged 11, have brown eyes…
- Discuss the use of percentages to promote the sales of goods, e.g. to indicate the extra amount in a packet.

Answer questions such as:

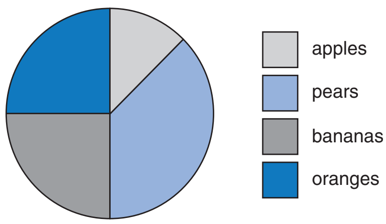
- Estimate the percentage of this line that is blue.

- 12% of a 125 g pot of yoghurt is whole fruit.
  How many grams are not whole fruit?

- 48% of the pupils at a school are girls.
  25% of the girls and 50% of the boys travel to school by bus.
  What percentage of the whole school travels by bus?

Use proportions to interpret pie charts. For example:

- Some people were asked which fruit they liked best.
  This chart shows the results.



Estimate:
a. the percentage of the people that liked oranges best;
b. the proportion that liked apples best;
c. the percentage that did not choose pears.

Link to problems involving percentages (pages 2–3).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
profit, loss, interest, service charge, tax, VAT…
unitary method…

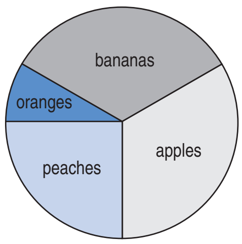
Use the equivalence of fractions, decimals and percentages to compare simple proportions and solve problems.

Apply understanding of simple percentages to other contexts, such as:

- the composition of alloys in science: for example, a 2p coin is 95% copper, 3.5% tin and 1.5% zinc;
- the age distribution of a population in geography;
- the elements of a balanced diet in nutrition;
- the composition of fabrics in design and technology: for example, a trouser fabric is 83% viscose, 10% cotton, 7% Lycra.

Answer questions such as:

- There is 20% orange juice in every litre of a fruit drink. How much orange juice is there in 2.5 litres of fruit drink? How much fruit drink can be made from 1 litre of orange juice?
- This chart shows the income that a market stall-holder got last week from selling different kinds of fruit.



  The stall-holder got £350 from selling bananas.
  Estimate how much she got from selling oranges.

- 6 out of every 300 paper clips produced by a machine are rejected.
  What is this as a percentage?
- Rena put £150 in her savings account.
  After one year, her interest was £12.
  John put £110 in his savings account.
  After one year, his interest was £12.
  Who had the better rate of interest, Rena or John?
  Explain your answer.

Link to problems involving percentages (pages 2–3).

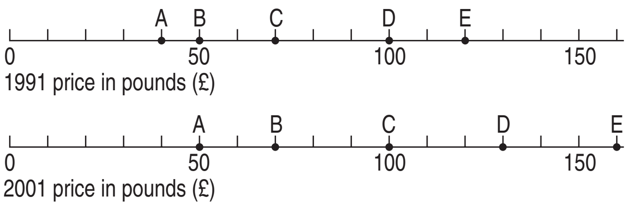
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
cost price, selling price, compound interest…

Recognise when fractions or percentages are needed to compare proportions and solve problems.

Answer questions such as:

- A slogan on a tube of mints says that it is 23% bigger. It contains 20 mints.
  How many mints are there in the normal tube?
- Which is the better buy:
  a 400 g pack of biscuits at 52p, or
  a pack of biscuits with 400g + 25% extra, at 57p?
- In a phone bill, VAT at 17.5% is added to the total cost of calls and line rental.
  What percentage of the total bill is VAT?
- In 1999, about 50% of the world's tropical rain forests had been destroyed.
  About 180 000 square kilometres are now destroyed each year.
  This represents about 1.2% of the remainder.
  Estimate the original area of the tropical rain forests.
- The prices of five items A, B, C, D and E in 1991 and 2001 are shown on these scales.



  Which of the items showed the greatest percentage increase in price from 1991 to 2001?

Link to problems involving percentages (pages 2–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Understand percentage as the number of parts per 100; recognise the equivalence of fractions, decimals and percentages; calculate percentages and use them to solve problems (continued)	

As outcomes, Year 8 pupils should, for example:

Find the outcome of a given percentage increase or decrease.

Understand that:

- If something increases by 100%, it doubles.
- If something increases by 500%, it increases by five times itself, and is then six times its original size.
- A 100% decrease leaves zero.
- An increase of 15% will result in 115%, and 115% is equivalent to 1.15.
- A decrease of 15% will result in 85%, and 85% is equivalent to 0.85.
- An increase of 10% followed by a further increase of 10% is not equivalent to an increase of 20%.

For example:

- An increase of 15% on an original cost of £12 gives a new price of
  £12 × 1.15 = £13.80
  or
  15% of £12 = £1.80 £12 + £1.80 = £13.80

- A decrease of 15% on the original cost of £12 gives a new price of
  £12 × 0.85 = £10.20
  or
  15% of £12 = £1.80 £12 – £1.80 = £10.20

Investigate problems such as:

- I can buy a bicycle for one cash payment of £119, or pay a deposit of 20% and then six equal monthly payments of £17.
  How much extra will I pay in the second method?

- A price is increased by 10% in November to a new price. In the January sales the new price is reduced by 10%. Is the January sale price more, less or the same as the price was in October? Justify your answer.

- At the end of a dinner the waiter added VAT of 17.5% and then a 12.5% service charge. The customer argued that the service charge should have been calculated first.
  Who was correct?
  Give mathematical reasons for your answer.

Link to enlargement and scale (pages 212–17), and area and volume (pages 234–41).

As outcomes, Year 9 pupils should, for example:

Use percentage changes to solve problems, choosing the correct numbers to take as 100%, or as a whole.

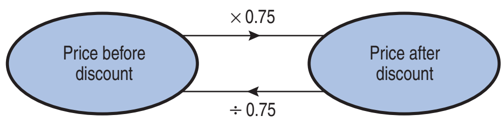
For example:

- There was a 25% discount in a sale.
  A boy paid £30 for a pair of jeans in the sale.
  What was the original price of the jeans?

  Using a unitary method

£30	represents 75%.
£30 ÷ 75	represents 1%.
£30 ÷ 75 × 100	represents 100%.

  Using inverse operations
  Let p be the original price.
  p × 0.75 = 30, so p = 30 ÷ 0.75 = 40



- An unstretched metal spring is 20 cm long.
  It is stretched to a length of 27cm.
  Find the percentage change in its length.

  The increase is $\frac{7}{20} = \frac{35}{100}$ or 35%.

Solve problems such as:

- A jacket is on sale at £45, which is 85% of its original price. What was its original price?

- I bought a fridge freezer in a sale and saved £49. The label said that it was a '20% reduction'.
  What was the original price of the fridge freezer?

- A stereo system has been reduced from £320 to £272. What is the percentage reduction?

- The number of people going to a cinema increased from 52 000 in 1998 to 71500 in 2001. Calculate the percentage increase in the number of people going to the cinema from 1998 to 2001.

- 12 500 people visited a museum in 2000.
  This was an increase of 25% on 1999.
  How many visitors were there in 1999?

- When heated, a metal bar increases in length from 1.25 m to 1.262m. Calculate the percentage increase correct to one decimal place.

- A woman deposits £75 in a bank with an annual compound interest rate of 6%. How much will she have at the end of 3 years? (The calculation 75 × $(1.06)^3$ gives the new amount.)

Link to proportionality (pages 78–9), enlargement and scale (page 212–17), and area and volume (pages 234–41).

Pupils should be taught to:

Understand the relationship between ratio and proportion, and use ratio and proportion to solve simple problems

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
ratio, proportion…
and the notation 3 : 2.

Proportion compares part to whole, and is usually expressed as a fraction, decimal or percentage. For example:

- If there are 24 fish in a pond, and 6 are gold and 18 are black, there are 6 gold fish out of a total of 24 fish. The proportion of gold fish is 6 out of 24, or 1 in 4, or $\frac{1}{4}$, or 25%, or 0.25.

Solve problems such as:

- Tina and Fred each have some Smarties in a jar. The table shows how many Smarties they have, and how many of these Smarties are red.

	Number of Smarties	Number of red Smarties
Tina	440	40
Fred	540	45

Who has the greater proportion of red Smarties, Tina or Fred?

Use direct proportion in simple contexts.
For example:

- Three bars of chocolate cost 90p. How much will six bars cost? And twelve bars?
- 1 litre of fruit drink contains 200 ml of orange juice. How much orange juice is there in 1.5 litres of fruit drink?
- £1 is worth 1.62 euros. How many euros will I get for £50?
- Here are the ingredients for fish pie for two people.

Fish pie for two people

250 g fish
400g potato
25g butter

I want to make a fish pie for three people. How many grams of fish should I use?

See Y456 examples (pages 26–7).

Link to problems involving proportion (pages 4–5).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
direct proportion…

Solve simple problems involving direct proportion.
For example:

- 5 miles is approximately equal to 8 km.
  Roughly, how many km are equal to 20 miles?
  Roughly, how many miles are equal to 24 km?

  1 mile $\approx \frac{8}{5}$ km

  20 miles $\approx \frac{8}{5} \times 20$ km = 32km

- 8 pizzas cost £16.
  What will 6 pizzas cost?

- 6 stuffed peppers cost £9.
  What will 9 stuffed peppers cost?

Link to problems involving proportion (pages 4–5).

Use a **spreadsheet** to explore direct proportion.
For example:

	A	B
1	No. of peppers	Cost (£)
2	1	=0.45*A2
3	2	=0.45*A3
4	3	=0.45*A4
5	4	=0.45*A5
6	5	=0.45*A6

	A	B
1	£	$
2	10	=1.62*A2
3	20	=1.62*A3
4	30	=1.62*A4
5	40	=1.62*A5
6	50	=1.62*A6

Link to conversion graphs (pages 172–3, 270–1), graphs of linear relationships (pages 164–5), and problems involving ratio and proportion (pages 4–5).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
proportionality, proportional to…
and the symbol $\propto$ (directly proportional to).

Identify when proportional reasoning is needed to solve a problem. For example:

- A recipe for fruit squash for six people is:

300 g	chopped oranges
1500ml	lemonade
750ml	orange juice

  Trina made fruit squash for ten people.
  How many millilitres of lemonade did she use?

  Jim used 2 litres of orange juice for the same recipe. How many people was this enough for?

Link to problems involving proportion (pages 4–5).

Use a **spreadsheet** to develop a table with a constant multiplier for linear relationships. Plot the corresponding graph using a **graph plotter** or **graphical calculator**.

Understand and use proportionality. Use
$y \propto x$ $y \propto x^2$ $y \propto 1/x$
to explore relationships between variables.

Use a **spreadsheet** to test whether one set of numbers is directly proportional to another, e.g.

	A	B	C	D	E	F	G	H
1	No. of litres	1	2	3	4	5	6	7
2	Price (p)	91	182	273	364	455	546	637
3	Price/litres	=B2/B1	=C2/C1	=D2/D1	=E2/E1	=F2/F1	=G2/G1	=H2/H1

Plot the corresponding graph using a **graph plotter**.

Compare with a non-linear relationship, such as
area of square = (side length)^2

Use proportionality in other contexts. For example, from science know that pressure is proportional to force and weight is proportional to mass.

Appreciate that some 'real-life' relationships, particularly in science, may appear to be directly proportional but are not. For example, consider:

- A plant grows 5 cm in 1 week.
  How much will it grow in 1 year?
- A man can run 1 mile in 4 minutes.
  How far can he run in 1 hour?

Link to graphs of functions (pages 170–1).

Pupils should be taught to:

Understand the relationship between ratio and proportion, and use ratio and proportion to solve simple problems (continued)

As outcomes, Year 7 pupils should, for example:

Understand the idea of a ratio and use ratio notation.

Ratio compares part to part. For example:

- If Lee and Ann divide £100 in the ratio 2 : 3, Lee gets 2 parts and Ann gets 3 parts. 1 part is £100 ÷ 5 = £20.
  So Lee gets £20 × 2 = £40 and Ann gets £20 × 3 = £60.

Know that the ratio 3 : 2 is not the same as the ratio 2:3.

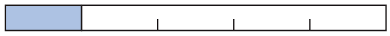
- If Lee and Ann divide £100 in the ratio 3 : 2,
  Lee gets £60 and Ann gets £40.

Simplify a (two-part) ratio to an equivalent ratio by cancelling, e.g.

- Which of these ratios is equivalent to 3 : 12?
  A. 3:1 B. 9:36 C. 4:13 D. 1:3

Link to fraction notation (pages 60–3).

Understand the relationship between ratio and proportion, and relate them both to everyday situations. For example:

- In this stick 
  the ratio of blue to white is one part to four parts or 1 : 4.
  The proportion of blue in the whole stick is 1 out of 5, and $\frac{1}{5}$ or 20% of the whole stick is blue.

Divide a quantity into two parts in a given ratio and solve simple problems using informal strategies. For example:

- A girl spent her savings of £40 on books and clothes in the ratio 1 : 3.
  How much did she spend on clothes?
- Coffee is made from two types of beans, from Java and Colombia, in the ratio 2 : 3.
  How much of each type of bean will be needed to make 500 grams of coffee?
- 28 pupils are going on a visit.
  They are in the ratio of 3 girls to 4 boys.
  How many boys are there?

See Y456 examples (pages 26–7).

Link to problems involving ratio (pages 4–5).

Use simple ratios when interpreting or sketching maps in geography or drawing to scale in design and technology.

As outcomes, Year 8 pupils should, for example:

Simplify a (three-part) ratio to an equivalent ratio by cancelling. For example:
- Write the ratio 12 : 9 : 3 in its simplest form.

Link to fraction notation (pages 60–3).

Simplify a ratio expressed in different units.
For example:
- 2 m : 50cm
- 450g:5kg
- 500mm:75cm:2.5m

Link to converting between measures (pages 228–9).

Consolidate understanding of the relationship between ratio and proportion. For example:

- In a game, Tom scored 6, Sunil scored 8, and Amy scored 10. The ratio of their scores was 6 : 8 : 10, or 3:4:5. Tom scored a proportion of $\frac{3}{12}$ or $\frac{1}{4}$ or 25% of the total score.

Divide a quantity into two or more parts in a given ratio. Solve simple problems using a unitary method.

- Potting compost is made from loam, peat and sand, in the ratio 7 : 3 : 2 respectively. A gardener used 1½ litres of peat to make compost.
  How much loam did she use? How much sand?
- The angles in a triangle are in the ratio 6:5:7. Find the sizes of the three angles.
- Lottery winnings were divided in the ratio 2:5. Dermot got the smaller amount of £1000.
  How much in total were the lottery winnings?

2 parts	=	£1000
1 part	=	£500
5 parts	=	£2500
Total	=	£1000 + £2500 = £3500

Link to problems involving ratio (pages 4–5).

Use ratios when interpreting or sketching maps or drawing to scale in geography and other subjects.

- A map has a scale of 1 : 10000. What distance does 5 cm on the map represent in real life?

Link to enlargement and scale (pages 212–17).

As outcomes, Year 9 pupils should, for example:

Simplify a ratio expressed in fractions or decimals.
For example:
- Write 0.5 : 2 in whole-number form.

Compare ratios by changing them to the form m : 1 or 1 :m. For example:
- The ratios of Lycra to other materials in two stretch fabrics are 2 : 25 and 3:40. By changing each ratio to the form 1:m, say which fabric has the greater proportion of Lycra.
- The ratios of shots taken to goals scored by two hockey teams are 17:4 and 13:3 respectively. By changing each ratio to the form m:1, say which is the more accurate team.

Interpret and use ratio in a range of contexts.
For example:

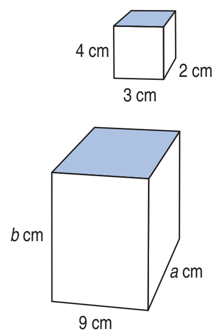
- Shortcrust pastry is made from flour and fat in the ratio 2 :1. How much flour will make 450g of pastry?
- An alloy is made from iron, copper, nickel and aluminium in the ratio 5:4:4:1. Find how much copper is needed to mix with 85g of iron.
- 2 parts of blue paint mixed with 3 parts of yellow paint makes green. A boy has 50ml of blue paint and 100ml of yellow. What is the maximum amount of green he can make?
- On 1st June the height of a sunflower was 1m. By 1st July, the height had increased by 40%. What was the ratio of the height of the sunflower on 1st June to its height on 1st July?

Link to problems involving ratio (pages 4–5).

Understand the implications of enlargement for area and volume. For example:

- Corresponding lengths in these similar cuboids are in the ratio 1 : 3.

  Find the values of a and b.
  Find the ratio of the areas of the shaded rectangles.
  Find the ratio of the volumes of the cuboids.



Link to enlargement and scale (pages 212–17).

Pupils should be taught to:

Consolidate understanding of the operations of multiplication and division, their relationship to each other and to addition and subtraction; know how to use the laws of arithmetic

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
operation, commutative, inverse, add, subtract, multiply, divide, sum, total, difference, product, multiple, factor, quotient, divisor, remainder…

Understand addition, subtraction, multiplication and division as they apply to whole numbers and decimals.

Multiplication

Understand that:

- Multiplication is equivalent to and is more efficient than repeated addition.
- Because multiplication involves fewer calculations than addition, it is likely to be carried out more accurately.

Compare methods and accuracy in examples such as:

- Find the cost of 38 items at £1.99 each.

Conclude it is easier to calculate £2 × 38 then compensate by 38p than to add £1.99 a total of 38 times, or calculate 1.99 × 38.

Understand the effect of multiplying by 0 and 1.

Division

Recognise that:

- 910 ÷ 13 can be interpreted as 'How many 13s in 910?', and calculated by repeatedly subtracting 13 from 910, or convenient multiples of 13.
- Division by 0 is not allowed.
- A quotient (the result obtained after division) can be expressed as a remainder, a fraction or as a decimal, e.g.
  90 ÷ 13 = 6 R 12
  or 90 ÷ 13 = $6\frac{12}{13}$
  or 90 ÷ 13 = 6.92 (rounded to two decimal places)
  The context often determines which of these is most appropriate.

Decide in the context of a problem how to express and interpret a quotient – that is:

- whether to express it with a remainder, or as a fraction, or as a decimal;
- whether to round it up or down;
- what degree of accuracy is required.

For example:

- Four small cars cost a total of £48 623. What should a newspaper quote as a typical cost of a small car?
  An appropriate answer is rounded: about £12000 each.

- 107 pupils and staff need to be taken to the theatre. How many 15-seater minibuses should be ordered?
  $7\frac{2}{15}$ minibuses is not an appropriate answer for this example. To round $7\frac{2}{15}$ down to 7 would leave 2 people without transport. 8 minibuses is the appropriate answer.

- How many boxes of 60 nails can be filled with 340 nails?
  340 ÷ 60 = 5 R 40 or $5\frac{2}{3}$, but the appropriate answer is obtained by rounding down to 5, ignoring the remainder.

See Y456 examples (pages 52–7).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
associative, distributive… partition…

Understand the operations of addition, subtraction, multiplication and division as they apply to positive and negative numbers.

Link to integers (pages 48–51).

Understand the operations of addition and subtraction as they apply to fractions.

Link to fractions (pages 66–9).

Understand that multiplying does not always make a number larger and that division does not always make a number smaller.

Recognise that:

- 9.1 ÷ 0.1 can be interpreted as 'How many 0.1s (or tenths) in 9.1?'
- 9.1 ÷ 0.01 can be interpreted as 'How many 0.01s (or hundredths) in 9.1?'

Link to multiplying and dividing by 0.1 and 0.01 (pages 38–9).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
reciprocal…

Understand the effect of multiplying and dividing by numbers between 0 and 1.

Understand the operations of multiplication and division as they apply to fractions.

Link to fractions (pages 66–9).

Understand that multiplying a positive number by a number between 0 and 1 makes it smaller and that dividing it by a number between 0 and 1 makes it larger. Use this to check calculations and to estimate the order of magnitude of an answer.

Generalise inequalities such as:
if $p > 1$ and $q > 1$, then $pq > p$.

Know the effect on inequalities of multiplying and dividing each side by the same negative number.

Know and understand that division by zero has no meaning. For example, explore dividing a number by successively smaller positive decimals approaching zero, then negative decimals approaching zero.

Link to multiplying and dividing by any integer power of 10 (pages 38–9), checking results (pages 110–11), and inequalities (pages 130–1).

Recognise and use reciprocals. Know that:

- A number multiplied by its reciprocal equals 1, e.g. the reciprocal of 4 is $\frac{1}{4}$ and of 7 is $\frac{1}{7}$.
- The reciprocal of a reciprocal gives the original number.

Find the reciprocal of a number and use the reciprocal key on a **calculator**, recognising that the answer may be inexact. For example:

- What is the reciprocal of:
  a. 0.3 b. 27 c. 0.0027?
- A * stands in the place of any missing digit.
  The reciprocal of a whole number between 0 and 100 is 0.02*26, to four significant figures.
  Find the number, and the missing digit.
- The reciprocal of a whole number between 100 and 1000 is 0.0012**5, to five significant figures. Find the number, and the missing digits.

Link to reciprocal function sequences (pages 108–9).

Pupils should be taught to:

Consolidate understanding of the operations of multiplication and division, their relationship to each other and to addition and subtraction; know how to use the laws of arithmetic (continued)

As outcomes, Year 7 pupils should, for example:

When dividing using a **calculator**, interpret the quotient in the context of a problem involving money, metric measures or time.

3.05

For example, depending on the context:

- A display of '3.05' could mean £3.05, 3 kilograms and 50 grams, or 3 hours and 3 minutes.
- A display of '5.2' could mean £5.20, 5 metres and 20 centimetres, or 5 hours and 12 minutes.

Relate division to fractions. Understand that:

- $\frac{1}{4}$ of 3.6 is equivalent to $3.6 \div 4$.
- $7 \div 8$ is equivalent to $\frac{7}{8}$.
- $\frac{50}{3}$ is equivalent to $50 \div 3$.

See Y456 examples (pages 54–7).

Link to finding fractions of numbers (pages 66–7).

Know how to use the **laws of arithmetic** to support efficient and accurate mental and written calculations, and calculations with a **calculator**.

Examples of commutative law

$4 \times 7 \times 5 = 4 \times 5 \times 7 = 20 \times 7 = 140$
or $\quad = 7 \times 5 \times 4 = 35 \times 4 = 140$

To find the area of a triangle, base 5 cm and height 6cm:
area = $\frac{1}{2} \times 5 \times 6 = \frac{1}{2} \times 6 \times 5 = 3 \times 5 = 15\text{cm}^2$

Example of associative law

$15 \times 33 = (5 \times 3) \times 33$ or $5 \times (3 \times 33) = 5 \times 99 = 495$

Example of distributive law

$$\begin{aligned} 3.7 \times 99 &= 3.7 \times (100 - 1) \\ &= (3.7 \times 100) - (3.7 \times 1) \\ &= 370 - 3.7 \\ &= 366.3 \end{aligned}$$

Link to algebraic operations (pages 114–17), and mental calculations (pages 92–7).

Inverses

Understand that addition is the inverse of subtraction, and multiplication is the inverse of division. For example:

- Put a number in a **calculator**. Add 472 (or multiply by 26). What single operation will get you back to your starting number?
- Fill in the missing number: $(\square \times 4) \div 8 = 5$.

Use inverses to check results. For example:

- $703 \div 19 = 37$ appears to be about right, because $36 \times 20 = 720$.

Link to inverse operations in algebra (pages 114–15), and checking results (pages 110–11).

As outcomes, Year 8 pupils should, for example:

Continue to use the **laws of arithmetic** to support efficient and accurate mental and written calculations, and calculations with a **calculator**.

For example, use mental or informal written methods to calculate:

- 484 × 25 = 484 × 100 ÷ 4 = 48 400 ÷ 4 = 12100
- 3.15 × 25 = 3.15 × 100 ÷ 4 = 315 ÷ 4 = 78.75
- 28 × 5 = 28 × 10 ÷ 2 = 280 ÷ 2 = 140
- 15 × 8 = 15 × 2 × 2 × 2 = 120
- 6785 ÷ 25 = 6785 ÷ 5 ÷ 5 = 1357 ÷ 5 = 271.4

Recognise the application of the distributive law when multiplying a single term over a bracket in number and in algebra.

Link to algebraic operations (pages 114–17), and mental calculations (pages 92–7).

Inverses

Use inverse operations. For example:

- Put a number in your **calculator**.
  Add 46, then multiply by 17.
  What must you do to get back to the starting number?
- Fill in the missing number:
  $\square^2 \div 4 = 16$

Use inverses to check results. For example:

- 6603 ÷ 18.6 = 355 appears to be about right, because 350 × 20 = 7000.

Link to inverse operations in algebra (pages 114–15), and checking results (pages 110–11).

As outcomes, Year 9 pupils should, for example:

Continue to use the **laws of arithmetic** to support efficient and accurate mental and written calculations, and calculations with a **calculator**.

Recognise the application of the distributive law when expanding the product of two linear expressions in algebra.

Link to algebraic operations (pages 114–21), and mental calculations (pages 92–7).

Inverses

Use inverse operations. For example:

- Put a number in your **calculator**. Cube it. What must you do to get back to the starting number?
- Put a number in your **calculator**. Find the square root. What must you do to get back to the starting number?

Explain why it may not be possible to get back exactly to the starting numbers using a calculator.

Use inverses to check results of calculations.

Link to algebraic operations (pages 114–21), and checking results (pages 110–11).

Pupils should be taught to:

Know and use the order of operations, including brackets

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
order of operations, brackets…

Know the conventions that apply when evaluating expressions:

- Contents of brackets are evaluated first.
- In the absence of brackets, multiplication and division take precedence over subtraction and addition.
- A horizontal line acts as a bracket in expressions such as $\frac{5+6}{2}$ or $\frac{a+b}{5}$.

brackets
⇩
powers or indices
⇩
multiplication (including 'of') and division
⇩
addition and subtraction

- With strings of multiplications and divisions, or strings of additions and subtractions, and no brackets, the convention is to work from left to right, e.g.
  $12 \div 4 \div 2 = 1.5$, not 6.

Calculate with mixed operations. For example:

- Find mentally or use jottings to find the value of:
  a. $16 \div 4 + 8 = 12$
  b. $16 + 8 \div 4 = 18$
  c. $14 \times 7 + 8 \times 11 = 186$
  d. $\frac{100}{4 \times 5} = 5$
  e. $32 + 13 \times 5 = 97$
  f. $(3^2 + 4^2)^2 = 625$
  g. $(5^2 - 7) / (2^2 - 1) = 6$

- Use a **calculator** to calculate with mixed operations, e.g.
  $(32 + 13) \times (36 - 5) = 1395$

- In algebra recognise that, for example, when $a = 4$,
  $3a^2 = 3 \times 4^2 = 3 \times 16 = 48$

Link to calculator methods (pages 108–9), order of algebraic operations (pages 114–15), and substitution in expressions and formulae (pages 138–41).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

Recognise that, for example:

$$\frac{100}{4 \times 5} = 100 \div 4 \div 5 = 5$$

or $$\frac{a}{b \times c} = a \div (b \times c) \text{ or } a \div b \div c$$

Calculate with more complex mixed operations, including using the bracket keys on a **calculator**. For example:

- Find the value of:
  a. $2.1 - (3.5 + 2.1) + (5 + 3.5) = 5$
  b. $\frac{(2 + 3)^2}{(14 - 9)^2} = \frac{5^2}{5^2} = 1$

- Find, to two decimal places, the value of:
  a. $(5.5 + 2)/7 = 1.07$ to 2 d.p.
  b. $\frac{8 + 4}{13 - 2} = 1.09$ to 2 d.p.
  c. $\frac{25}{6 \times 93} = 0.04$ to 2 d.p.
  d. $\sqrt{(26^2 - 14^2)} = 21.91$ to 2 d.p.

Evaluate expressions using nested brackets, such as:
$120 \div \{30 - (2 - 7)\}$

Understand that the position of the brackets is important. For example:
- Make as many different answers as possible by putting brackets into the expression
  $3 \times 5 + 3 - 2 \times 7 + 1$
  For example:
  a. $3 \times (5 + 3) - (2 \times 7) + 1 = 11$
  b. $3 \times (5 + 3) - 2 \times (7 + 1) = 8$
  c. $(3 \times 5) + 3 - (2 \times 7) + 1 = 5$
  d. $(3 \times 5) + (3 - 2) \times 7 + 1 = 23$
  e. $(3 \times 5) + (3 - 2) \times (7 + 1) = 23$
  f. $(3 \times 5) + 3 - (2 \times 7 + 1) = 3$

Link to calculator methods (pages 108–9), order of algebraic operations (pages 114–15), substitution in expressions and formulae (pages 138–41).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years.

Understand the effect of powers when evaluating an expression. For example:

- Find the value of:
  a. $36 \div (3 + 9) - 7 + 3 \times (8 \div 2)^3 = 188$
  b. $\frac{7 \times 8^2}{7 \times 2} = \frac{8^2}{2} = 32$
  c. $\frac{(7 \times 8)^2}{7 \times 2} = \frac{7 \times 8 \times 7 \times 8}{7 \times 2} = 7 \times 8 \times 4 = 224$
  d. $-7^2 + 5 = {}^{-}44$
  e. $({}^{-}7)^2 + 5 = 54$
  f. $(4/3)^2 = 4^2 \div 3^2 = 16/9 = 1\,7/9$

Calculate with more complex mixed operations, including using the bracket keys on a **calculator**. For example:

- Find the value of:
  $-(251 \times 3 + 281) + 3 \times 251 - (1 - 281) = {}^{-}1$

- Find, to two decimal places, the value of:
  a. $\frac{(12 - 5)^2(7 - 3)^2}{(8 - 5)^3} = \frac{7^2 \times 4^2}{3^3} = 29.04$ to 2 d.p.
  b. $\frac{(16 - 9)^2(17 - 15)^2}{3(16 - 11)^3} = \frac{7^2 \times 2^2}{3 \times 5^3} = 0.52$ to 2 d.p.

- In algebra recognise that when $a = 2$,
  a. $3a^2 - 9 = 3(2^2) - 9 = 3$
  b. $3(a^2 - 9) = 3(4 - 9) = {}^{-}15$
  c. $(3a)^2 - 9 = 6^2 - 9 = 27$

  Recognise that $({}^{-}a)^2 \neq -a^2$.

Link to calculator methods (pages 108–9), order of algebraic operations (pages 114–15), substitution in expressions and formulae (pages 138–41).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate the rapid recall of number facts and use known facts to derive unknown facts

Use, read and write, spelling correctly:
increase, decrease, double, halve, complement, partition…

Addition and subtraction facts
Know with rapid recall addition and subtraction facts to 20.

Complements
Derive quickly:
- whole-number complements in 100 and 50,
  e.g. 100 = 63 + 37, 50 = $^{-}$17 + 67
- decimal complements in 1 (one or two decimal places),
  e.g. 1 = 0.8 + 0.2, 1 = 0.41 + 0.59

Doubles and halves
Derive quickly:
- doubles of two-digit numbers including decimals,
  e.g. 23 × 2, 3.8 × 2, 0.76 × 2
- doubles of multiples of 10 to 1000,
  e.g. 670 × 2, 830 × 2
- doubles of multiples of 100 to 10 000,
  e.g. 1700 × 2, 6500 × 2

and all the corresponding halves.

Multiplication and division facts
Know with rapid recall multiplication facts up to 10 × 10, and squares to at least 12 × 12.
Derive quickly the associated division facts, e.g. 56 ÷ 7, √81.

Use knowledge of place value to multiply and divide mentally any number by 10, 100, 1000, or by a small multiple of 10.
For example:
- 4.3 × 100
- 60 ÷ 1000
- 1.6 × 20 = 16 × 2 = 32
- ☐ ÷ 100 = 4.7

Use knowledge of multiplication facts and place value
to multiply mentally examples such as:
- 0.2 × 8
- 0.04 × 9
- 8 × 0.5
- 7 × 0.03
- ☐ × 0.2 = 10
- 80 × ☐ = 8

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

Use known facts to derive unknown facts
For example, generate constant-step sequences, such as:
- Start at 108, the rule is 'add 8'.
- The start number is 5, target is 33. What is the rule?

Complements
Derive quickly:
- complements in 1, 10, 50, 100, 1000.

Solve mentally equations such as:
- 100 = x + 37
- 50 – x = 28
- 10 = 3.62 + x
- 220 = 1000 – x

Doubles and halves
Use doubling and halving methods to multiply and divide by powers of 2. For example:
- 18 × 16 = 18 × 2 × 2 × 2 × 2
- 180 ÷ 8 = 180 ÷ 2 ÷ 2 ÷ 2

Link to using the laws of arithmetic (pages 84–5).

Multiplication and division facts
Derive the product and quotient of multiples of 10 and 100 (whole-number answers). For example:
- 30 × 60
- 900 × 20
- 1400 ÷ 700
- 6300 ÷ 30

Use knowledge of place value to multiply and divide whole numbers by 0.1 and 0.01. For example:
- 47 × 0.1
- 9 × 0.01
- 432 × 0.01
- 8 ÷ 0.1
- 16 ÷ 0.1
- 37 ÷ 0.01

Extend to decimals, such as:
- 0.5 × 0.1
- 0.1 × ☐ = 0.08
- 5.2 ÷ 0.01
- ☐ ÷ 0.01 = 3

Use knowledge of multiplication and division facts and place value to:
derive products involving numbers such as 0.4 and 0.04. For example:
- 4 × 0.6 = 4 × 6 ÷ 10 = 24 ÷ 10 = 2.4
- 0.7 × 0.9 = 7 × 9 ÷ 100 = 0.63
- 0.04 × 8 = 4 × 8 ÷ 100 = 0.32
- ☐ × ☐ = 0.08

divide mentally by 2, 4 and 5. For example:
- 0.2 ÷ 4 = 2 ÷ 4 ÷ 10 = 0.5 ÷ 10 = 0.05
- 0.03 ÷ 5 = 3 ÷ 5 ÷ 100 = 0.6 ÷ 100 = 0.006
- ☐ ÷ ☐ = 0.4

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years.

Use known facts to derive unknown facts
For example:
- Derive 36 × 24 from 36 × 25.

Multiplication and division facts
Derive the product and quotient of multiples of 10, 100 and 1000. For example:
- 600 × 7000
- 48 000 ÷ 800
- 400 ÷ 8000
- 60 ÷ 90000

Use knowledge of place value to multiply and divide decimals by 0.1 and 0.01. For example:
- 0.47 × 0.1
- 9.6 × 0.01
- 0.0432 × 0.01
- 0.01 × ☐ = 1.7
- 0.8 ÷ 0.1
- 0.016 ÷ 0.1
- 3.7 ÷ 0.01
- ☐ ÷ 0.01 = 3.2

Consolidate knowledge of multiplication and division facts and place value to multiply and divide mentally. For example:
- 0.24 × 0.4 = 24 × 4 ÷ 1000 = 96 ÷ 1000 = 0.096
- 800 × 0.7 = 80 × 7 = 56 × 10 = 560
- 72 ÷ 0.9 = 72 ÷ 9 × 10 = 8 × 10 = 80
- 0.48 ÷ 0.6 = 4.8 ÷ 6 = 48 ÷ 6 ÷ 10 = 8 ÷ 10 = 0.8
- 720 ÷ 0.03 = 72 000 ÷ 3 = 24 000
- ☐ × ☐ × ☐ = 0.08

Link to using the laws of arithmetic (pages 84–5).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate the rapid recall of number facts and use known facts to derive unknown facts (continued)

Factors, powers and roots

Know or derive quickly:

- prime numbers less than 30;
- squares of numbers 0.1 to 0.9, and of multiples of 10 to 100, and the corresponding roots;
- pairs of factors of numbers to 100.

Calcuate mentally:

- $4^2 + 9$
- $(4 + 3)^2$
- $4^2 + 5^2$
- $5^2 - 7$
- $\sqrt{(9 + 7)}$
- $\sqrt{(40 - 2^2)}$
- What is the fourth square number?

Solve mentally:

- $3a = 15$
- $x^2 = 49$
- $n(n + 1) = 12$

Link to multiples, factors and primes (pages 52–5), and powers and roots (pages 56–9).

Fraction, decimal and percentage facts

See pages 70–1.

Measurements

Recall and use formulae for:

- the perimeter and area of a rectangle.

Calculate simple examples mentally.

Recall:

- relationships between units of time;
- relationships between metric units of length, mass and capacity (e.g. between km, m, cm and mm).

Convert between units of measurement. For example:

- Convert 38 cm into mm.
- Convert 348p into pounds.
- Convert 45 minutes into seconds.

See Y456 examples (pages 38–9, 58–9, 90–3).

Link to measures and mensuration (pages 228–31).

As outcomes, Year 8 pupils should, for example:

Factors, powers and roots

Know or derive quickly:

- cubes of numbers from 1 to 5, and 10, and the corresponding roots;
- the prime factorisation of numbers to 30.

Calcuate mentally:

- $\sqrt{(24 + 12)}$
- $\sqrt{(89 - 25)}$
- $(7 + 4)^2$
- $(12 + 9 - 18)^2$

Solve mentally:

- $3a - 2 = 31$
- $n(n - 1) = 56$

Link to multiples, factors and primes (pages 52–5), and powers and roots (pages 56–9).

Fraction, decimal and percentage facts

See pages 70–1.

Measurements

Recall and use formulae for:

- the perimeter and area of a rectangle;
- the area of a triangle;
- the volume of a cuboid.

Calculate simple examples mentally.

Know and use rough metric equivalents for:

1 mile, 1 yard, 1 pound (lb), 1 gallon, 1 pint,

and rough imperial equivalents for:

1 km, 1m, 1kg, 1 litre.

For example, use 5 miles ≈ 8 kilometres to work out:

- The signpost said that it was 50 miles to London. How many kilometres is that, approximately?
- The jogger was pleased that she had run 32 km. About how many miles is this?

Convert between units of time. For example:

- How many minutes in: 3 hours, 4.5 hours, 2.25 hours, 5 hours 25 minutes?
- Change to hours and minutes: 120 minutes, 75 minutes, 300 minutes.
- How many hours in: 3 days, $5\frac{1}{4}$ days, 1 week 2 days, …?
- How many days in: 36 hours, 100 hours, the last 3 months of the year?
- How many days to Christmas? Your birthday?

Link to measures and mensuration (pages 228–31).

As outcomes, Year 9 pupils should, for example:

Factors, powers and roots

Find mentally:

- the HCF and LCM of pairs of numbers such as 36 and 48, 27 and 36;
- products of small integer powers, such as $3^3 \times 4^2 = 27 \times 16 = 432$;
- factor pairs for a given number.

Calcuate mentally:

- $(23 - 15 + 4 - 8)^3$
- $\sqrt[3]{(89 + 36)}$

Solve mentally:

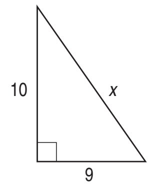
- $(3 + x)^2 = 25$
- $(12 - x)^2 = 49$

Identify numbers from property questions, such as:

- This number is a multiple of 5. It leaves remainder 1 when divided by 4. What could it be?
- This number has a digit sum of 6. It is divisible by 7. What is it?

Know simple Pythagorean triples such as 3, 4, 5, or 5, 12, 13, and their multiples.

- Apply Pythagoras' theorem:
  $x^2 = 9^2 + 10^2 = 181$
  $x = \sqrt{181}$



Measurements

Recall and use formulae for:

- the perimeter of a rectangle and circumference of a circle;
- the area of a rectangle, triangle, parallelogram, trapezium, circle;
- the volume of a cuboid and a prism.

Calculate simple examples mentally.

Link to measures and mensuration (pages 228–31), and use of compound measures in science.

Know that speed = distance/time.
Use this to derive facts from statements such as:

- A girl takes 20 minutes to walk to school, a distance of 1.5 km. Find her average speed in km/h.

Solve problems such as:

- £1 is equivalent to 1.65 euros. £1 is also equivalent to 1.5 US dollars ($1.5). How many euros are equivalent to $6?
- A car travels 450km on 50 litres of fuel. How many litres of fuel will it use to travel 81km?

Link to speed and solving problems involving constant rates of change (pages 232–3).

Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings

As outcomes, Year 7 pupils should, for example:

Strategies for mental addition and subtraction

Count forwards and backwards from any number.
For example:
- Count on in 0.1s from 4.5.
- Count back from 4.05 in 0.01s.
- Count on from and back to zero in steps of ¾.

Identify positions of 0.1s and 0.01s on a number line.

Use a **spreadsheet** to replicate cells, e.g. to 'count' from 1 in steps of 1.

	A	B	C	D	E	F	G
1	1	= A1+1	= B1+1	= C1+1	= D1+1	= E1+1	= F1+1

	A	B	C	D	E	F	G
1	1	2	3	4	5	6	7

Modify the spreadsheet to count from 0.5 in steps of 0.1.

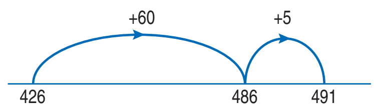
Add and subtract several small numbers.
For example:
- 4 + 8 + 12 + 6 + 13
- 5 – 4 + 8 – 10 – 7

Extend to adding and subtracting several small multiples of 10:
- 40 + 30 + 20
- 60 + 50 – 30

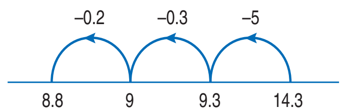
Continue to add and subtract any pair of two-digit whole numbers, such as 76 + 58, 91 – 47.
Extend to:
- adding and subtracting a two-digit whole number to or from a three-digit whole number;
- adding and subtracting decimals such as:
  8.6 ± 5.7   0.76 ± 0.58   0.82 ± 1.5
  by considering
  86 ± 57   76 ± 58   82 ± 150

Use jottings such as an empty number line to support or explain methods for adding and subtracting mentally. Choose an appropriate method, such as one of the following:

Partition and deal with the most significant digits first.
For example:
- 426 + 65 = (426 + 60) + 5 = 486 + 5 = 491



- 14.3 – 5.5 = 14.3 – 5 – 0.3 – 0.2 = 9 – 0.2 = 8.8



As outcomes, Year 8 pupils should, for example:

Strategies for mental addition and subtraction

Consolidate and use addition and subtraction strategies from previous years. For example:

Add and subtract mentally pairs of integers.
Use strategies for addition and subtraction to add and subtract pairs of integers. For example:

- $^-3 + {}^-5 = \dots$ $\quad$ $^-13 + {}^-25 = \dots$
  $^-46 + {}^-59 = \dots$ $\quad$ $^-100 + {}^-99 = \dots$
  $^-9 - {}^-14 = \dots$ $\quad$ $^-43 - {}^-21 = \dots$
  $^-37 - {}^-25 = \dots$ $\quad$ $^-7 - {}^-7 = \dots$

- The result of subtracting one integer from another is $^-29$.
  What could the integers be?

- □ + □ = $^-46$

Add mentally several positive or negative numbers, including larger multiples of 10. For example:
- $5 + {}^-4 + 8 + {}^-10 + {}^-7$
- $250 + 120 - 190$

Calculate a mean using an assumed mean.
For example:
- Find the mean of 18.7, 18.4, 19.1, 18.3 and 19.5.
  Use 19.0 as the assumed mean.
  The differences are $^-0.3$, $^-0.6$, 0.1, $^-0.7$ and 0.5, giving a total difference of $^-1.0$.
  The actual mean is $19.0 - (1.0 \div 5) = 18.8$.

Link to integers (pages 48–9).

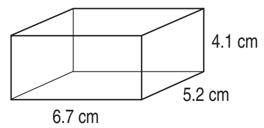
Add and subtract pairs of numbers of the same order (both with two significant figures). For example:
- 360 + 250
- 4800 – 1900
- 7.8 + 9.3
- 0.081 – 0.056

As outcomes, Year 9 pupils should, for example:

Strategies for mental addition and subtraction

Consolidate and use addition and subtraction strategies from previous years. For example:

- Find the length of wire in this framework.



$4(6.7) + 4(5.2) + 4(4.1) = 4 \times 16 = 64$ cm

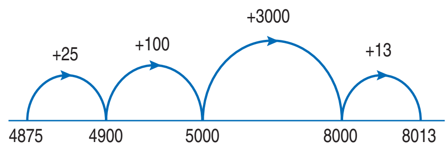
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

Mental addition and subtraction strategies (continued)

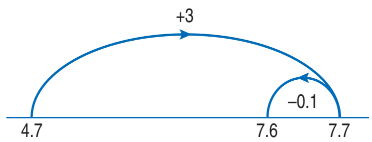
Find a difference by counting up from the smaller to the larger number. For example:

- 8013 – 4875 = 25 + 100 + 3000 + 13 = 3138

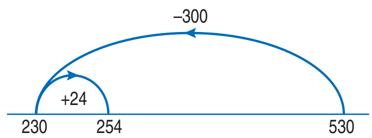


Use compensation, by adding or subtracting too much, and then compensating. For example:

- 4.7 + 2.9 = 4.7 + 3 – 0.1 = 7.7 – 0.1 = 7.6



- 530 – 276 = 530 – 300 + 24 = 230 + 24 = 254



Recognise special cases. For example:

Near doubles
- 8.5 + 8.2 = 16.7 (double 8.2 plus 0.3)
- 427 + 366 = 793 (double 400 plus 27 minus 34)

'Nearly' numbers
Add and subtract near 10s and near 100s, by adding or subtracting a multiple of 10 or 100 and adjusting. For example:

• 48 + 39	• 84 – 29
• 92 + 51	• 70 – 51
• 76 + 88	• 113 – 78
• 427 + 103	• 925 – 402
• 586 + 278	• 350 – 289

Use the relationship between addition and subtraction.
For example, recognise that knowing one of:

2.4 + 5.8 = 8.2	5.8 + 2.4 = 8.2
8.2 – 5.8 = 2.4	8.2 – 2.4 = 5.8

means that you also know the other three.

See Y456 examples (pages 40–7).

As outcomes, Year 8 pupils should, for example:	As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

Strategies for multiplication and division

Use factors. For example:

- 3.2 × 30 — 3.2 × 10 = 32; 32 × 3 = 96
- 156 ÷ 6 — 156 ÷ 3 = 52; 52 ÷ 2 = 26

Use partitioning. For example:

For multiplication, partition either part of the product:

- 7.3 × 11 = (7.3 × 10) + 7.3
  = 73 + 7.3
  = 80.3

For division, partition the dividend (the number that is to be divided by another):

- 430 ÷ 13 — 400 ÷ 13 = 30 R 10
  30 ÷ 13 = 2 R 4
  430 ÷ 13 = 32 R 14
  = 33 R 1

Recognise special cases where doubling or halving can be used. For example:

To multiply by 50, first multiply by 100 and then divide by 2. For example:

- 1.38 × 50 — 1.38 × 100 = 138; 138 ÷ 2 = 69

Double one number and halve the other. For example:

- 6 × 4.5 — 3 × 9 = 27
  12 × 7.5 — 6 × 15 = 3 × 30 = 90

Use the relationship between multiplication and division.
For example, knowing one of these facts means you also know the other three:

2.4 × 3 = 7.2	3 × 2.4 = 7.2
7.2 ÷ 3 = 2.4	7.2 ÷ 2.4 = 3

See Y456 examples (pages 60–5).

As outcomes, Year 8 pupils should, for example:

Strategies for multiplication and division

Use factors. For example:

- 22 × 0.02 — 22 × 0.01 = 0.22; 0.22 × 2 = 0.44
- 420 ÷ 15 — 420 ÷ 5 = 84; 84 ÷ 3 = 28
- 126 ÷ 18 — 126 ÷ 6 = 21; 21 ÷ 3 = 7

Use partitioning. For example, for multiplication, partition either part of the product:

- 13 × 1.4 = (10 × 1.4) + (3 × 1.4) = 14 + 4.2 = 18.2
- 7.3 × 21 = (7.3 × 20) + 7.3 = 146 + 7.3 = 153.3

Use knowledge of place value to multiply and divide mentally any number by 0.1 or 0.01. For example:

- 3.6 × 0.1 — 3.6 ÷ 10
- 99.2 × 0.01 — 99.2 ÷ 100
- $^{-}1.8$ ÷ 0.1 — $^{-}1.8$ × 10
- 0.35 ÷ 0.01 — 0.35 × 100

99.2 × 0.01   0.992
× 10 ↓   ↓ × 100   ↑ ÷1000
992 × 1 = 992

Recognise special cases where doubling or halving can be used. For example:

Extend doubling and halving methods to include decimals and negative numbers. For example:

- 3.4 × 4.5 — 1.7 × 9 = 15.3
- 8.12 × 2.5 — 4.06 × 5 = 20.3
- 22 × 3.01 — 11 × 6.02 = 66.22
- $^{-}17$ × 1.5 — $^{-}8.5$ × 3 = $^{-}25.5$
- $^{-}8.4$ × $^{-}1.25$ — $^{-}4.2$ × $^{-}2.5$ = $^{-}2.1$ × $^{-}5$ = 10.5

Multiply by near 10s.
For example:

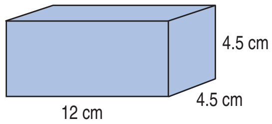
- 23 × 11 = 230 × 10 + 23 = 253
- 75 × 29 = 75 × 30 – 75 = 2175
- 8 × $^{-}19$ = 8 × ($^{-}20$ + 1) = $^{-}160$ + 8 = $^{-}152$

As outcomes, Year 9 pupils should, for example:

Strategies for multiplication and division

Consolidate and use multiplication and division strategies from previous years. For example:

- Find the volume of this square-based cuboid.



4.5 × 4.5 × 12 $= \frac{9}{2} \times \frac{9}{2} \times 12$
= 9 × 9 × 3
= 243 cm^3

or

4.5 × 4.5 × 12 = (4.5 × 12) × 4.5
= 54 × 4.5
= 216 + 27
= 243 cm^3

Pupils should be taught to:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

As outcomes, Year 7 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know or derive quickly:

- simple decimal/fraction/percentage equivalents, such as:
  - $\frac{1}{4} = 0.25$ or 25% — 0.23 is equivalent to 23%
  - $\frac{7}{10} = 0.7$ or 70% — 57% is equivalent to 0.57 or $\frac{57}{100}$
- simple addition facts for fractions, such as:
  - $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ — $\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$
- some simple equivalent fractions for $\frac{1}{4}$ and $\frac{1}{2}$, such as:
  - $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{50}{100}$
  - $\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{16} = \frac{5}{20} = \frac{25}{100}$

Strategies for finding equivalent fractions, decimals and percentages

For example:

- Convert $\frac{1}{8}$ into a decimal.
  (Know that $\frac{1}{4} = 0.25$ so $\frac{1}{8}$ is $0.25 \div 2 = 0.125$.)
- Express $\frac{3}{5}$ as a percentage.
  (Know that $\frac{3}{5} = \frac{6}{10}$ or $\frac{60}{100}$, so it is equivalent to 60%.)
- Express 23% as a fraction and as a decimal.
  (Know that 23% is equivalent to $\frac{23}{100}$ or 0.23.)
- Express 70% as a fraction in its lowest terms.
  (Know that 70% is equivalent to $\frac{70}{100}$, and cancel this to $\frac{7}{10}$.)

Use known facts such as $\frac{1}{5} = 0.2$ to convert fractions to decimals mentally. For example:

$\frac{3}{5} = 0.2 \times 3 = 0.6$

Find simple equivalent fractions.
For example:

- State three fractions equivalent to $\frac{3}{5}$, such as:
  $\frac{6}{10}$, $\frac{30}{50}$, $\frac{24}{40}$
- Fill in the boxes:
  $\frac{3}{4} = \frac{\square}{8} = \frac{\square}{12} = \frac{\square}{16} = \frac{\square}{20}$
  $\frac{7}{\square} = \frac{21}{30}$

Strategies for calculating fractions and percentages of whole numbers and quantities. For example:

- $\frac{1}{8}$ of 20 = 2.5 (e.g. find one quarter, halve it)
- 75% of 24 = 18 (e.g. find 50% then 25% and add the results)
- 15% of 40 (e.g. find 10% then 5% and add the results)
- 40% of 400 kg (e.g. find 10% then multiply by 4)

- 60 pupils go to the gym club.
  25% of them are girls.
  How many are boys?

See Y456 examples (pages 24–5, 32–3).

Link to finding fractions and percentages of quantities

As outcomes, Year 8 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know or derive quickly:

- decimal/fraction/percentage equivalents such as:
  $\frac{1}{8}$ = 0.125 or $12\frac{1}{2}$% $\frac{3}{5}$ = 0.6 or 60%
  $1\frac{3}{4}$ = 1.75 or 175% $\frac{1}{3} \approx$ 0.33 or $33\frac{1}{3}$%
- the simplified form of fractions such as:
  $\frac{3}{15} = \frac{1}{5}$ $\frac{14}{21} = \frac{2}{3}$

Know that $\frac{1}{3}$ is $0.\dot{3}$ and $\frac{2}{3}$ is $0.\dot{6}$.

Know that 0.03 is $\frac{3}{100}$ or 3%.

Strategies for finding equivalent fractions, decimals and percentages

For example:

- Express 136% as a decimal.
  (Know that 136% is equivalent to $\frac{136}{100}$ or 1.36.)
- Express 55% as a fraction in its lowest terms.
  (Know 55% is equivalent to $\frac{55}{100}$, cancel to $\frac{11}{20}$.)
- Express $\frac{13}{20}$ as a percentage.
  (Work out that $\frac{13}{20} = \frac{65}{100}$, so it is equivalent to 65%.)
- Convert $\frac{4}{25}$ into a decimal.
  (Work out that $\frac{4}{25} = \frac{16}{100}$, so it is equivalent to 0.16.)
- Convert 0.45 into a fraction.
  (Know that 0.45 = $\frac{45}{100}$, and simplify this to $\frac{9}{20}$.)
- Express 0.06 as a percentage.
  (Recognise this as $\frac{6}{100}$ or 6%.)

Use known facts such as $\frac{1}{8}$ = 0.125 to convert fractions to decimals mentally. For example:
$\frac{5}{8}$ = 0.125 × 5 = 0.625

Convert between improper fractions and mixed numbers. For example:

- Convert $7\frac{1}{3}$ into an improper fraction.
- Convert $\frac{36}{5}$ into a mixed number.

Strategies for calculating fractions and percentages of whole numbers and quantities. For example:

- $\frac{3}{5}$ of 20 = 12   (e.g. find one fifth, multiply by 3)
- $1\frac{1}{2}$ of 16 = 24   (e.g. find one half, add it to 16)
- 125% of 240   (e.g. find 25%, add it to 240)
- 35% of 40   (e.g. find 10% then 30% then 5%, add the last two results)

- There is a discount of 5% on a £45 coat in a sale. By how much is the coat's price reduced?
  (e.g. 1% = 45p   so 5% = (45 × 5)p = £2.25
  or   10% = £4.50   so 5% = £2.25)

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

As outcomes, Year 9 pupils should, for example:

Recall of fraction, decimal and percentage facts

Know that 0.005 is half of one per cent,
so that   37.5% = 37% + 0.5%
or   0.37 + 0.005 = 0.375

Strategies for finding equivalent fractions, decimals and percentages

For example:

- Express 0.625 as an equivalent percentage.
  (Recognise this as 62%, plus half of one per cent, or 62.5%.)
- Express 10.5 as an equivalent percentage.
  (Recognise this as 1000% plus 50%, or 1050%.)

Simplify fractions by cancelling highest common factors mentally. For example:

- Simplify:
  $\frac{85}{100}$ $\frac{630}{720}$

Strategies for calculating fractions and percentages of numbers and quantities. For example:

- $\frac{2}{5}$ of 20.5 = 8.2
  (e.g. find one fifth, multiply by 2)
- $\frac{3}{8}$ of 6400 = 2400
  (e.g. find one eighth, multiply by 3)

- Find 20% of £3.50.
- Find 35% of £5.
- Increase 480 kg by 20%.
- Decrease 500mm by 12%.
- 25% of a number is 12. What is the number?

Link to finding fractions and percentages of quantities (pages 66–7, 72–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Consolidate and extend mental methods of calculation, accompanied where appropriate by suitable jottings (continued)

Word problems and puzzles (all four operations)

Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

- Arrange the digits 3, 5 and 2 to make the largest possible odd number.
- Write in figures the number two and a quarter million.
- A girl scored 67 in her first innings and 128 in her second innings. What was her total score?
- Pencils cost 37p each.
  How many pencils can you buy with £3.70?
- A 55 g bag of crisps has 20% fat. How much fat is that?
- A boy saved £215. He bought a Walkman for £69.
  How much money did he have left?
- A girl used 2 metres of wood to make 5 identical shelves.
  How long was each shelf?
- Estimate the value of 51 × 19.
- Find two numbers whose sum is 14 and whose product is 48.
- There are 12 green buttons and 4 white buttons in a tin.
  I choose one button at random from the tin.
  What is the probability it is a white button?

Written questions

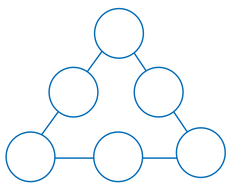
- Sandy and Michael dug a neighbour's garden.
  They were paid £32 to share for their hours of work.
  Sandy worked for 6 hours. Michael worked for 2 hours.
  How much should Sandy get paid?
- The mean of a, b and c is 6. a is 5 and b is 11. What is c?
- Tony, David and Estelle are playing a team game.
  They need to get a mean of 75 points to win.
  Tony scores 63 points, Estelle scores 77 points and David scores 77 points. Have they scored enough points to win?
- What is the value of 6n + 3 when n = 2.5?

Solve problems or puzzles such as:

- Three consecutive integers add up to 87.
  What are they?

- Choose from 1, 2, 3, 4 and 5 to place in the boxes.
  In any question, you cannot use a number more than once.
  a. □ – □ + □ = 5
  b □ + □ – □ = 4
  c. □ × □ – □ = 3
  d. (□ + □) ÷ □ = 2
  e. (□ + □) ÷ (□ + □) = 1

- Use each of the numbers
  1, 2, 4, 6, 8, 12 once.
  Write one number in each circle.
  The product of the three numbers on each side of the triangle must be 48.



- Write any number up to 40.
  Multiply its last digit by 4 then add the other digit to this.
  Repeat the process until you get back to the original.
  What is the longest chain you can make?

As outcomes, Year 8 pupils should, for example:

Word problems and puzzles (all four operations)

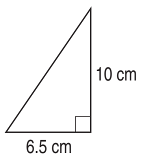
Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

- Write in figures the number that is one less than seven and a half million.
- Two angles fit together to make a straight line. One of them is 86°. What is the other?
- 1 ounce is about 28 grams. About how many grams are 3 ounces?
- Four oranges cost 37p. What do 12 oranges cost?
- How many metres are there in 2.5 kilometres?
- A person's heart beats 70 times in 1 minute. How many times does it beat in 1 hour?
- Carpet tiles are 50 cm by 50cm. How many are needed to cover one square metre?
- Estimate the value of 502 ÷ 49.
- Solve 45 + x = 92.
- The probability that it will rain in August is 0.05. What is the probability it will not rain in August?

Written questions

- 14 × 39 = 546. What is 14 × 3.9?
- Four sunflowers have heights of 225 cm, 199cm, 185cm and 239cm. What is their mean height?
- The sum of p and q is 12. The product of p and q is 27. Calculate the values of p and q.
- Find 25% of 10% of £400.
- Calculate the area of this triangle.



Solve problems or puzzles such as:

- Make 36 using any combination of +, –, ×, ÷, and brackets, and each of 1, 3, 3 and 5 once.
- The numbers 3 and 10 are written on the front of two cards. There is a different number on the back of each card.
  When the two cards are on the table, the sum of the two numbers showing is 12, 13, 14 or 15.
  What two numbers are on the back of the cards?
- Use each of the digits 1, 2, 3, 4, 5, 6, 7 once. Write them in the boxes to make this statement true:
  □□ + □□ + □□ + □ = 100
- Take a pair of consecutive integers.
  Square each of them.
  Find the difference of the two squares.
  Repeat with different pairs of consecutive integers.
  Repeat with a pair of numbers whose difference is 2, or 3, or 4 …

As outcomes, Year 9 pupils should, for example:

Word problems and puzzles (all four operations)

Apply mental skills to solving simple problems, using jottings if appropriate. For example:

Oral questions

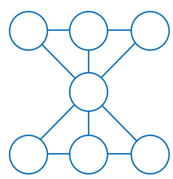
- Two years ago Jim's height was 1.48 metres. Now Jim's height is 1.7 metres. How much has Jim grown?
- Two of the angles of a triangle are 47° and 85°. Calculate the third angle.
- You get $56 for £40. How many dollars do you get for £100?
- 75 miles per hour is about the same as 33 metres per second. About how many metres per second is 50 miles per hour?
- In a raffle, half of the tickets are bought by men. One third are bought by women. The rest are bought by children. What fraction of the tickets are bought by children?
- The ratio of men to women in a room is 3 to 5. There are 12 men. How many women are there?
- x = 2 and y = 3. Work out the value of x to the power y plus y to the power x.

Written questions

- The probability that a train will be late is 0.03. Of 50 trains, how many would you expect to be late?
- Find 1% of 2% of £1000.
- Some girls and boys have £32 between them. Each boy has £4 and each girl has £5. How many boys are there?

Solve problems or puzzles such as:

- You can use four 8s to make 10, e.g. (8 + 8)/8 + 8. Using any of +, –, ×, ÷ and brackets, and eight 8s, make the number 1000.
- Find two numbers:
  whose sum is 0.8 and whose product is 0.15;
  whose sum is $^{-}11$ and whose product is 28;
  whose difference is 4 and whose quotient is 3;
  whose difference is 2 and whose quotient is $^{-}1$.
- The product of two numbers is six times their sum. The sum of their squares is 325. What are the two numbers?
- Use each of the prime numbers 5, 7, 11, 13, 17, 19, 23 once. Write one in each circle so that the three primes in each line add up to the same prime number.



Pupils should be taught to:

Make and justify estimates and approximations (of numbers and calculations)

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
guess, estimate, approximate, roughly, nearly, approximately, too many, too few, enough, not enough… and the symbol ≈.

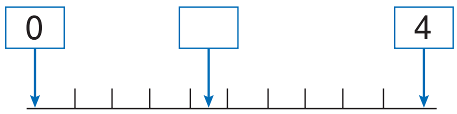
Understand that there are occasions when there is no need to calculate an exact answer and an estimate is sufficient.

Understand that the context affects the method used for estimating. For example:

- If I have £10 and some shopping to do, I need to round the amounts up in order to check I have enough money.
- If I estimate how much paint I need to paint a room, I need to round up so I have enough paint.

Estimate the position of a point on a marked scale, given the values of the end points. For example:

- Estimate the number that the arrow is pointing to:



if the end points of the scale are 0 and 4;
if the end points are $^{-}5$ and 5;
if the end points are 2.7 and 4.7.

Know that there are different ways for finding an approximate answer. For example:

- An approximate answer for 404 – 128 can be
  400 – 100 = 300 or 400 – 130 = 270
  Which is the better estimate?
- An approximate answer for 7.5 × 2.5 can be
  7 × 3 = 21
  or 8 × 2 = 16
  or between 7 × 2 = 14 and 8 × 3 = 24.
  Use a **calculator** to check which is the closer estimate.

Recognise what makes a 'good approximation'.
Answer questions such as:

- Which is the best approximation for 40.8 – 29.7?
  A. 408 – 297   C. 41 – 30
  B. 40 – 29   D. 4.0 – 2.9
- Which is the best approximation for 9.18 × 3.81?
  A. 10 × 3   C. 9 × 3
  B. 10 × 4   D. 9 × 4

See Y456 examples (pages 10–11).

Link to rounding (pages 42–5), and checking results of calculations (pages 110–11).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

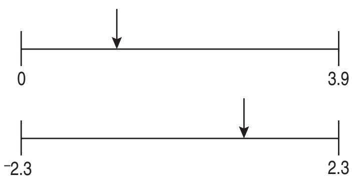
Begin to recognise when an exact answer is not needed and an estimate is sufficient, or when an exact answer is needed and an estimate is insufficient.

Estimate large numbers by estimating a small proportion then scaling up, in mathematics and other subjects. For example, estimate:

- the number of clover plants on a lawn;
- the daily volume of water a typical person drinks.

Estimate the position of a point on an unmarked scale. For example:

- Estimate the number that the arrow is pointing to.



Estimate squares and square roots. For example:

- Estimate $\sqrt{30}$.
  $\sqrt{25} < \sqrt{30} < \sqrt{36}$, so $5 < \sqrt{30} < 6$, and $\sqrt{30} \approx 5.5$
- Estimate $(3.718)^2$.
  $3^2 < (3.7)^2 < 4^2$, so $9 < (3.7)^2 < 16$, and $(3.7)^2 \approx 14$

Know that:

- Sometimes there are different ways for finding an approximate answer.
- More than one approximation to the same calculation is possible.

For example:

- $467 \times 24 \approx 400 \times 25 = 10\,000$
  $467 \times 24 \approx 500 \times 20 = 10000$
- $677 \div 48 \approx 600 \div 40 = 15$
  $677 \div 48 \approx 700 \div 50 = 14$

Recognise what makes a 'good approximation'. Answer questions such as:

- Approximate: $\frac{127 \times 31}{19}$
- Explain how to estimate an answer to
  $(17.8 - 4.6) \div (11.4 + 9.7)$
  before you use a **calculator** to work it out.

Check the position of a decimal point in a multiplication by estimating the answer, e.g.

- $67.3 \times 98.02 \approx 70 \times 100 = 7000$
  and, using a **calculator**,
  $67.3 \times 98.02 = 6596.746$

Link to rounding (pages 42–5), and checking results of calculations (pages 110–11).

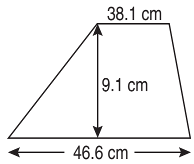
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years. Understand the difference between and when to use $\approx$, $=$, $\neq$, $\equiv$.

Recognise when an exact answer is not needed and an estimate is sufficient, or when an exact answer is needed and an estimate is insufficient.

Find approximate areas by rounding lengths. For example, the approximate area of this trapezium is:

$\frac{1}{2} \times (40 + 50) \times 9\ \text{cm}^2$
$= \frac{1}{2} \times 810\text{cm}^2$
$= 405\text{cm}^2$



Know that 3, $\frac{22}{7}$ and 3.14 are all approximations to π. Use these values to calculate approximations to the areas and circumferences of circles.

Make and justify estimates of calculations such as:

- $(2095 \times 302) + 396$
- $3.75 \times (2.36 - 0.39)$
- $\frac{103 \times 0.44}{\sqrt{(16.1)}}$

for example, by rounding to one significant figure.

Recognise the effects of rounding up and down on a calculation. Discuss questions such as:

- Why is $6 \div 2$ a better approximation for $6.59 \div 2.47$ than $7 \div 2$?

Recognise when approximations to the nearest 10, 100 or 1000 are good enough, and when they are not.

Check the position of a decimal point in a multiplication by estimating the answer, e.g.

- $48.6 \times 0.078 \approx 50 \times 0.1 = 50 \div 10 = 5$
  and, using a **calculator**,
  $48.6 \times 0.078 = 3.7908$

Link to rounding (pages 42–5), checking results of calculations (pages 110–11), finding the area of a circle using an approximation to π (pages 236–7).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use efficient column methods for addition and subtraction of whole numbers, and extend to decimals

Continue to use and refine efficient methods for column addition and subtraction, while maintaining accuracy and understanding. Extend to decimals with up to two decimal places, including:

- sums and differences with different numbers of digits;
- totals of more than two numbers.

For example:

- 671.7 – 60.2
- 45.89 + 653.7
- 764.78 – 56.4
- 543.65 + 45.8
- 1040.6 – 89.09
- 76.56 + 312.2 + 5.07

See Y456 examples (pages 48–51).

Refine written methods of multiplication and division of whole numbers to ensure efficiency, and extend to decimals

Multiplication

Use written methods to support, record or explain multiplication of:

- a three-digit number by a two-digit number;
- a decimal with one or two decimal places by a single digit.

- 6.24 × 8 is approximately 6 × 8 = 48.

×	6	0.2	0.04	Answer
8	48	1.6	0.32	49.92

Progress from the 'grid' method (see Year 6) to using a standard procedure efficiently and accurately, with understanding.

- 673 × 24 is approximately 700 × 20 = 14 000.

```
                  673
              ×    24
673 × 20        13460
673 ×  4         2692
                16152
                 1 1
```

- 6.24 × 8 is approximately 6 × 8 = 48 and is equivalent to 624 × 8 ÷ 100.

```
      624
  ×     8
     4992    4992 ÷ 100 = 49.92
      1 3
```

- 642.7 × 3 is approximately 600 × 3 = 1800 and is equivalent to 6427 × 3 ÷ 10.

```
     6427
  ×     3
    19281    19281 ÷ 10 = 1928.1
     1  2
```

See Y456 examples (pages 66–7).

Link to estimating calculations (pages 102–3, 110–11), and multiplying by powers of 10 (pages 38–9).

As outcomes, Year 8 pupils should, for example:

Consolidate the methods learned and used in previous years, and extend to harder examples of sums and differences with different numbers of digits.

For example:

- 44.8 + 172.9 + 87.36
- 5.05 + 3.9 + 8 + 0.97
- 14 – 3.98 – 2.9
- 32.7 + 57.3 – 45.17
- 18.97 + 2.9 – 17.36 – 28.4 + 5.04

Multiplication

Use written methods to multiply by decimals with up to two decimal places. Consider the approximate size of the answer in order to check the magnitude of the result. For example:

- 23.4 × 4.5 is approximately 23 × 5 = 115.

×	20	3	0.4	Check
4	80	12	1.6	93.6
0.5	10	1.5	0.2	+ 11.7
	90	13.5	1.8	105.3

Use a standard procedure to improve efficiency, maintaining accuracy and understanding.

- 1.89 × 23 is approximately 2 × 20 = 40, and is equivalent to 1.89 × 100 × 23 ÷ 100, or 189 × 23 ÷ 100.

```
                 189
               ×  23
189 × 20        3780
189 ×  3         567
                4347
                1 1
Answer: 4347 ÷ 100 = 43.47
```

- 23.4 × 4.5 is approximately 23 × 5 = 115, and is equivalent to 23.4 × 10 × 4.5 × 10 ÷ 100, or 234 × 45 ÷ 100.

```
                 234
               ×  45
234 × 40        9360
234 ×  5        1170
               10530
                 1
Answer: 10 530 ÷ 100 = 105.3
```

Link to estimating calculations (pages 102–3, 110–11), and multiplying by powers of 10 (pages 38–9).

As outcomes, Year 9 pupils should, for example:

Use a standard column procedure for addition and subtraction of numbers of any size, including a mixture of large and small numbers with differing numbers of decimal places.

For example:

- 6543 + 590.005 + 0.0045
- 5678.98 – 45.7 – 0.6

Multiplication

Use a standard column procedure for multiplications equivalent to three digits by two digits. Understand where to put the decimal point for the answer. Consider the approximate size of the answer in order to check the magnitude of the result. For example:

- 64.2 × 0.43 ≈ 60 × 0.5 = 30, and is equivalent to 642 × 43 ÷ 1000.

```
      642
    ×  43
    25680
     1926
    27606
     1 1
Answer: 27606 ÷ 1000 = 27.606
```

Where appropriate, round the answer to a suitable number of decimal places or significant figures. For example:

- 0.0721 × 0.036 ≈ 0.07 × 0.04 = 0.0028, and is equivalent to 721 × 36 ÷ 10 000000, or 0.0025956, or 0.0026 correct to 4 d.p.
- 5.16 × 3.14 ≈ 5 × 3 = 15, and is equivalent to 516 × 314 ÷ 10000, or 16.2024, or 16.2 correct to 3 s.f.

Link to estimating calculations (pages 102–3, 110–11), and multiplying by powers of 10 (pages 38–9).

Pupils should be taught to:

Refine written methods of multiplication and division of whole numbers to ensure efficiency, and extend to decimals with two places (continued)

As outcomes, Year 7 pupils should, for example:

Division

Use written methods to support, record or explain division of:
- a three-digit number by a two-digit number;
- a decimal with one or two decimal places by a single digit.

Progress from informal methods to using a standard algorithm efficiently and accurately, and with understanding.

For example:

- 3199 ÷ 7 is approximately 2800 ÷ 7 = 400.

```
  7)3199
  – 2800      7 × 400
     399
  – 350       7 ×  50
      49
  –   49      7 ×   7
       0
       Answer:  457
```

Refine methods to improve efficiency while maintaining accuracy and understanding.

- 109.6 ÷ 8 is approximately 110 ÷ 10 = 11.

```
   8)109.6                      8)109.6
 –    80.0   8 × 10           – 104.0   8 × 13
      29.6                         5.6
 –    24.0   8 ×  3                5.6   8 × 0.7
       5.6                         0.0
 –     5.6   8 × 0.7
       0.0
    Answer: 13.7
```

- 239.22 ÷ 6 is approximately 200 ÷ 5 = 40.

```
  6)239.22
 – 180.00        6 ×   30
    59.22
 –  54.00    6 ×  9
     5.22
 –   4.80    6 ×  0.8
     0.42
 –   0.42    6 ×  0.07
     0.00
       Answer:  39.87
```

See Y456 examples (pages 68–9).

Link to estimating calculations (pages 102–3), and multiplying and dividing by powers of 10 (pages 38–9).

As outcomes, Year 8 pupils should, for example:

Division

Use a standard procedure for divisions equivalent to three digits by two digits, by transforming to an equivalent calculation with a non-decimal divisor.
Consider the approximate size of the answer in order to check the magnitude of the result.

For example:

- 91.8 ÷ 17 is approximately 100 ÷ 20 = 5.

```
17)91.8
–   85.0      17 × 5
    6.8
–       6.8        17 × 0.4
    0.0
            Answer: 5.4
```

- 87.5 ÷ 16 is approximately 90 ÷ 15 = 6.

```
16)87.50
– 80.00       16 × 5
   7.50
–       6.40       16 × 0.4
   1.10
–       0.96       16 × 0.06
   0.14
            Answer: 5.46 R 0.14
                    5.5 to 1 d.p.
```

- 428 ÷ 3.4 is approximately 400 ÷ 4 = 100
  and is equivalent to 4280 ÷ 34.

```
34)4280
–   3400      34 × 100
    880
–       680        34 ×  20
    200
–       170        34 ×   5
     30.0
–          27.2    34 ×   0.8
      2.80
–          2.72 34 ×      0.08
      0.08

            Answer: 125.88 R 0.08
                    125.9 to 1 d.p.
```

Link to estimating calculations (pages 102–3), and multiplying and dividing by powers of 10 (pages 38–9).

As outcomes, Year 9 pupils should, for example:

Division

Use a standard procedure for divisions involving decimals by transforming to an equivalent calculation with a non-decimal divisor.
Consider the approximate size of the answer in order to check the magnitude of the result.

For example:

- 361.6 ÷ 0.8 is equivalent to 3616 ÷ 8.
- 547.4 ÷ 0.07 is equivalent to 54 740 ÷ 7.
- 0.048 ÷ 0.0035 is equivalent to 480 ÷ 35.
- 0.593 ÷ 6.3 is equivalent to 5.93 ÷ 63.

Where appropriate, round the answer to a suitable number of decimal places or significant figures.
For example:

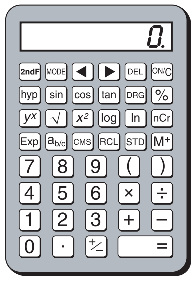
- 0.0821 ÷ 0.78 ≈ 0.08 ÷ 0.8 = 0.1
  and is equivalent to
  (821 ÷ 10 000) ÷ (78 ÷ 100)
  or
  (821 ÷ 78) ÷ 100
  or
  0.105 correct to 3 s.f.

Link to estimating calculations (pages 102–3), and multiplying and dividing by powers of 10 (pages 38–9)

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Carry out more complex calculations using the facilities on a calculator

Use, read and write, spelling correctly:
calculator, display, key, enter, clear, memory...



Know how to:

- Key in money calculations, and measurements of time, e.g. 4 hours 15 minutes is keyed in as 4.25 hours.
- Input a negative number.
- Use the bracket keys and select the correct key sequence to carry out calculations involving more than one step, e.g. to calculate 364 ÷ (23 + 17).
- Find whole-number remainders after division.
- Convert units of time, e.g. 1000 minutes to hours and minutes.
- Use the square and square root keys.
- Consider the approximate size of an answer before and after a calculation and, where necessary, check it appropriately, e.g. by performing the inverse operation.

For example:

- Use a **calculator** to work out:

  a. 7.6 – (3.05 – 1.7) b. $\frac{8.4 - 3.7}{8.4 + 3.7}$

Interpret the display on a calculator in different contexts (fractions, decimals, money, metric measures, time)

Know how to:

- Recognise a negative number in the display.
- Recognise how brackets are displayed.
- Interpret the display in the context of a problem, e.g. 109.2 may mean £109.20 in the context of money, 109 metres and 20 centimetres in the context of length, and 109 minutes and 12 seconds in the context of time.
- Read the display of, say, 91.333 3333 after dividing 822 by 9 as '91 point three recurring', and know that 0.3333333 represents one third.
- Interpret a rounding error, e.g. when calculating 2 ÷ 7 × 7 some calculators may display 1.999999 instead of 2.

For example:

- Convert 950 hours to days and hours.
  The display after dividing 950 by 24 will be 39.583 333. Subtract 39 from the answer to give the fraction of a day, then multiply by 24 to convert the fraction of a day back to hours.

See Y456 examples (pages 70–1).

Link to rounding numbers to one decimal place (pages 42–5).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
sign change key…

Know how to:
- Use the sign change or +/- key where appropriate.
- Use the memory and/or bracket keys, and select the correct key sequence to carry out complex calculations.
- Key in fractions, recognise the equivalent decimal form, and use this to compare and order fractions.
- Use the fraction key, including to enter time, e.g. 3 hours 25 minutes = $3\frac{25}{60}$ hours.
- Use the cube and cube root keys, if available.
- Consider the approximate size of an answer before and after a calculation and, where necessary, check it appropriately.

Use a **calculator** to evaluate correctly complex expressions such as those with brackets or where the memory function could be used.

For example:

- Use a calculator to work out

  $4 \times (6.78)^2$

Know how to:
- Recognise recurring decimals when they are rounded on the calculator, e.g. 2 ÷ 3 is displayed as 0.666 66667.
- Recognise that if, for example, $\sqrt{3}$ is shown to be 1.732051 then $(1.732051)^2 \approx 3$.

Link to rounding numbers to one or two decimal places (pages 42–5), converting fractions to decimals (pages 64–5), working with integers, powers and roots (pages 48–59).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
constant… reciprocal…

Know how to:
- Use the constant, π, sign change, power (x^y), root and fraction keys to evaluate expressions.
- Use the reciprocal key ($\frac{1}{x}$).

For example:
- Add on 101 repeatedly using the constant key. How long is the digit pattern maintained? Explain why.
- Find the circumference of a circle with radius 8 cm to two decimal places.
- Calculate 6^7, $\sqrt[4]{625}$, $\sqrt{(57.6/\pi)}$, $\sqrt{(15.5^2 - 3.7^2)}$.
- Use a calculator to work out the answer as a fraction for $\frac{12}{19} + \frac{17}{22}$.

Use a **calculator** to evaluate more complex expressions such as those with nested brackets or where the memory function could be used.

For example:

- Use a calculator to work out:

  a. $\dfrac{45.65 \times 76.8}{1.05 \times (6.4 - 3.8)}$

  b. $4.6 + (5.7 - (11.6 \times 9.1))$

  c. $\{(4.5)^2 + (7.5 - 0.46)\}^2$

  d. $\dfrac{5 \times \sqrt{(4.5^2 + 6^2)}}{3}$

Understand how a **scientific calculator** presents large and small numbers in standard form, linking to work in science.

Link to multiplying by powers of 10 and writing numbers in standard form (page 39).

Use a **calculator** to investigate sequences involving a reciprocal function, such as:

$$x \rightarrow \frac{1}{x - 1}$$

Link to reciprocals (pages 82–3).

Link to rounding numbers to one or two decimal places (pages 42–5), converting fractions to decimals (pages 64–5), working with integers, powers and roots (pages 48–59).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use checking procedures, including working the problem backwards and considering whether the result is the right order of magnitude

Use the context of a problem to check whether an answer is sensible. For example:

- Check that the sum of two odd numbers, positive or negative, is an even number.
- When multiplying two large numbers together, check the last digit, e.g. 239 × 46 must end in a '4' because 6 × 9 = 54.
- Having multiplied a number by, for example, 3, the sum of the digits should be divisible by 3.

Discuss questions such as:

- A girl worked out the cost of 8 bags of apples at 47p a bag. Her answer was £4.06.
  Without working out the answer, say whether you think it is right or wrong.
- A boy worked out how many 19p stamps you can buy for £5. His answer was 25.
  Do you think he was right or wrong? Why?
- I buy six items costing 76p, 89p, 36p, £1.03, 49p and 97p. I give the shop assistant a £10 note and get £3.46 change. I immediately think the change is wrong. Without calculating the sum, explain why you think I am right.
- A boy worked out £2.38 + 76p on a calculator. The display showed 78.38.
  Why did the calculator give the wrong answer?

Use rounding to approximate and judge whether the answer is the right order of magnitude. For example:

• 2605 – 1897	is about	3000 – 2000
• 245 × 19	is about	250 × 20
• 786 ÷ 38	is about	800 ÷ 40
• 12% of 192	is about	10% of 200
• 1.74 × 16	lies between	1 × 16 = 16 and 2 × 16 = 32

Check by doing the inverse operation.
For example, use a **calculator** to check:

• 43.2 × 26.5 = 1144.8	with	1144.8 ÷ 43.2
• $\frac{3}{5}$ of 320 = 192	with	192 × 5 ÷ 3
• 3 ÷ 7 = 0.428 5714…	with	7 × 0.4285714

Check by doing an equivalent calculation.
For example, check:

• 592 × 9 = 5328	with	(600 – 8) × 9 = 5400 – 72
	or	592 × (10 – 1) = 5920 – 592
• 44 × 99 = 4356	with	44 × (100 – 1) = 4400 – 44
	or	(40 + 4) × 99 = 3960 + 396

See Y456 examples (pages 72–3).

Link to making estimates and approximations of calculations (pages 102–3).

As outcomes, Year 8 pupils should, for example:

Use the context of a problem to check whether an answer is sensible. For example:

- When calculating a mean, check that it is within the range of the data. For example, the mean of 34, 21, 65, 89, 43, 29, 76, 79 must lie between 21 and 89.
- When using measurements, check the magnitude of the answer in the context of the problem.

Discuss questions such as:

- Will the answer to 75 ÷ 0.9 be smaller or larger than 75?
- A class of pupils was asked whether they preferred pop or classical music. They said:

Prefer classical	21%
Prefer pop	67%
Don't know	13%

  All results are correct to the nearest per cent but the three percentages add to 101%.
  Is this possible?
- Without using a calculator, pick out a possible answer to the calculation. Explain your choice.
  a. 47 × 59
     3443 or 2773 or 2887
  b. 456 × 0.48
     218.9 or 636 or 322.7

Use rounding to approximate and judge whether the answer is the right order of magnitude. For example:
- √7 lies between √4 and √9
  i.e. between 2 and 3
- Round to the nearest ten, e.g.
  $^{62}/_{9}$ is approximately $^{60}/_{10}$ = 6.
- Round to 'nice' numbers, e.g.
  $^{62}/_{9}$ is approximately $^{63}/_{9}$ = 7.

Check by doing the inverse operation.
For example, use a **calculator** to check:
- √7 = 2.645 75… with $(2.64575)^2$

Link to making estimates and approximations of calculations (pages 102–3), and checking the solution of an equation by substitution (pages 122–5).

As outcomes, Year 9 pupils should, for example:

Use the context of a problem to check whether an answer is sensible.

Discuss questions such as:

- The price of an audio system is reduced by 10%. Two months later the price increases by 10%. It does not return to its original price.
  Is this possible?
- Without using a calculator, pick out a possible answer to the calculation from the three possible answers given.
  a. $(398)^2$
     158 404 or 6344 or 161484
  b. 365 ÷ 0.43
     849 or 84.9 or 157
  c. 67 ÷ 0.083
     87.2 or 8.72 or 807.2
  Explain your choice in each case.
- Can a square have an exact area of 32 cm²? What about a circle?

Link to making estimates and approximations of calculations (pages 102–3), checking the solution of an equation by substitution (pages 122–5), and checking that the sum of probabilities for all outcomes is 1 (pages 278–9).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use letter symbols and distinguish their different roles in algebra

Use, read and write, spelling correctly:
algebra, unknown, symbol, variable… equals… brackets…
evaluate, simplify, substitute, solve…
term, expression, equation… squared… commutative…

Reinforce the idea of an **unknown**. Answer questions such as:
- 5 + □ = 17
- 3 × □ – 5 = 7
- ▼ + ♦ = 4. What numbers could ▼ and ♦ be?
- The product of two numbers is 24. What could they be?

Know that letters are used to stand for numbers in algebra. Begin to distinguish between different uses of letters.
For example:
- In the equation 3n + 2 = 11, n is a particular unknown number, but in the equation a + b = 12, a and b can take many different values.

Recognise algebraic conventions, such as:
- 3 × n or n × 3 can be thought of as '3 lots of n', or n + n + n, and can be shortened to 3n.
- In the expression 3n, n can take any value, but when the value of an expression is known, an equation is formed, i.e. if 3n is 18, the equation is written as 3n = 18.

Understand the meaning of and begin to **use simple expressions with brackets,** e.g. 3(n + 2) meaning 3 × (n + 2), where the addition operation is to be performed first and the result of this is then multiplied by 3.

Use the equals sign appropriately and correctly.
- Recognise that if a = b then b = a, and that a + b = c can also be written as c = a + b.
- Avoid errors arising from misuse of the sign when setting out the steps in a calculation, e.g. incorrectly writing
  38 + 29 = 38 + 30 = 68 – 1 = 67

Use letter symbols to write expressions in meaningful contexts.
For example:

add 7 to a number	n + 7
subtract 4 from a number	n – 4
4 minus a number	4 – n
a number multiplied by 2 and then 5 added	(n × 2) + 5 or 2n + 5
a number divided by 2	n ÷ 2 or n/2
a number plus 7 and then multiplied by 10	(n + 7) × 10 or 10(n + 7)
a number multiplied by itself	n × n or n^2

Understand the difference between expressions such as:

2n and n + 2	3(c + 5) and 3c + 5
n^2 and 2n	$2n^2$ and $(2n)^2$

Link to formulating expressions and formulae (pages 122–5).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
algebraic expression, formula, function…
partition, multiply out… cubed, to the power of…

Know that an algebraic expression is formed from letter symbols and numbers, combined with operation signs such as +, –, ×, ( ), ÷ and /.

Use letter symbols in different ways and begin to distinguish their different roles. For example:

- In the equation 4x + 3 = 47, x is a particular unknown number.
- In the formula V = IR, V, I and R are variable quantities, related by the formula.
  In the formula F = $^{9C}/_{5}$ + 32, once C is known, the value of F can be calculated.
- In the function y = 3x – 4, for any chosen value of x, the related value of y can be calculated.

Know how multiplication and division are represented in algebraic expressions. For example:

- 2 × n is written as 2n.
- p × q is written as pq.
- a × (b + c) is written as a(b + c).
- (x + y) ÷ z is written as $\frac{x + y}{z}$.

Use the equals sign appropriately and correctly.

- Know that the symbol = denotes equality.
- Avoid misuse of the equals sign when working through a sequence of steps, e.g. incorrectly writing 56 + 37 = 56 + 30 = 86 + 7 = 93.
- Avoid interpreting the equals sign as 'makes', which suggests it means merely the answer to a calculation, as in 3 × 2 + 7 = 13.

Begin to interpret the equals sign more broadly, including in equations with expressions on each side. For example:

- Recognise equalities in different forms, such as:
  a + b = c + d   8 = 15 – 3x
  and know that they can be written as:
  a + b = c + d or c + d = a + b
  8 = 15 – 3x or 15 – 3x = 8
- Know that expressions such as 2a + 2 and 2(a + 1) always have the same value for any value of a.

Link to constructing and solving equations (pages 122–5).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
identity, identically equal, inequality… subject of the formula… common factor, factorise… index law… linear, quadratic, cubic… and the identity sign (≡).

Explain the distinction between equations, formulae and functions. For example:

- In the equation 5x + 4 = 2x + 31, x is a particular unknown number.
- In the formula v = u + at, v, u, a and t are variable quantities, related by the formula. Once the values of three of the variables are known, the fourth value can be calculated.
- In the function y = 8x + 11, for any chosen value of x, the related value of y can be calculated.

Know that an inequality or ordering is a statement that one expression is greater or less than another. For example:

- x ≥ 1 means that x is greater than or equal to 1.
- y ≤ 2 means that y is less than or equal to 2.

An inequality remains true if the same number is added to or subtracted from each side, or if both sides are multiplied or divided by the same positive number. Multiplying or dividing by a negative number reverses the sense of the inequality.

Know the meaning of an identity and use the ≡ sign.
In an identity, the expressions on each side of the equation always take the same value, whatever numbers are substituted for the letters in them; the expressions are said to be identically equal.
For example:

- 4(a + 1) ≡ 4a + 4 is an identity, because the expressions 4(a + 1) and 4a + 4 always have the same value, whatever value a takes.

Link to constructing and solving equations (pages 122–9).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Know that algebraic operations follow the same conventions and order as arithmetic operations; use index notation and the index laws

Know that algebraic operations follow the same conventions and order as arithmetic operations.
Begin to generalise from arithmetic that multiplication and division have precedence over addition and subtraction.
For example:

- In the expression 2 + 5a, the multiplication is to be performed first.

Recognise that calculators that allow a whole calculation to be displayed apply the conventions of arithmetic, so 2 + 3 × 4 will be evaluated as 14, because 3 × 4 is evaluated first; other calculators may give 20 unless brackets are used: 2 + (3 × 4).

Know that the commutative and associative laws apply to algebraic expressions as they do to arithmetic expressions, so:

• 2 + 3 = 3 + 2	a + b = b + a
• 2 × 3 = 3 × 2	a × b = b × a or ab = ba
• 2 + (3 + 4) = (2 + 3) + 4	a + (b + c) = (a + b) + c
• 2 × (3 × 4) = (2 × 3) × 4	a × (b × c) = (a × b) × c
	or a(bc) = (ab)c

Link to arithmetic operations (pages 84–5).

Inverses
Understand addition and subtraction as the inverse of each other, and multiplication and division as the inverse of each other. Generalise from arithmetic that:

- a + b = 5 implies b + a = 5, using the commutative law, and the corresponding inverse relationships
  5 – b = a and 5 – a = b.
- Similarly, a × b = 24 implies that
  b × a = 24, b = 24/a and a = 24/b.

Verify by substituting suitable sets of numbers.

Begin to apply inverse operations when two successive operations are involved. For example:

- The inverse of multiplying by 6 and adding 4 is subtracting 4 and dividing by 6, i.e.
  if a × 6 + 4 = 34, then (34 – 4)/6 = a.

Alternatively:

- I think of a number, multiply by 6 and add on 4.
  The answer is 34.
  What was the original number?

Use a **calculator** to verify, pressing the equals key between each operation. For example, starting with 5:

[5] [×] [6] [=] [+] [4] [=] [−] [4] [=] [÷] [6] [=]

should result in the same number as originally entered, i.e. 5. Check and explain what happens when the = key is not pressed at the various stages.

Link to arithmetic operations (pages 84–5).

As outcomes, Year 8 pupils should, for example:

Recognise that algebraic operations follow the same conventions and order as arithmetic operations.
Know that contents of brackets are evaluated first, and that multiplication and division are carried out before addition and subtraction. For example:
- In 7 – 5s, the multiplication is performed first.
- In $6 - s^2$, the square is evaluated first.
- In 3(x – 2), the expression in the brackets is evaluated first.

Use index notation for small positive integer powers.
Know that expressions involving repeated multiplication of the same number, such as:

n × n   n × n × n   n × n × n × n

are written as n^2, n^3 and n^4, and are referred to as n squared, n cubed, n to the power of 4, etc.
Know why the terms squared and cubed are used for to the power of 2 and to the power of 3.

Understand the different meanings of expressions such as:

$2n$ and n^2 $3n$ and n^3

Simplify expressions such as:

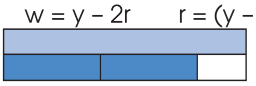
$2x^2 + 3x^2$ $n^2 \times n^3$ $p^3 \div p^2$

Understand and use inverse operations.
Recognise that any one of the equations:

- a + b = c, b + a = c, c – a = b and c – b = a
- ab = c, ba = c, b = c/a and a = c/b

implies each of the other three in the same set, as can be verified by substituting suitable sets of numbers into the equations.

Use coloured rods, e.g. white (1 unit), red (2 units) and yellow (5 units), to express relationships such as:
- y = 2r + w   w = y – 2r   r = (y – w)/2



Apply inverse operations when two successive operations are involved. For example:
- The inverse of dividing by 4 and subtracting 7 is adding 7 and multiplying by 4, i.e.
  if m/4 – 7 = n, then m = 4(n + 7).

Use a **spreadsheet** to verify this, entering different numbers in column A, including negative numbers and decimals. If column C always equals column A, the inverse is probably correctly expressed.

	A	B	C
1	Number	=A1/4–7	=(B1+7)*4
2		=A2/4–7	=(B2+7)*4
3		=A3/4–7	=(B3+7)*4
4		=A4/4–7	=(B4+7)*4
5		=A5/4–7	=(B5+7)*4

	A	B	C
1	1	–6.75	1
2	2	–6.5	2
3	3	–6.25	3
4	4	–6	4
5	5	–5.75	5

Link to arithmetic operations (pages 84–5).

As outcomes, Year 9 pupils should, for example:

Apply simple instances of the index laws for multiplication and division of small integer powers.
For example:

- $n^2 \times n^3 = n^{2+3} = n^5$
- $p^3 \div p^2 = p^{3-2} = p$

See page 59.

Know and use the general forms of the index laws for multiplication and division of positive integer powers.

$p^a \times p^b = p^{a+b}$ $p^a \div p^b = p^{a-b}$ $(p^a)^b = p^{ab}$

Begin to extend understanding of index notation to negative and fractional powers and recognise that the index laws can be applied to these as well.

See page 59.

Pupils should be taught to:

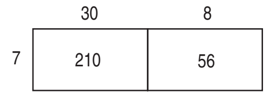
Simplify or transform algebraic expressions

As outcomes, Year 7 pupils should, for example:

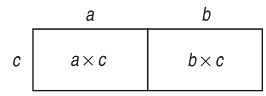
Simplify linear expressions by collecting like terms; begin to multiply a single term over a bracket.

Understand that partitioning a number helps to break a multiplication into a series of steps. For example:

- By partitioning 38, 38 × 7 becomes
  $(30 + 8) \times 7 = 30 \times 7 + 8 \times 7$



  Generalise, from this and similar examples, to:
  $(a + b) \times c = (a \times c) + (b \times c)$
  or $ac + bc$



Link to written methods for multiplication (pages 104–5).

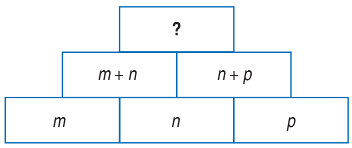
Recognise that letters stand for numbers in problems.
For example:

- Simplify expressions such as:

  a. $a + a + a = 3a$
  b. $b + 2b + b = 4b$
  c. $x + 6 + 2x = 3x + 6$
  d. $3n + 2n = 5n$
  e. $3(n + 2) = 3n + 6$

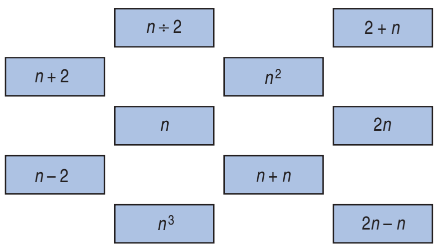
  and $a/a = 1$, $2a/a = 2$, … and $4a/2 = 2a$, $6a/2 = 3a$, etc.

- The number in each cell is the result of adding the numbers in the two cells beneath it.



  Write an expression for the number in the top cell.
  Write your expression as simply as possible.

- Here are some algebra cards.



  a. Which card will always give the same answer as n/2?
  b. Which card will always give the same answer as n × n?
  c. Two cards will always give the same answer as 2 × n.
  Which cards are they?
  d. When the expressions on three of the cards are added together they will always have the same answer as 5n.
  Which cards are they?
  e. Write a new card that will always give the same answer as 3n + 2n.

- Draw some shapes that have a perimeter of $6x + 12$.

- The answer is $2a + 5b$. What was the question?

As outcomes, Year 8 pupils should, for example:

Simplify or transform linear expressions by collecting like terms; multiply a single term over a bracket.

Understand the application of the distributive law to arithmetic calculations such as:

- $7 \times 36 = 7(30 + 6) = 7 \times 30 + 7 \times 6$
- $7 \times 49 = 7(50 - 1) = 7 \times 50 - 7 \times 1$

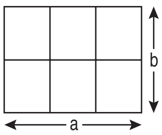
Know and use the distributive law for multiplication:

- over addition, namely $a(b + c) = ab + ac$
- over subtraction, namely $a(b - c) = ab - ac$

Recognise that letters stand for numbers in problems. For example:

- Simplify these expressions:
  a. $3a + 2b + 2a - b$
  b. $4x + 7 + 3x - 3 - x$
  c. $3(x + 5)$
  d. $12 - (n - 3)$
  e. $m(n - p)$
  f. $4(a + 2b) - 2(2a + b)$

- Write different equivalent expressions for the total length of the lines in this diagram.

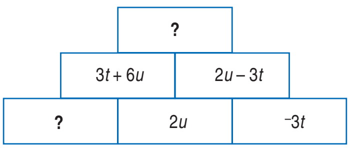


  Simplify each expression as far as possible.
  What did you discover?

- In a magic square the sum of the expressions in each row, column and diagonal is the same. Show that this square is a magic square.

$m - p$	$m + p - q$	$m + q$
$m + p + q$	m	$m - p - q$
$m - q$	$m - p + q$	$m + p$

- The number in each cell is made by adding the numbers in the two cells beneath it.
  Fill in the missing expressions.
  Write each expression as simply as possible.



As outcomes, Year 9 pupils should, for example:

Simplify or transform expressions by taking out single-term common factors.

Continue to use the distributive law to multiply a single term over a bracket.

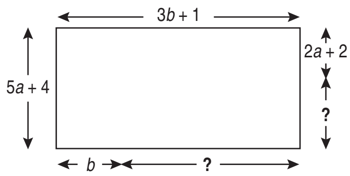
Extend to taking out single-term common factors.

Recognise that letters stand for numbers in problems. For example:

- Simplify these expressions:
  $3(x - 2) - 2(4 - 3x)$
  $(n + 1)^2 - (n + 1) + 1$

- Factorise:
  $3a + 6b = 3(a + 2b)$
  $x^3 + x^2 + 2x = x(x^2 + x + 2)$

- Write an expression for each missing length in this rectangle. Write each expression as simply as possible.



- The area of a rectangle is $2x^2 + 4x$.
  Suggest suitable lengths for its sides.
  What if the perimeter of a rectangle is $2x^2 + 4x$?

- Prove that the sum of three consecutive integers is always a multiple of 3.
  Let the integers be n, n + 1 and n + 2.
  Sum $= n + (n + 1) + (n + 2)$
  $= 3n + 3$
  $= 3(n + 1)$, which is a multiple of 3.

- Think of a number, multiply by 3, add 15, divide by 3, subtract 5. Record your answer.
  Try other starting numbers. What do you notice?
  Use algebra to prove the result.

- What is the smallest value you can get for $x^2 - x$ if x is an integer?
  What is the smallest value if x does not have to be an integer?
  Use a **spreadsheet** to help.

- Prove that the value of $x^3 - x + 9$ is divisible by 3 for any integer value of x.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Simplify or transform algebraic expressions (continued)	

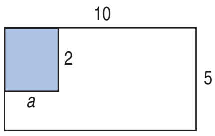
As outcomes, Year 8 pupils should, for example:

Explore general algebraic relationships.

For example:

- By dividing this shape into rectangles in different ways, form different equivalent expressions for the unshaded area, for example:

  $50 - 2a$
  $2(10 - a) + 30$
  $3a + 5(10 - a)$



  Then multiply out and simplify the expressions to show in a different way that they are equivalent.

- Use a **spreadsheet** to verify that $2a + 2b$ has the same value as $2(a + b)$ for any values of a and b, e.g. set up the expressions in columns C and D.

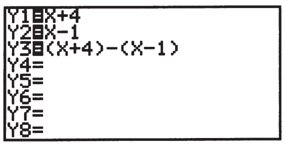
	A	B	C	D
1			=2*A1+2*B1	=2*(A1+B1)

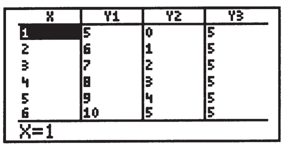
	A	B	C	D
1	3	7	20	20

  Repeatedly enter a different pair of numbers in columns A and B. Try positive and negative numbers, and whole numbers and decimals.

  Recognise that if equivalent expressions have been entered in C1 and D1, then columns C and D will always show the same number.

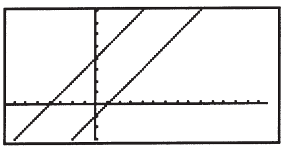
- Use a **graphical calculator** to verify that $(x + 4) - (x - 1) = 5$, for any value of x.





  Use the table to confirm that $4 - {}^{-}1 = 5$.

  Draw the graphs of the two straight lines $y = x + 4$ and $y = x - 1$.



  Confirm that the vertical distance beween the two lines is always 5, i.e. $(x + 4) - (x - 1) = 5$.

As outcomes, Year 9 pupils should, for example:

Add simple algebraic fractions.

Generalise from arithmetic that:

$$\frac{a}{b} \pm \frac{c}{d} = \frac{ad \pm bc}{bd}$$

- This is what a pupil wrote. Show that the pupil was wrong.

  For all numbers t and w

  $$\frac{1}{t} + \frac{1}{w} = \frac{2}{t + w}$$

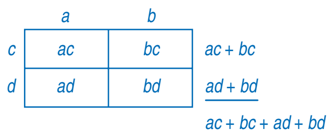
Square a linear expression, and expand and simplify the product of two linear expressions of the form x ± n.

Apply the distributive law to calculations such as:

- $53 \times 37 = (50 + 3)(30 + 7)$
  $= 50 \times 30 + 3 \times 30 + 50 \times 7 + 3 \times 7$
- $57 \times 29 = (50 + 7)(30 - 1)$
  $= 50 \times 30 + 7 \times 30 - 50 \times 1 - 7 \times 1$

Derive and use identities for the product of two linear expressions of the form $(a + b)(c \pm d)$:

- $(a + b)(c + d) \equiv ac + bc + ad + bd$
- $(a + b)(a + b) \equiv a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$
- $(a + b)(c - d) \equiv ac + bc - ad - bd$
- $(a + b)(a - b) \equiv a^2 - ab + ab - b^2 = a^2 - b^2$



For example:

- Multiply out and simplify:

  $(p - q)^2 = (p - q)(p - q)$
  $= p^2 - pq - pq + q^2$
  $= p^2 - 2pq + q^2$

  $(3x + 2)^2 = (3x + 2)(3x + 2)$
  $= 9x^2 + 6x + 6x + 4$
  $= 9x^2 + 12x + 4$

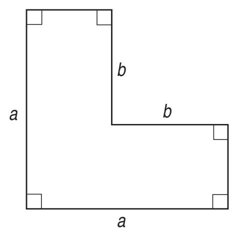
  $(x + 4)(x - 3) = x^2 + 4x - 3x - 12$
  $= x^2 + x - 12$

Use geometric arguments to prove these results.
For example:

- Multiply out and simplify these expressions to show that they are equivalent.

  $a^2 - b^2$
  $a(a - b) + b(a - b)$
  $2b(a - b) + (a - b)(a - b)$
  $(a - b)(a + b)$

  By formulating the area of this shape in different ways, use geometric arguments to show that the expressions are equivalent.

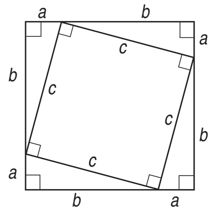


Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Simplify or transform algebraic expressions (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

- Four identical right-angled triangles with hypotenuse of length c are placed as shown.

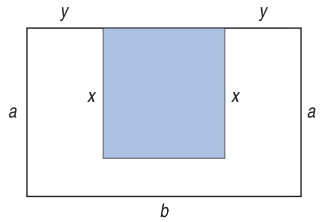
  

  Show that the inner shape formed is a square of side c, and the outer shape formed is a square of side a + b.

  Use the diagram to deduce a proof of Pythagoras' theorem:

  $c^2 = (a + b)^2 - 4(\frac{1}{2}ab)$
  $= (a^2 + 2ab + b^2) - 2ab$
  $= a^2 + b^2$

- Imagine a room with an area of carpet (shaded).

  By dividing the room into rectangles in different ways, find different equivalent expressions for the floor area with no carpet, for example:

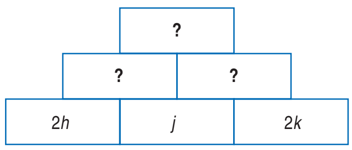
  $A = 2xy + b(a - x)$
  $A = 2ay + (a - x)(b - 2y)$

  Multiply out the expressions to confirm that they are equivalent.

Solve problems such as:

- Prove that the product of two odd numbers is always odd.

- In this diagram, h, j and k can be any integers. The missing number in each cell is found by adding the two numbers beneath it. Prove that the number in the top cell will always be even.

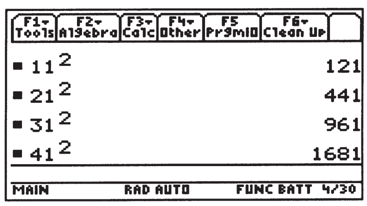
  

  What if j is replaced by j + 1?
  What if 2h is replaced by h?

- Show that:

  $(n + 1)^2 = n^2 + 2n + 1$

  Use this result to calculate 91^2, 801^2.

Pupils should be taught to:

Construct and solve linear equations, selecting an appropriate method

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
equation, solution, unknown, solve, verify, prove, therefore (∴).

Construct and solve simple linear equations with integer coefficients, the unknown on one side only.

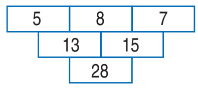
Choose a suitable unknown and form expressions leading to an equation. Solve the equation by using inverse operations or other mental or written methods.

For example:

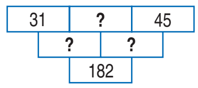
- I think of a number, subtract 7 and the answer is 16.
  What is my number?
  Let n be the number.
  $n - 7 = 16$
  $\therefore\ n = 16 + 7 = 23$

- A stack of 50 sheets of card is 12 cm high.
  How thick is one sheet of card?
  Let d cm be the thickness of each sheet.
  $50d = 12$
  $\therefore\ d = 12/50 = 24/100 = 0.24$
  The thickness of each sheet is 0.24 cm.

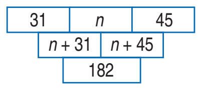
- In this diagram, the number in each cell is formed by adding the two numbers above it.



  What are the missing numbers in this diagram?



  Let n be the number in the top centre cell. Form the first row and the subsequent row.
  It follows that:



  $n + 31 + n + 45 = 182$
  $\therefore\ 2n = 106$
  $n = 53$

  What if the top three numbers are swapped around?
  What if you start with four numbers?

- I think of a number, multiply it by 6 and add 1.
  The answer is 37. What is my number?

- There are 26 biscuits altogether on two plates.
  The second plate has 8 fewer biscuits than the first plate.
  How many biscuits are there on each plate?

- Find the angle a in a triangle with angles a, a + 10, a + 20.

- Solve these equations:

a. $a + 5 = 12$	c. $7h - 3 = 20$	e. $2c + 3 = 19$
b. $3m = 18$	d. $7 = 5 + 2z$	f. $6 = 2p - 8$

Check solutions by substituting into the original equation.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
linear equation…

Consolidate forming and solving linear equations with an unknown on one side.

Choose a suitable unknown and form expressions leading to an equation. Solve the equation by removing brackets, where appropriate, collecting like terms and using inverse operations.

For example:

- There are 376 stones in three piles. The second pile has 24 more stones than the first pile. The third pile has twice as many stones as the second.
  How many stones are there in each pile?

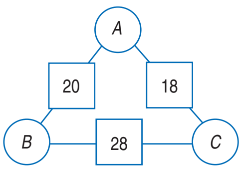
  Let s stand for the number in the first pile.

Pile 1	Pile 2	Pile 3	Total
s	s + 24	2(s + 24)	376

  $$\begin{aligned} s + (s + 24) + 2(s + 24) &= 376 \\ \therefore\ s + s + 24 + 2s + 48 &= 376 \\ 4s + 72 &= 376 \\ 4s &= 376 - 72 = 304 \\ s &= 76 \end{aligned}$$

- In an arithmagon, the number in a square is the sum of the numbers in the two circles on either side of it.

  In this triangular arithmagon, what could the numbers A, B and C be?

  Let x stand for the number in the top circle.
  Form expressions for the numbers in the other circles, (20 – x) and (18 – x). Then form an equation in x and solve it.

  $$\begin{aligned} (20 - x) + (18 - x) &= 28 \\ \therefore\ 38 - 2x &= 28 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

  So A = 5, B = 15, C = 13.

- On Dwain's next birthday, half of his age will be 16.
  How old is Dwain now?

- Solve these equations:
  a. $5x = 7$
  b. $3 = \frac{12}{n}$
  c. $2(p + 5) = 24$
  d. $2.4z + 5.9 = 14.3$
  e. $4(b - 1) + 5(b + 1) = 100$

Check solutions by substituting into the original

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
inequality, region… and, or…

Construct and solve linear equations with negative signs anywhere in the equation, negative solution…

Solve linear equations using inverse operations, by transforming both sides in the same way or by other methods.

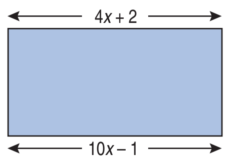
For example:

- Compare different ways of solving 'think of a number' problems and decide which would be more efficient – retaining brackets and using inverse operations, or removing brackets first.
  For example:
  I think of a number, add 3, multiply by 4, add 7, divide by 9, then multiply by 15.
  The final answer is 105.
  What was the number that I thought of?

- Jack, Jo and Jim are sailors.
  They were shipwrecked on an island with a monkey and a crate of 185 bananas.
  Jack ate 5 more bananas than Jim.
  Jo ate 3 more bananas than Jim.
  The monkey ate 6 bananas.
  How many bananas did each sailor eat?

- The length of a rectangle is three times its width.
  Its perimeter is 24 centimetres.
  Find its area.

- The area of this rectangle is 10 cm^2.

  Calculate the value of x and use it to find the length and width of the rectangle.

- In ΔABC, ∠B is three quarters of ∠A, and ∠C is one half of ∠A. Find all the angles of the triangle.

- Solve these equations:
  a. $3c - 7 = {}^{-}13$
  b. $1.7m^2 = 10.625$
  c. $4(z + 5) = 8$
  d. $4(b - 1) - 5(b + 1) = 0$
  e. $\frac{12}{(x + 1)} = \frac{21}{(x + 4)}$

Check solutions by substituting into the original equation.

Pupils should be taught to:

Construct and solve linear equations, selecting an appropriate method (continued)

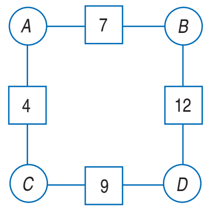
As outcomes, Year 7 pupils should, for example:

Explore ways of constructing simple equations to express relationships, and begin to recognise equivalent statements. For example:

- In an arithmagon, the number in a square is the sum of the numbers in the two circles either side of it.

  In this square arithmagon, what could the numbers A, B, C and D be?

  Can you find any relationships between A, B, C and D?



Some results

A	B	C	D
3	4	1	8
2	5	2	7
1	6	3	6
4	3	0	9
0	7	4	5

Some relationships

A + B = 7
C + D = 9
A + B + C + D = 16
D – A = 5
B – C = 3
B = C + 3

Recognise that statements such as B – C = 3 and B = C + 3 express the same relationship in different ways.

As outcomes, Year 8 pupils should, for example:

Explore alternative ways of solving simple equations, e.g. deciding whether or not to remove brackets first. For example:

- $$\begin{aligned} 2(x + 5) &= 36 \\ x + 5 &= 18 \\ x &= 13 \end{aligned} \quad \text{or} \quad \begin{aligned} 2(x + 5) &= 36 \\ 2x + 10 &= 36 \\ 2x &= 26 \\ x &= 13 \end{aligned}$$

Begin to understand that an equation can be thought of as a balance where, provided the same operation is performed on both sides, the resulting equation remains true. For example:

- Start with a true statement, such as:
  $52 - 7 = 41 + 4$
  Make the same change to both sides, e.g. subtract 4:
  $52 - 7 - 4 = 41$
  Check that this statement is true.

  Then start with a simple equation, such as:

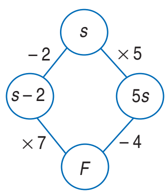
	y	=	$x + 4$
add 3	$y + 3$	=	$x + 7$
double	$2(y + 3)$	=	$2(x + 7)$
subtract d	$2(y + 3) - d$	=	$2(x + 7) - d$

  Check that the resulting equation is true by substituting numbers which fit the original, e.g. $x = 1$, $y = 5$.

Form linear equations (unknown on both sides) and solve them by transforming both sides in the same way. Begin to recognise what transformations are needed and in what order. For example:

- Jill and Ben each have the same number of pens. Jill has 3 full boxes of pens and 2 loose pens. Ben has 2 full boxes of pens and 14 loose pens. How many pens are there in a full box?
  $3n + 2 = 2n + 14$
- In the two-way flow diagram, find the starting number s that has to be entered, so that you reach the same finishing number F, whichever route is followed.

  $$\begin{aligned} 7(s - 2) &= 5s - 4 \\ \therefore\ 7s - 14 &= 5s - 4 \\ 7s &= 5s - 4 + 14 \\ 7s &= 5s + 10 \\ 2s &= 10 \\ s &= 5 \end{aligned}$$



- Solve these equations:
  a. $3x + 2 = 2x + 5$
  b. $5z - 7 = 13 - 3z$
  c. $4(n + 3) = 6(n - 1)$

Check solutions by substituting into the original equation.

As outcomes, Year 9 pupils should, for example:

Form linear equations (unknown on both sides) and solve them by transforming both sides in the same way. For example:

- Multiplying a number by 2 and then adding 5 gives the same answer as subtracting the number from 23. What is the number?
- Françoise and Jeanette have 250 euros between them. Jeanette gave Françoise 50 euros. Françoise now has four times as many euros as Jeanette. How many euros has Françoise?
- The sum of the ages of a mother and her daughter is 46. In three years' time the mother will be three times as old as her daughter is then. How old is the daughter now?
- Solve these equations:
  a. $7(s + 3) = 45 - 3(12 - s)$
  b. $3(2a - 1) = 5(4a - 1) - 4(3a - 2)$
  c. $2(m - 0.3) - 3(m - 1.3) = 4(3m + 3.1)$
  d. $\frac{3}{4}(c - 1) = \frac{1}{2}(5c - 3)$
  e. $\frac{x - 3}{2} = \frac{x - 2}{3}$

Check solutions by substituting into the original equation.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Solve a pair of simultaneous linear equations	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Solve a pair of simultaneous linear equations by eliminating one variable.

Know that **simultaneous equations** are true at the same time and are satisfied by the same values of the unknowns involved, and that linear simultaneous equations may be solved in a variety of ways.

Substitute from one equation into another.
For example:

- x and y satisfy the equation $5x + y = 49$.
  They also satisfy the equation $y = 2x$.
  Find x and y.
  Write down another equation satisfied by x and y.

  Method
  From the second equation, $y = 2x$.
  Substituting into the first equation gives
  $5x + 2x = 49$
  So $x = 7$ and $y = 14$.
  Other equations might be $x + y = 21$, $y = x + 7$.

- Solve the equations:
  a. $5p - q = 30$, $q = 3p$
  b. $3x - 5y = 22$, $x = 3y + 2$

Extend the substitution method to examples where one equation must be rearranged before the substitution can be made. For example:

- Solve the equations: $x - 2y = 5$, $2x + 5y = 100$
  From the first equation, $x = 5 + 2y$.
  Substituting into the second equation gives
  $2(5 + 2y) + 5y = 100$

Add or subtract equations. For example:

- Solve the equations:
  $x + 3y = 56$
  $x + 6y = 101$

  Method
  $x + 3y = 56$
  $x + 6y = 101$
  Compare the two equations and deduce that
  $3y = 45$ so $y = 15$.
  Substituting into the first equation gives $x = 11$.

- Solve the equations:
  a. $4x + y = 44$, $x + y = 20$
  b. $5x + y = 17$, $5x - y = 3$

Extend to adding or subtracting equations in order to eliminate one variable. For example:

- $2x + y = 17$, $3x + 2y = 28$

Multiply by 2:	$4x + 2y$	$= 34$
Subtract:	$3x + 2y$	$= 28$
	x	$= 6$

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Solve a pair of simultaneous linear equations (continued)	

As outcomes, Year 8 pupils should, for example:

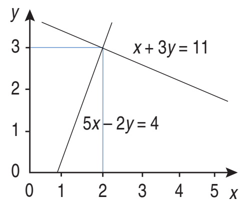
As outcomes, Year 9 pupils should, for example:

Use a graphical method and check algebraically.

Use pencil and paper or a **graph plotter** or **graphical calculator** to draw the graph of each equation.

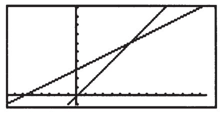
- Solve:

  $x + 3y = 11$
  $5x - 2y = 4$



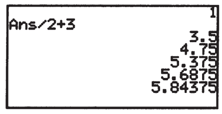
  The intersection of the two lines, the point (2, 3), gives an approximate solution to the equations.

- $y = x$
  $y = \frac{x}{2} + 3$



  Link the solution x = 6, y = 6 of this pair of equations to finding the limit of the sequence 'divide by 2, add 3', starting with 1, represented as

  $x \rightarrow \frac{x}{2} + 3$



Recognise that:

- The point at which the graphs of two equations intersect lies on both lines; its coordinates give the simultaneous solution of the two equations.
- Equations such as $x + y = 7$ and $3x + 3y = 21$ have an infinite number of solutions, since they are represented by the same graph.
- Equations such as $y = 4x + 5$ and $y = 4x + 10$ have no simultaneous solution, since their graphs are parallel lines which never meet.
- If the graphs of three (or more) equations in two unknowns pass through a common point then the equations have a common solution, given by the coordinates of the point. If the graphs are not coincident then the equations they represent have no common solution.

Form and solve linear simultaneous equations to solve problems. For example:

- In five years' time, Ravi's father will be twice as old as Ravi. In 13 years' time, the sum of their ages will be 100. How old is Ravi now?
- A nursery was asked to plant a number of trees in a number of days. If the nursery plants 240 trees per day, then 400 fewer trees than planned will be planted. If it plants 280 trees daily, it will plant 200 more than planned.
  How many trees should the nursery plant?
  How many days will it take?
- Find x and y when $x + 3y = 10$ and $2x + y = 5$.
  Invent a problem that could give rise to these two simultaneous equations.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Solve linear inequalities in one variable, and represent the solution set on a number line; begin to solve inequalities in two variables	

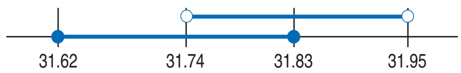
As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Solve linear inequalities in one variable; represent the solution set on a number line. For example:

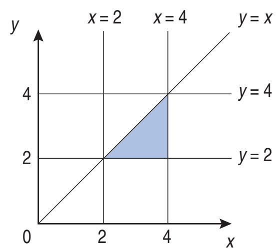
- An integer n satisfies $^-8 < 2n \le 10$.
  List all the possible values of n.
  ($n = {}^-3, {}^-2, {}^-1, 0, 1, 2, 3, 4, 5$)
- Find all the possible integer values of q that satisfy
  $2q < 17$ and $3q > 7$.
- Show the solution of the inequality $^-2 < z \le 3$ on a number line, for example:

- Solve these inequalities, marking the solution set on a number line:
  a. $3n + 4 < 17$ and $n > 2$
  b. $2(x - 5) \le 0$ and $x > {}^-2$
- The variable y satisfies each of these inequalities:
  $5 - 2y \le 13$
  $4y + 6 \le 10$
  Mark the solution set for y on a number line.
- The variable b satisfies each of these inequalities:
  $31.62 \le b \le 31.83$ and $31.95 > b > 31.74$
  Mark the solution set for b on a number line.



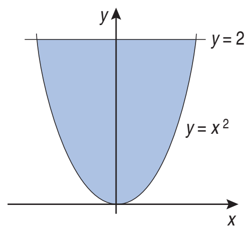
Begin to solve inequalities in two variables.
For example:

- This pattern is formed by straight-line graphs of equations in the first quadrant.



Write three inequalities to describe fully the shaded region.

- The shaded region is bounded by the line $y = 2$ and the curve $y = x^2$.



Circle two inequalities that fully describe the shaded region.

$y < x^2$	$x \le 0$	$y \le 2$	$y < 0$
$y \ge x^2$	$x > 0$	$y > 2$	$y \ge 0$

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Use systematic trial and improvement methods and ICT tools to find solutions, or approximate solutions, to non-linear equations	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Use algebraic methods to solve simple non-linear equations. For example:

- Solve these equations exactly.
  Each has two solutions.

  a. $c^2 + 24 = 60$
  b. $\frac{9}{y+2} = y + 2$
  c. $x^2 - 5 = 220$
  d. $3 = \frac{12}{x^2}$

Use systematic trial and improvement methods and ICT tools to find solutions, or approximate solutions, to equations.

Use a **calculator** to answer questions such as:

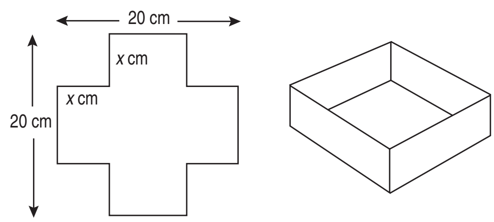
- Solve these equations, giving answers correct to two decimal places:
  a. $s^3 = 30$
  b. $a^3 + a = 50$
  c. $z^3 - z = 70$
  d. $5.5p^3 = 9.504$

- The product of three consecutive odd numbers is 205 143. Find the numbers.

- A cuboid has a square cross-section (side x cm), height 20cm and total surface area 800cm². Form and solve an equation in x, giving the answer correct to one decimal place.

Set up an equation for a problem and find an approximate solution using a **spreadsheet** or a **graph plotter**. For example:

- A small open box is made by starting with a sheet of metal 20 cm by 20cm, cutting squares from each corner and folding pieces up to make the sides.



  The box is to have a capacity of 450 cm³ to its rim. Use a **spreadsheet** to find what size of square, to the nearest millimetre, should be cut from the corners.

  Use a **graphical calculator** or **graph plotting program** to find what size of square should be cut from the corners to make a box with the maximum possible volume.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Use systematic trial and improvement methods and ICT tools to find solutions, or approximate solutions, to non-linear equations (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

- The length of one side of a rectangle is y.
  This equation shows the area of the rectangle.

  $$y(y + 2) = 67.89$$

  Find the value of y. Show your working.

  You may find this table helpful.

y	$y + 2$	$y(y + 2)$	
8	10	80	too large

- Complete the table below for values of x and y for the equation $y = x^3 - x - 10$.

x	0	1	2	3	4	5	6
y							

  The value of y is 0 for a value of x between 2 and 3.
  Find the value of x, to 1 decimal place, that gives the value of y closest to 0.

  Use trial and improvement.

  Using a **spreadsheet,** the solution lies between 2 and 3.

	A	B
1	x	$x^3 - x - 10$
2	2	=A2*A2*A2–A2–10
3	=A2+0.1	=A3*A3*A3–A3–10
4	=A3+0.1	=A4*A4*A4–A4–10
5	=A4+0.1	=A5*A5*A5–A5–10
6	=A5+0.1	=A6*A6*A6–A6–10
7	=A6+0.1	=A7*A7*A7–A7–10
8	=A7+0.1	=A8*A8*A8–A8–10
9	=A8+0.1	=A9*A9*A9–A9–10
10	=A9+0.1	=A10*A10*A10–A10–10
11	=A10+0.1	=A11*A11*A11–A11–10
12	=A11+0.1	=A12*A12*A12–A12–10

	A	B
1	x	$x^3 - x - 10$
2	2	–4
3	2.1	–2.839
4	2.2	–1.552
5	2.3	–0.133
6	2.4	1.424
7	2.5	3.125
8	2.6	4.976
9	2.7	6.983
10	2.8	9.152
11	2.9	11.489
12	3	14

  The table shows that y has the value 0 for a value of x between 2.3 and 2.4.

  Adjust column A to examine values of y for these values of x. This confirms that x = 2.31 gives the value of y closest to 0, that is, x = 2.3 to 1 d.p.

	A	B
1	x	$x^3 - x - 10$
2	2.3	–0.133
3	2.31	0.016391
4	2.32	0.167168
5	2.33	0.319337
6	2.34	0.472904
7	2.35	0.627875
8	2.36	0.784256
9	2.37	0.942053
10	2.38	1.101272
11	2.39	1.261919
12	2.4	1.424

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Set up and use equations to solve word and other problems involving direct proportion	

As outcomes, Year 8 pupils should, for example:

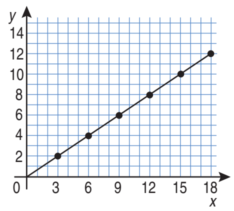
Begin to use graphs and set up equations to solve simple problems involving direct proportion.

Discuss practical examples of direct proportion, such as:

- the number of euros you can buy for any amount in pounds sterling (no commission fee);
- the number of kilometres equal to a given number of miles, assuming 8 km to every 5 miles;
- the cost of tennis balls, originally at £3 each, offered at £2 each in a sale. For example, generate sets of proportional pairs by multiplying and test successive ratios to see if they are equal:

Original price £3 $\xrightarrow{\times 2}$ £6 $\xrightarrow{\times 3}$ £9 $\xrightarrow{\times 4}$ £12...
Sale price £2 $\xrightarrow{\times 2}$ £4 $\xrightarrow{\times 3}$ £6 $\xrightarrow{\times 4}$ £8...

Check by drawing a graph, using pencil and paper or **ICT**.

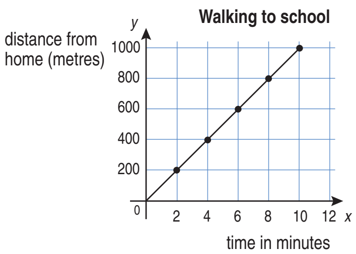


Observe that the points lie in a straight line through the origin, and that the sale price (y) is related to the original price (x) by the formula:

$y = \frac{2}{3}x$

Discuss the distance travelled in a given time, assuming constant speed. For example:

- Consider a walking speed of 200 metres every 2 minutes.



Generate (distance, time) pairs.
Observe that distance/time is constant.
Use this relationship to find the distance walked in:
a. 12 minutes, b. 7 minutes.

Link to ratio and proportional reasoning in number (pages 78–81).

As outcomes, Year 9 pupils should, for example:

Solve problems involving proportion using algebraic methods, relating algebraic solutions to graphical representations of the equations. For example:

Understand direct proportion as equality of ratios.
If variables x and y take these values:

x	1	2	3	4	5	6	...
y	3.5	7	10.5	14	17.5	21	...

- the ratios of corresponding values of y and x are equal:

$$\frac{y}{x} = \frac{3.5}{1} = \frac{7}{2} = \frac{10.5}{3} = \frac{14}{4} = \frac{17.5}{5} = \frac{21}{6} = \ldots = 3.5$$

- y is said to be directly proportional to x;
- the relationship between the variables is expressed by $y = 3.5x$;
- the graph of y against x will be a straight line through the origin.

Check whether two sets of values are in direct proportion by comparing corresponding ratios.

Use algebraic methods to solve problems such as:

- Green paint is made by mixing 11 parts of blue paint with 4 parts of yellow paint.
  How many litres of blue paint would be needed to mix with 70 litres of yellow paint?

  <u>Algebraic method</u>
  Let b be the number of litres of blue paint needed.

  $$\frac{\text{blue}}{\text{yellow}} \qquad \frac{b}{70} = \frac{11}{4}$$

  How many litres of blue paint would be needed to make up 100 tins of green paint?

  $$\frac{\text{blue}}{\text{green}} \qquad \frac{b}{100} = \frac{11}{15}$$

Link proportionality to work in science.
Appreciate that where data are obtained from experimental measurements:

- limitations in measuring mean that exactly equal ratios are unlikely and plotted points may only approximate to a straight line;
- significant deviations of individual observations often indicate experimental error, such as misreading a value.

Link to ratio and proportion in number (pages 78–81), gradients of lines of the form y = mx (pages 166–7), graphs of real situations (pages 172–7), lines of best fit (pages 266–7), and scale factor (pages 212–15).

Pupils should be taught to:

Use formulae from mathematics and other subjects

As outcomes, Year 7 pupils should, for example:

Substitute positive integers into simple linear expressions.

For example:

- Substitute positive integer values into:

  $x + y - z$ $3(x + y)$ $20/x$
  $9y - x$ $2(8 - x)$ $x/2 - 6$

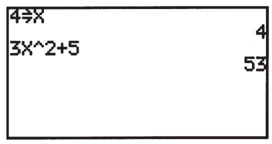
- Check that these statements are true for particular values by substituting the values into each expression and its simplification.

  $a + a + a = 3a$ $3n + 2n = 5n$ $6n/2 = 3n$
  $b + 2b + b = 4b$ $3(n + 2) = 3n + 6$

- Use a **spreadsheet**.
  Enter a formula such as 3A + B in column C. Find six different ways of putting different numbers in columns A and B to produce, say, 56 in column C.

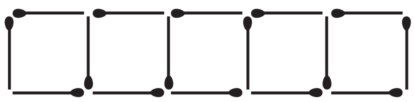
  Try other formulae in column C.

- Use a **graphical calculator** to substitute numbers in expressions such as $3x^2 + 5$.



- If n is an integer, the expressions
  $2n + 1$, $2n + 3$, $2n + 5$, $2n + 7$ and $2n + 9$
  represent a set of five consecutive odd numbers.
  Explain why.
  What value of n would produce the set of odd numbers 9, 11, 13, 15, 17?

- The expression $3s + 1$ gives the number of matches needed to make a row of s squares.



  How many matches are needed to make a row of 13 squares?

As outcomes, Year 8 pupils should, for example:

Substitute positive and negative numbers into linear expressions and positive integers into simple expressions involving powers.

For example:

- Find the value of these expressions when $a = 4$.
  $3a^2 + 4$ $\quad$ $2a^3$
- Find the value of these expressions when $x = 2.5$.
  $4x + 3$ $\quad$ $2 - 3x$ $\quad$ $7(x - 1)$
- Find the value of y when $x = {}^{-}3$.
  $y = \frac{2x + 3}{x}$ $\quad$ $y = \frac{x - 1}{x + 1}$
- Use a short **computer program**. For example:

```
10 CLS
20 INPUT A
30 INPUT B
40 C = 6*A–2*B
50 PRINT C
60 PRINT
```

```
TO EXPRESSION :A :B
MAKE "C 6*:A–2*:B
PRINT :C
END
```

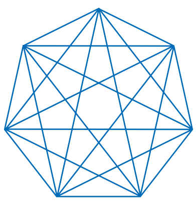
  Find different ways of inputting different values for A and B to print a particular value for C.
  Try different formulae.

- Use a **spreadsheet** to explore what happens when different values are substituted in an expression. For example:

	A	B	C
1	–3	7	=A1*A1–B1
2	–2	2	=A2*A2–B2
3	–1	8	=A3*A3–B3
4	0	–3	=A4*A4–B4
5	1	1	=A5*A5–B5
6	2	–4	=A6*A6–B6
7	3	5	=A7*A7–B7

- The number of diagonals in a polygon with n sides is given by the expression

  $\frac{n(n - 3)}{2}$



  How many diagonals are there in a polygon with 20 sides?

- In this magic square, choose different values for m, p and q and substitute them.

$m - p$	$m + p - q$	$m + q$
$m + p + q$	m	$m - p - q$
$m - q$	$m - p + q$	$m + p$

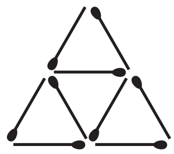
  What values for m, p and q will give a magic square filled with the numbers 1 to 9?

As outcomes, Year 9 pupils should, for example:

Substitute positive and negative numbers into linear expressions and expressions involving powers.

For example:

- Find the value of these expressions
  $3x^2 + 4$ $\quad$ $4x^3 - 2x$
  when $x = {}^{-}3$, and when $x = 0.1$.
- Find the values of a and b when $p = 10$.
  $a = \frac{3p^3}{2}$ $\quad$ $b = \frac{2p^2(p - 3)}{7p}$
- A triangle of matches is made like this.



  If the triangle has R rows, the number of matches needed is

  $\frac{1}{2}(3R^2 + 3R)$

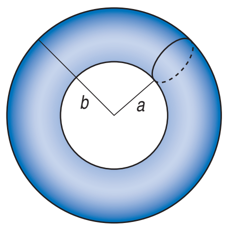
  How many matches are needed for a triangle with 17 rows?

- A sphere with diameter 3.6 cm is made using clay. The volume of a sphere is

  $\frac{\pi d^3}{6}$, where d is the diameter.

  Work out the volume of clay in the sphere. Give your answer to a sensible degree of accuracy.

  More clay is used to make this shape, a torus, with radii $a = 4.5$ and $b = 7.5$.



  Its volume is $\frac{1}{4}\pi^2(a + b)(b - a)^2$.
  Work out the volume of clay used.

- Here are two formulae.

  $P = s + t + \frac{5\sqrt{(s^2 + t^2)}}{3}$ $\quad$ $A = \frac{1}{2}st + \frac{(s^2 + t^2)}{9}$

  Work out the values of P and A when $s = 1.7$ and $t = 0.9$.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use formulae from mathematics and other subjects (continued)

Explain the meaning of and substitute integers into formulae expressed in words, or partly in words, such as:

- number of days = 7 times the number of weeks
- cost = price of one item × number of items
- age in years = age in months ÷ 12
- pence = number of pounds × 100
- area of rectangle = length times width
- cost of petrol for a journey
  = cost per litre × number of litres used

Progress to substituting into formulae such as:

- conversion of centimetres c to metres m:
  $m = c \div 100$
- the area A of a rectangle of length l and width w:
  $A = lw$
- the perimeter P of a rectangle of length l and width w:
  $P = 2l + 2w$ or $P = 2(l + w)$
- the area A of a right-angled triangle with base b and height h:
  $A = \frac{1}{2}bh$

As outcomes, Year 8 pupils should, for example:

Explain the meaning of and substitute numbers into formulae such as:

- the volume V of a cuboid of length l, breadth b and height h:
  $V = lbh$
- the surface area S of a cuboid with width w, depth d and height h:
  $S = 2dw + 2dh + 2hw$

Answer questions such as:

- The voltage V in an electrical circuit, with current I and resistance R, is given by the formula:
  $V = IR$
  What is V when I = 5 and R = 7?
  What is R when V = 42 and I = 3?

In simple cases, find an unknown where it is not the subject of the formula and where an equation must be solved. For example:

- The formula for the change £C from £50 for d compact discs at £7 each is C = 50 – 7d.

  If C = 15, what is d?

- The formula for the perimeter P of a rectangle l by w is
  $P = 2(l + w)$.

  If P = 20 and l = 7, what is w?

As outcomes, Year 9 pupils should, for example:

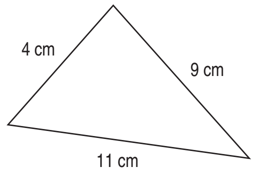
Explain the meaning of and substitute numbers into formulae such as:

- A formula to convert C degrees Celsius to F degrees Fahrenheit:
  $F = \frac{9C}{5} + 32$ or $F = \frac{9}{5}(C + 40) - 40$
- The Greek mathematician Hero showed that the area A of a triangle with sides a, b and c is given by the formula

  $A = \sqrt{s(s - a)(s - b)(s - c)}$,

  where $s = \frac{1}{2}(a + b + c)$.

  Use Hero's formula to find the area of this triangle.

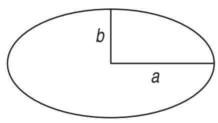


Find an unknown where it is not the subject of the formula and where an equation must be solved. For example:

- The surface area S of a cuboid of length l, width w and height h is
  $S = 2lw + 2lh + 2hw$

  What is h if S = 410, l = 10 and w = 5?

- The area A cm^2 enclosed by an ellipse is given by
  $A = \pi ab$



  Calculate to one decimal place the length a cm, if b = 3.2 and A = 25.

In simple cases, change the subject of a formula, using inverse operations. For example:

- Make I or R the subject of the formula
  $V = IR$
- Make l or w the subject of the formula
  $P = 2(l + w)$
- Make b or h the subject of the formulae
  $A = \frac{1}{2}bh$ $V = lbh$
- Make r the subject of the formulae
  $C = 2\pi r$ $A = \pi r^2$
  $V = \pi r^2 h$ $V = \frac{4}{3}\pi r^3$
- Make u, a or t the subject of the formula
  $v = u + at$
- Make C the subject of the formula
  $F = \frac{9C}{5} + 32$

- Make l the subject of the formula
  $T = 2\pi\sqrt{(l/g)}$
- Make u or v the subject of the formula
  $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use formulae from mathematics and other subjects (continued)

Derive simple algebraic expressions and formulae.
Check for correctness by substituting particular values.

For example:

- You have p pencils.
  a. Rashida has twice as many pencils as you have. How many pencils does Rashida have?
  b. You give away 2 pencils. How many pencils do you have left?
  c. Rashida shares her pencils equally between herself and 4 other friends. How many pencils do they each get?

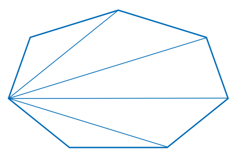
- Jo plants cherry trees, plum trees, apple trees and pear trees. n stands for the number of cherry trees Jo plants.
  a. Jo plants the same number of plum trees as cherry trees. How many plum trees does she plant?
  b. Jo plants twice as many apple trees as cherry trees. How many apple trees does she plant?
  c. Jo plants 7 more pear trees than cherry trees. How many pear trees does she plant?
  d. How many trees does Jo plant altogether? Write your answer as simply as possible.

Derive formulae such as:

- the number c of connecting lines joining m dots to n dots:
  $c = m \times n$

- the number s of 1-metre square concrete slabs that will surround a rectangular ornamental pond that is 1 metre wide and m metres long:
  $s = 2m + 6$

- the number D of (non-intersecting) diagonals from a single vertex in a polygon with n sides:
  $D = n - 3$



Link to finding the nth term of a sequence (pages 154–7).

Use a **spreadsheet** to construct simple formulae to model situations such as:

- the number of pints in a gallon;
- a currency conversion chart for use when going on a foreign holiday.

	A	B	C	D	E	F	G	H
1	£ (pounds sterling)	10	=B1+10	=C1+10	=D1+10	=E1+10	=F1+10	=G1+10
2	$ (US dollars)	=B1*1.6	=C1*1.6	=D1*1.6	=E1*1.6	=F1*1.6	=G1*1.6	=H1*1.6

	A	B	C	D	E	F	G	H
1	£ (pounds sterling)	10	20	30	40	50	60	70
2	$ (US dollars)	16	32	48	64	80	96	112

As outcomes, Year 8 pupils should, for example:

Derive algebraic expressions and formulae. Check by substituting particular values.

For example:

- Mr Varma bought n apples and some oranges.
  a. He had 4 times as many oranges as apples. How many oranges did he have?
  b. He had 3 oranges left after making a pudding. How many oranges did he use?
  c. He used half the apples in a pie and his son ate one. How many apples were left?

- José has x euros and Juan has y euros. Write equations for each of these statements.
  a. José and Juan have a total of 2000 euros.
  b. Juan has four times as many euros as José.
  c. If Juan gave away 400 euros he would then have three times as many euros as José.
  d. If Juan gave 600 euros to José they would both have the same number of euros.
  e. Half of José's euros is the same as two fifths of Juan's.

Derive formulae such as:

- the number f of square faces that can be seen by examining a stack of n cubes:
  $f = 4n + 2$
- the sum S of the interior angles of a polygon with n sides:
  $S = (n - 2) \times 180°$
- the area A of a parallelogram with base b and height h:
  $A = b \times h$
- the number n half way between two numbers n_1 and n_2:
  $n = \frac{n_1 + n_2}{2}$

Link to finding the nth term of a sequence (pages 154–7).

Use a **spreadsheet** to construct simple formulae to model situations such as:

- petrol for a car that uses 1 litre for every 8 miles;
- sale prices at 5% discount.

	A	B	C
1	Original price (£)	5% discount (£)	Sale price (£)
2	50	=0.05*A2	=A2–B2
3	=A2+5	=0.05*A3	=A3–B3
4	=A3+5	=0.05*A4	=A4–B4
5	=A4+5	=0.05*A5	=A5–B5
6	=A5+5	=0.05*A6	=A6–B6

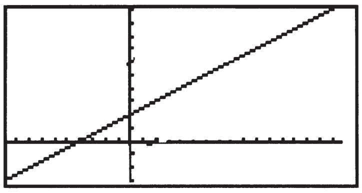
	A	B	C
1	Original price (£)	5% discount (£)	Sale price (£)
2	50	2.5	47.5
3	55	2.75	52.25
4	60	3	57
5	65	3.25	61.75
6	70	3.5	66.5

As outcomes, Year 9 pupils should, for example:

Derive more complex algebraic expressions and formulae. Check by substituting particular values.

For example:

- To cook a chicken allow 20 minutes per ½ kg and another 20 minutes.
  A chicken weighs x kg.
  Write an expression to show the number of minutes m to cook a chicken.



Link to conversion graphs (pages 172–3), and graphs of the form y = mx + c (pages 164–9).

Derive formulae such as:

- Euler's formula for a plane network, where N = number of nodes, R = number of regions and A = number of arcs:
  $N + R = A + 2$
- the area A of a trapezium with parallel sides a and b, and height h:
  $A = \frac{1}{2}(a + b) \times h$
- the area A of an annulus with outer radius r_1 and inner radius r_2:
  $A = \pi(r_1^2 - r_2^2)$
- the perimeter p of a semicircle with radius r:
  $p = r(\pi + 2)$

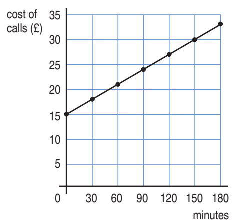
Link to finding the nth term of a sequence (pages 154–7).

Use a **spreadsheet** to construct formulae to model situations such as:

- the height of bounce when a rubber ball is dropped from varying heights;
- a mobile phone tariff based on a monthly rental charge plus the cost per minute of calls.

	A	B	C	D
1	Rent per month (£)	Cost of calls per minute (£)	Number of minutes	Tost cost (£)
2				=A2+(B2*C2)

	A	B	C	D
1	Rent per month (£)	Cost of calls per minute (£)	Number of minutes	Tost cost (£)
2	£15	£0.1	30	£18



Pupils should be taught to:

Generate and describe sequences

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
sequence, term, nth term, consecutive, rule, relationship, generate, predict, continue… increase, decrease… finite, infinite…

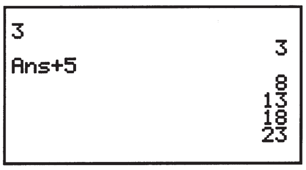
Know that:
- A number sequence is a set of numbers in a given order.
- Each number in a sequence is called a term.
- Terms next to each other are called consecutive and are often separated by commas, e.g. 6, 8, 10 and 12 are consecutive terms in a sequence of even numbers.

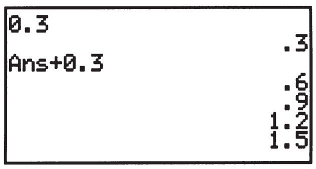
Know that a sequence can have a finite or an infinite number of terms, and give simple examples. For example:
- The sequence of counting numbers, 1, 2, 3, 4, 5, …, is infinite; dots indicate that counting continues indefinitely.
- The sequence of two-digit even numbers, 10, 12, 14, …, 98, is finite; dots indicate that the sequence continues in the same way until the final value 98 is reached.

Give simple examples of number sequences that:
- follow a simple rule that is easy to describe (e.g. odd numbers on house doors, square numbers);
- follow a more complex rule (e.g. time of sunset each day);
- follow an irregular pattern, affected by different factors (e.g. the maximum temperature each day);
- consist of a random set of numbers (e.g. numbers in the lottery draw).

Explore and predict terms in sequences generated by counting in regular steps, e.g. describe the sequences in words then use a simple **computer program** or **graphical calculator** to generate them and similar sequences.
Extend to decimals and negative numbers. For example:
- 8, 16, 24, 32, 40, …
- 5, 13, 21, 29, 37, …
- 89, 80, 71, 62, 53, …

- Start at 0 and count on in steps of 0.5. Each term is a multiple of 0.5. — 0.5, 1, 1.5, 2, …
- Start at 41 and count back in steps of 5. Each term is a multiple of 5 plus 1. — 41, 36, 31, 26, …





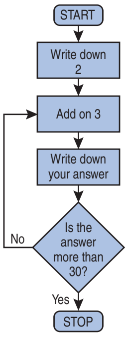
Begin to categorise familiar types of sequence. For example:
- Sequences can be ascending (the terms get bigger), or descending (the terms get smaller).
- Some sequences increase or decrease by equal steps.
- Some sequences increase or decrease by unequal steps.

See Y456 examples (pages 16–17).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
difference pattern, general term, T(n)…
flow chart… linear sequence, arithmetic sequence…

Generate sequences from **flow charts**. For example:

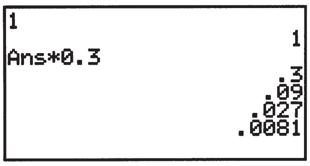


Continue familiar sequences.
For example:

- Square numbers 1, 4, 9, 16, …
- Powers of 10 10, 100, 1000, 10 000, …
- Powers of 2 64, 32, 16, …

Generate sequences by multiplying or dividing by a constant factor. For example:

- 1, $^{-}2$, 4, $^{-}8$, 16, $^{-}32$, …
- 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, …
- 1, 0.5, 0.25, 0.125, …



Generate sequences by counting forwards or backwards in increasing or decreasing steps.
For example:

- Start at 30 and count forwards by 1, 2, 3, … to give 31, 33, 36, …
- Start at 1 and count forwards by 2, 3, 4, … to give the triangular numbers 1, 3, 6, 10, …

Know that, unless a rule is specified, sequences may not continue in the most obvious way and 'spotting a pattern' may lead to incorrect conclusions.
For example:

- Explain why 1, 2, 4, … may continue 1, 2, 4, 8, 16, … or 1, 2, 4, 7, 11, … or in other ways.
- Explain why 1, 2, 3, … may continue 1, 2, 3, 4, 5, … or 1, 2, 3, 5, 8, … or in other ways.

Begin to appreciate that:

- Seeing a pattern in results enables predictions to be made, but any prediction must always be checked for correctness.
- A satisfactory conclusion is reached only when a general explanation of either a term-to-term or a position-to-term rule is given and the pattern can be justified.

As outcomes, Year 9 pupils should, for example:

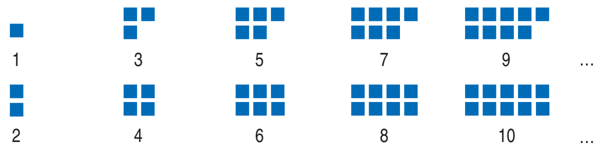
Use vocabulary from previous years and extend to:
quadratic sequence…
first difference, second difference…

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Generate and describe sequences (continued)

Generate and describe simple integer sequences and relate them to geometrical patterns. For example:

- Odd and even numbers

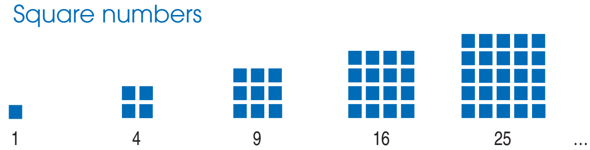


Know that:
Odd numbers are so called because, when arranged in pairs, there is always an odd one left unmatched.
Even numbers are so called because they can be arranged in matched pairs.

- Multiples of 3

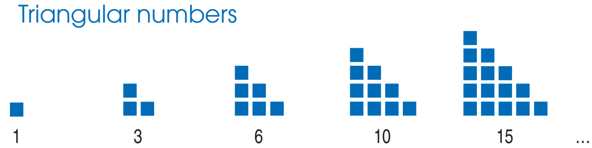
Know that multiples of 3 can be arranged in rectangular patterns of width 3.

- Square numbers



Know that square numbers make square patterns of dots and that 1 is counted as the first square number.

- Triangular numbers



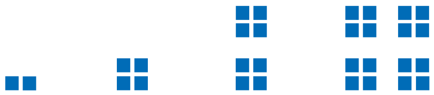
Know that triangular numbers can be arranged in triangular patterns of dots and that 1 is counted as the first triangular number.

Link to work on integers, powers and roots (pages 48–59).

As outcomes, Year 8 pupils should, for example:

Generate and describe integer sequences, relating them to geometrical patterns. For example:

- Powers of 2



Visualise powers of 2 as:
a row of 2 dots,
a square of 4 dots,
a column of two 4-dot squares,
a square of four 4-dot squares…

- Growing rectangles

Classify familiar sequences according to whether they are ascending or descending and whether they do so by equal or unequal steps.

Know that an arithmetic sequence is generated by starting with a number a, then adding a constant number d to the previous term. For example:

- If a = 2 and d = 3, the sequence is
  2, 5, 8, 11, 14, …
- If a = 4 and d = ⁻2, the sequence is
  4, 2, 0, ⁻2, ⁻4, …

Link to work on integers, powers and roots (pages 48–59).

As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:

Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence, on paper and using ICT

As outcomes, Year 7 pupils should, for example:

Generate terms of a sequence given a rule for finding each term from the previous term, on paper and using **ICT**.
For example:

- Generate the first few terms of these sequences and then describe them in words:

1st term	Term-to-term rule
10	add 3 Each term is 3 more than the one before.
100	subtract 5 Each term is 5 less than the one before.
2	double Each term is double the one before.
5	multiply by 10 Each term is 10 times the one before.

- Here is a rule for a sequence: To find the next term add 3.
  There are many sequences with this rule.
  Is it possible to find one for which:
  a. all the numbers are multiples of 3?
  b. all the numbers are odd?
  c. all the numbers are multiples of 9?
  d. none of the numbers is a whole number?
  Explore with a **graphical calculator**.

Generate a sequence given a rule for finding each term from its position in the sequence. For example:

- The nth term of a sequence is $n + 3$. Write the first five terms.
- The nth term of a sequence is:
  a. $n + 7$ b. $105 - 5n$ c. $2n - 0.5$ d. $4n$
  Write the first five terms of each sequence.
  Describe each sequence in words (e.g. odd numbers), or using a rule for generating successive terms (e.g. first term is 1, rule is 'add 2').

Recognise that sequences of multiples can be generated in two ways:

- They can be generated by using a term-to-term rule of repeated addition of the number, e.g. for multiples of 6:
  $6, \quad 6 + 6, \quad 6 + 6 + 6, \quad 6 + 6 + 6 + 6, \quad \ldots$
  1st term = 6 Term-to-term rule 'add 6'
  The difference between consecutive terms is always 6.

	A	B	C	D	E	F
1	Position	1	2	3	4	5
2	Term	6	=B2+6	=C2+6	=D2+6	=E2+6

- Each multiple can be calculated from its position, using a position-to-term rule:

1st term	2nd term	3rd term	…	nth term
1×6	2×6	3×6	…	$n \times 6$ (or $6n$)

	A	B	C	D	E	F
1	Position	1	2	3	4	5
2	Term	=B1*6	=C1*6	=D1*6	=E1*6	=F1*6

  The nth term can be found more quickly using a position-to-term rule, particularly for terms a long way into the sequence, such as the 100th term.

As outcomes, Year 8 pupils should, for example:

Generate terms of a sequence given a rule for finding each term from the previous term, on paper and using **ICT**. For example:

- Generate and describe in words these sequences.

1st term(s)	Term-to-term rule
8	subtract 4
1	add consecutive odd numbers, starting with 3
28	halve
1 000000	divide by 10
1, 2, …	add the two previous terms

- Here is a rule to find the next term of a sequence:
  Add ☐.
  Choose a first term for the sequence and a number to go in the box in such a way to make all the terms of the sequence:
  a. even;
  b. odd;
  c. multiples of 3;
  d. all numbers ending in the same digit.
  Explore with a **graphical calculator**.

Generate a sequence given a rule for finding each term from its position in the sequence, referring to terms as T(1) = first term, T(2) = second term, …, T(n) = nth term. For example:

- The nth term of a sequence is 2n, i.e. T(n) = 2n. Write the first five terms.

- Write the first five terms of a sequence whose nth term or T(n) is:
  a. 5n + 4 c. 99 – 9n e. 3n – 0.1
  b. 105 – 5n d. n + ½ f. n × 0.1

Use a **spreadsheet** or **graphical calculator** to find particular terms such as:
- the 24th multiple of 13 in the sequence 13, 26, 39, …
- the 100th multiple of 27 in the sequence 27, 54, 71, …

- Write the nth multiple of 18 in the sequence 18, 36, 54, …

As outcomes, Year 9 pupils should, for example:

Generate terms of a sequence given a rule for finding each term from the previous term, on paper and using **ICT**. For example:

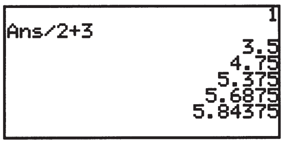
Review general properties of linear sequences of the form an + b, where a and b take particular values, e.g. 2n + 5, 3n – 7, 10 – 4n.

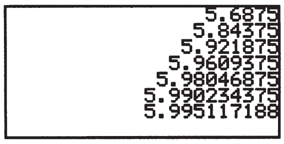
- The sequence can also be defined by
  1st term, T(1) = a + b
  term-to-term rule, 'add a to the previous term'
  so T(2) = a + T(1), etc.
- If the constant difference a between successive terms is positive, the sequence is ascending; if a is negative, the sequence is descending.

- Here is a rule for a sequence:
  To find the next term of the sequence add ☐.
  Choose a first term for the sequence and a number to go in the box in such a way that:
  a. every other number is an integer;
  b. every fourth number is an integer;
  c. there are exactly ten two-digit numbers in the sequence;
  d. every fourth number is a multiple of 5.
  Explore with a **graphical calculator**.

- The nth term of a sequence is given by
  T(n) = 2n + (n – 1)(n – 2)(n – 3)(n – 4)
  Write down the first five terms of the sequence.

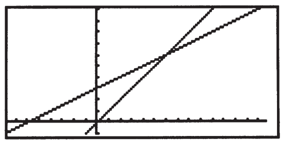
- The sequence 'divide by 2, add 3', starting with 1, can be represented as $x \rightarrow \frac{x}{2} + 3$.





Link the apparent limiting value of 6 to the solution of the equation $x = \frac{x}{2} + 3$

and to the intersection of the graphs of y = x and $y = \frac{x}{2} + 3$



Explore the effect of varying the numbers 2 and 3 on the limiting value.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence (continued)

Use a **spreadsheet** to generate tables of values and explore **term-to-term** and **position-to-term** relationships.
For example:

- The 1st term is 100, and the rule is 'subtract 5'.

	A	B	C	D	E	F	G	H
1	Position	1	=B1+1	=C1+1	=D1+1	=E1+1	=F1+1	=G1+1
2	Term	100	=B2–5	=C2–5	=D2–5	=E2–5	=F2–5	=G2–5

	A	B	C	D	E	F	G	H
1	Position	1	2	3	4	5	6	7
2	Term	100	95	90	85	80	75	70

Arrange a sequence in a table. For example:

- Multiples of 3:

Position	1	2	3	4	5	…	n
Term	3	6	9	12	15	…	3 × n

As outcomes, Year 8 pupils should, for example:

Use a **spreadsheet** to generate tables of values and explore **term-to-term** and **position-to-term** linear relationships. For example:

- The nth term is $3n + 7$.

	A	B
1	Position	Term
2	1	=A2*3+7
3	=A2+1	=A3*3+7
4	=A3+1	=A4*3+7
5	=A4+1	=A5*3+7
6	=A5+1	=A6*3+7
7	=A6+1	=A7*3+7
8	=A7+1	=A8*3+7
9	=A8+1	=A9*3+7
10	=A9+1	=A10*3+7

	A	B
1	Position	Term
2	1	10
3	2	13
4	3	16
5	4	19
6	5	22
7	6	25
8	7	28
9	8	31
10	9	34

Arrange a sequence in a table, referring to terms as T(1) = first term, T(2) = second term, …, T(n) = nth term. For example, for multiples of 3:

Position n	1		2		3		4		5		…	n
T(n)	3		6		9		12		15		…	T(n)
Difference		3		3		3		3			…	

Explain the effect on a sequence of multiples if a constant number is added to or subtracted from each term. For example:

- $T(n) = 2n + b$:
  $2n – 1$ generates the odd numbers, starting at 1, because each is one less than an even number.
  $2n + 1$ generates odd numbers, starting at 3.
  $2n + 2$ generates even numbers, starting at 4.
  $2n – 6$ generates even numbers, starting at $^{-}4$.

- $T(n) = 3n + b$:
  If b is a multiple of 3, this generates multiples of 3, starting at different numbers.
  Otherwise, it generates a sequence with a difference of 3 between consecutive terms.

- $T(n) = 10n + b$:
  If b is between 0 and 9, this generates numbers whose units digit is b.

Explain how descending sequences can be generated by subtracting multiples from a constant number: $T(n) = b – an$. For example:

- $T(n) = 6 – n$ generates descending integers:
  5, 4, 3, 2, 1, 0, $^{-}1$, $^{-}2$, $^{-}3$, $^{-}4$, $^{-}5$, …
- $T(n) = 110 – 10n$ generates the 10 times table backwards.

Explore difference patterns between terms of a linear sequence of the form $an + b$. Observe that the differences are constant and equal to a. Use this result

As outcomes, Year 9 pupils should, for example:

Begin to generate a quadratic sequence given a rule for finding each term from its position. For example:

- Generate the first ten terms of these sequences:
  $T(n) = n^2$ $T(n) = 2n^2 + 1$ $T(n) = n^2 – 2$

- The nth term of a sequence is $n/(n^2 + 1)$.
  The first term of the sequence is ½.
  Write the next three terms.
  Use a **spreadsheet** to generate terms beyond the third term.

Use **ICT** to generate non-linear sequences. For example:

- Use a **computer program** to generate sequences where the second row of differences (i.e. the increment in the step) is constant, as in:

	1st term	Initial step (1st difference)	Step increment (2nd difference)
a.	1	1	1
b.	58	$^{-}1$	$^{-}3$
c.	7	18	$^{-}2$
d.	100	$^{-}17$	5

- Generate a familiar sequence, e.g. square or triangular numbers.

- Generate the first few terms of the sequence of square numbers, $T(n) = n^2$, and examine term-to-term difference patterns:

1		4		9		16		25		36		…	
	3		5		7		9		11		…		
		2		2		2		2		…			

  Observe that the terms increase by successive odd numbers, and that the second row of differences in always 2.

- Explore the sequences generated by these formulae for T(n), by finding some terms and examining difference patterns:
  $3n^2 + 2$ $4 – 3n^2$ $n^2 + n$
  $n(n – 1)/2$ $4n – 6$ $n^2 + 4n – 6$

Know that rules of the form $T(n) = an^2 + bn + c$ generate quadratic sequences. Know that the second row of differences is constant and equal to 2a. Use this result when searching for a quadratic rule.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Generate terms of a sequence using term-to-term and position-to-term definitions of the sequence (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Find the next term and the nth term of a sequence where the rule is quadratic or closely related to $T(n) = n^2$.

Prove that, for a sequence of the form

$T(1) = a + b + c, \quad T(n) = an^2 + bn + c$

the second differences between successive terms are constant and equal to $2a$.

- Find the nth term of the sequence

  6, 15, 28, 45, 66, …

  ($T(1) = 6$, $T(n) = T(n - 1) + 9 + 4(n - 2)$)

  Pattern of differences:

  6 15 28 45 66 91 …

  9 13 17 21 25 …

  4 4 4 4

  The second differences are 4, so the sequence is of the form $T(n) = an^2 + bn + c$, and $a = 2$.

  $T(1) = 6$, so $2 + b + c = 6$

  $T(2) = 15$, so $8 + 2b + c = 15$

  Subtracting $T(1)$ from $T(2)$ gives $6 + b = 9$,
  so $b = 3$ and $c = 1$.
  So $T(n) = 2n^2 + 3n + 1$.

- Find the nth term of the sequence

  2, 5, 10, 17, 26, …

  ($T(1) = 2$, $T(n) = T(n - 1) + 3 + 2(n - 2)$)

  What is the nth term if:
  a. $T(1) = 0$? b. $T(1) = 9$?

Explore how some fraction sequences continue.
For example:

- $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots, 2^{-n}, \ldots$
- $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots, \frac{n}{(n+1)}, \ldots$
- $\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \ldots$
  (from the Fibonacci sequence)

- A flea sits in middle of circular table of radius 1 m. It takes a series of jumps towards the edge of the table. On the first jump it jumps half way to the edge of the table. On each succeeding jump it jumps half the remaining distance.
  Investigate the total distance travelled after 1, 2, 3, … jumps.
  How long will it take the flea to reach the edge of the table?

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

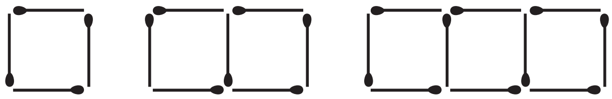
Find the nth term, justifying its form by referring to the context in which it was generated

Generate sequences from simple practical contexts.

- Find the first few terms of the sequence.
- Describe how it continues by reference to the context.
- Begin to describe the general term, first using words, then symbols; justify the generalisation by referring to the context.

For example:

- Growing matchstick squares

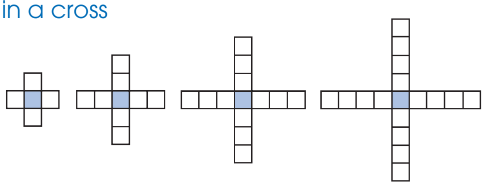


Number of squares	1	2	3	4	…
Number of matchsticks	4	7	10	13	…

Justify the pattern by explaining that the first square needs 4 matches, then 3 matches for each additional square, or you need 3 matches for every square plus an extra one for the first square.

In the nth arrangement there are 3n +1 matches.
Possible justification:
Every square needs three matches, plus one more for the first square, which needs four. For n squares, the number of matches needed is 3n, plus one for the first square.

- Squares in a cross



Size of cross	1	2	3	4	…
Number of squares	5	9	13	17	…

Justify the pattern by explaining that the first cross needs 5 squares, then 4 extra for the next cross, one on each 'arm', or start with the middle square, add 4 squares to make the first pattern, 4 more to make the second, and so on.

In the nth cross there are 4n + 1 squares.

Possible justification:
The crosses have four 'arms', each increasing by one square in the next arrangement. So the nth cross has 4n squares in the arms, plus one in the centre.

- Making different amounts using 1p, 2p and 3p stamps

Amount	1p	2p	3p	4p	5p	6p	…
No. of ways	1	2	3	4	5	7	…

From experience of practical examples, begin to appreciate that some sequences do not continue in an 'obvious' way and simply 'spotting a pattern' may lead to incorrect results.

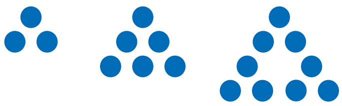
As outcomes, Year 8 pupils should, for example:

Generate sequences from practical contexts.

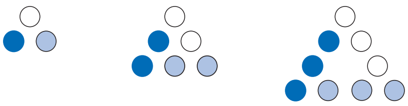
- Find the first few terms of the sequence; describe how it continues using a term-to-term rule.
- Describe the general (nth) term, and justify the generalisation by referring to the context.
- When appropriate, compare different ways of arriving at the generalisation.

For example:

- Growing triangles



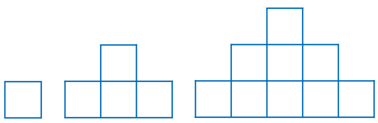
This generates the sequence: 3, 6, 9…
Possible explanations:
We add three each time because we add one more dot to each side of the triangle to make the next triangle.
It's the 3 times table because we get…



3 lots of 1 3 lots of 2 3 lots of 3 etc.

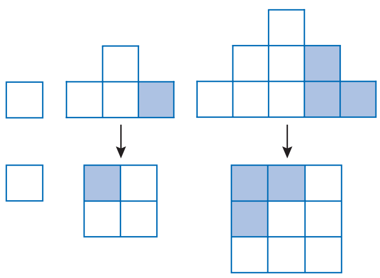
The general (nth) term is $3 \times n$ or $3n$.
Possible justification:
This follows because the 10th term would be '3 lots of 10'.

- 'Pyramid' of squares



This generates the sequence: 1, 4, 9, …
Possible explanation:
The next 'pyramid' has another layer, each time increasing by the next odd number 3, 5, 7, …

The general (nth) term is $n \times n$ or n^2.
Possible justification:
The pattern gives square numbers. Each 'pyramid' can be rearranged into a square pattern, as here:



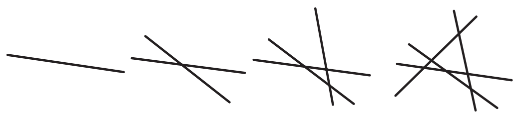
As outcomes, Year 9 pupils should, for example:

Generate sequences from practical contexts.

- Find the first few terms of the sequence; describe how it continues using a term-to-term rule.
- Use algebraic expressions to describe the nth term, justifying them by referring to the context.
- When appropriate, compare different ways of arriving at the generalisation.

For example:

- Maximum crossings for a given number of lines

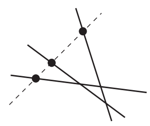


Number of lines	1	2	3	4	…
Maximum crossings	0	1	3	6	…

Predict how the sequence might continue and test for several more terms.

Discuss and follow an explanation, such as:

A new line must cross all existing lines.

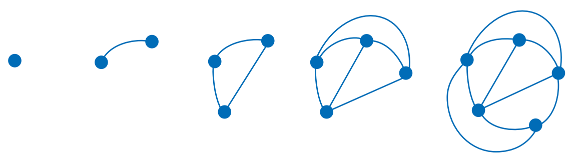


So when a new line is added, the number of extra crossings will equal the existing number of lines, e.g. when there is one line, an extra line adds one crossing, when there are two lines, an extra line adds two crossings, and so on.

No. of lines	1	2	3	4	5	…
Max. crossings	0	1	3	6	10	…
Increase		1	2	3	4	…

- Joining points to every other point

Joins may be curved or straight. Keeping to the rule that lines are not allowed to cross, what is the maximum number of joins that can be made?



No. of points	1	2	3	4	5 …
Maximum joins	0	1	3	6	9 …

Predict how the sequence might continue, try to draw it, discuss and provide an explanation.

Pupils should be taught to:

Find the nth term, justifying its form by referring to the context in which it was generated (continued)

As outcomes, Year 7 pupils should, for example:

Begin to find a simple rule for the nth term of some simple sequences.

For example, express in words the nth term of counting sequences such as:

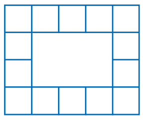
- 6, 12, 18, 24, 30, … nth term is six times n
- 6, 11, 16, 21, 26, … nth term is five times n plus one
- 9, 19, 29, 39, 49, … nth term is ten times n minus one
- 40, 30, 20, 10, 0, … nth term is fifty minus ten times n

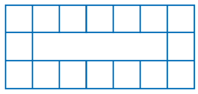
Link to generating sequences using term-to-term and position-to-term definitions (pages 148–51).

As outcomes, Year 8 pupils should, for example:

- Paving stones

  Investigate the number of square concrete slabs that will surround rectangular ponds of different sizes. Try some examples:





2 by 3 pond1 by 5 pond
needs 14 slabs needs 16 slabs

  Collect a range of different sizes and count the slabs. Deduce that, for an l by w rectangle, you need 2l + 2w + 4 slabs.
  Possible justification:
  You always need one in each corner, so it's twice the length plus twice the width, plus the four in the corners.

  Other ways of counting could lead to equivalent forms, such as 2(l + 2) + 2w or 2(l + 1) + 2(w + 1).

  Confirm that these formulae give consistent values for the first few terms.

Check predicted terms for correctness.

Link to simplifying and transforming algebraic expressions (pages 116–19).

Begin to use linear expressions to describe the nth term of an arithmetic sequence, justifying its form by referring to the activity from which it was generated.

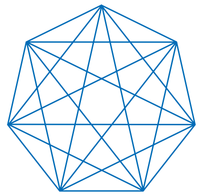
Develop an expression for the nth term of sequences such as:

- 7, 12, 17, 22, 27, … 2 + 5n
- 100, 115, 130, 145, 160, … 15n + 85
- 2.5, 4.5, 6.5, 8.5, 10.5, … (4n + 1)/2
- $^{-}12$, $^{-}7$, $^{-}2$, 3, 8, … 5n – 17
- 4, $^{-}2$, $^{-}8$, $^{-}14$, $^{-}20$, … 10 – 6n

As outcomes, Year 9 pupils should, for example:

- Diagonals of polygons

  How many diagonals does a polygon have altogether (crossings allowed)?



No. of sides	1	2	3	4	5	6	7	…
No. of diagonals	–	–	0	2	5	9	14	…

  Explain that, starting with 3 sides, the terms increase by 2, 3, 4, 5, …

  Follow an explanation to derive a formula for the nth term, such as:
  Any vertex can be joined to all the other vertices, except the two adjacent ones.
  In an n-sided polygon this gives n – 3 diagonals, and for n vertices a total of n(n – 3) diagonals. But each diagonal can be drawn from either end, so this formula counts each one twice.
  So the number of diagonals in an n-sided polygon is ½n(n – 3).

  Confirm that this formula gives consistent results for the first few terms.

Know that for sequences generated from an activity or context:

- Predicted values of terms need to be checked for correctness.
- A term-to-term or position-to-term rule needs to be justified by a mathematical explanation derived from consideration of the context.

Use linear expressions to describe the nth term of an arithmetic sequence, justifying its form by referring to the activity or context from which it was generated.

Find the nth term of any linear (arithmetic) sequence. For example:

- Find the nth term of 21, 27, 33, 39, 45, …

  The difference between successive terms is 6, so the nth term is of the form T(n) = 6n + b.
  T(1) = 21, so 6 + b = 21, leading to b = 15.
  T(n) = 6n + 15
  Check by testing a few terms.

- Find the nth term of these sequences:
  54, 62, 70, 78, 86, …
  68, 61, 54, 47, 40, …
  2.3, 2.5, 2.7, 2.9, 3.1, …
  $^{-}5$, $^{-}14$, $^{-}23$, $^{-}32$, $^{-}41$, …

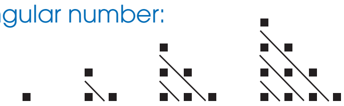
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Find the *n*th term, justifying its form by referring to the context in which it was generated (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Explore spatial patterns for triangular and square numbers. For example:

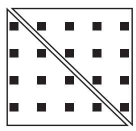
- Generate a pattern for specific cases of T(n), the nth triangular number:



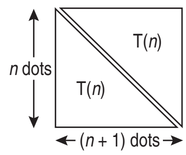
By considering the arrangement of dots, express T(n) as the sum of a series:

T(n) = 1 + 2 + 3 + … + n

By repeating the triangular pattern to form a rectangle, deduce a formula for T(n):

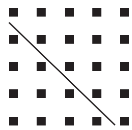


T(4) + T(4)
= 4 × 5



T(n) + T(n) = n(n + 1) or T(n) = ½n(n + 1)
Use this result to find the sum of the first 100 whole numbers: 1 + 2 + 3 + … + 100.

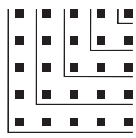
- Split a square array of dots into two triangles:

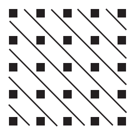


$T(4) + T(5) = 5^2$

Deduce the result $T(n - 1) + T(n) = n^2$.
Test it for particular cases.

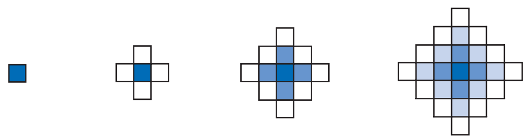
Consider other ways of dividing a square pattern of dots. For example:





Deduce results such as $1 + 3 + 5 + 7 + 9 = 5^2$.
Generalise to a formula for the sum of the first n odd numbers: $1 + 3 + 5 + … + (2n - 1) = n^2$.
Say what can be deduced from the other illustration of dividing the square.

- Certain 2-D 'creatures' start as a single square cell and grow according to a specified rule. Investigate the growth of a creature which follows the rule 'grow on sides':



Stage 1	Stage 2	Stage 3	Stage 4 …
1 cell	5 cells	13 cells	25 cells …

Investigate other rules for the growth of creatures.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

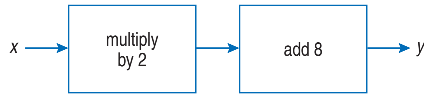
Express functions and represent mappings

Use, read and write, spelling correctly:
input, output, rule, function, function machine, mapping…

Express simple functions at first in words then using symbols.
For example:

Explore simple function machines by:
- finding outputs (y) for different inputs (x);
- finding inputs for different outputs.

For example:



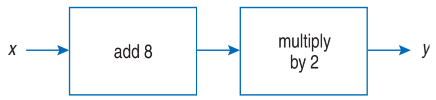
- What happens when the input is 5?
- What input gives an output of 40?

Descibe the effect of this function machine as $x \times 2 + 8 = y$.

Produce a table of inputs and outputs, such as:

x	1	2	3	4	5	6	7
y	10	12	14				

- What happens if the order of the machines is changed?



Descibe the effect of this function machine as $(x + 8) \times 2 = y$.

Participate in a function guessing game. For example:
- Find the secret rule that connects the blue number on the left to the black number on the right, e.g.
  (blue number − 1) × 2 = black number.

3	→	**4**
7	→	**12**
5	→	**8**
9	→	**16**

Participants in the game remain silent, but can:
A. offer a blue number, **or**
B. offer a black number for a given blue number, **or**
C. write in words what they think the rule is.

Know that the function which generates the output for a given input, can be expressed either by using a mapping arrow (→) or by writing an equation. For example:

blue number → (blue number − 1) × 2
or (blue number − 1) × 2 = black number
or black number = (blue number − 1) × 2

Draw simple mapping diagrams, e.g. for $x \to x + 2$.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
linear function…

Express simple functions in symbols.
For example:

Generate sets of values for simple functions using a function machine or a spreadsheet. For example:

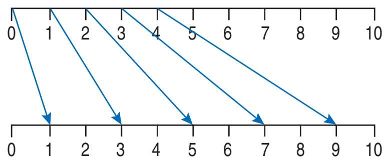
- Use a **spreadsheet** to produce a table of inputs and outputs, e.g. $x \to 2x + 8$ or $y = 2x + 8$.

	A	B
1	x	y
2	1	=A2*2+8
3	=A2+1	=A3*2+8
4	=A3+1	=A4*2+8
5	=A4+1	=A5*2+8
6	=A5+1	=A6*2+8
7	=A6+1	=A7*2+8
8	=A7+1	=A8*2+8

	A	B
1	x	y
2	1	10
3	2	12
4	3	14
5	4	16
6	5	18
7	6	20
8	7	22

Extend to negative and non-integral values.

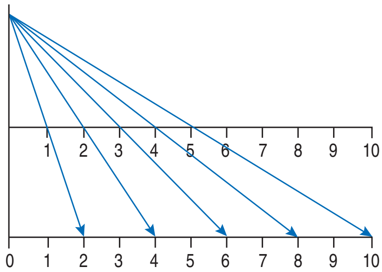
Draw mapping diagrams for simple functions.
For example, $x \to 2x + 1$:



Extend the mapping to include:
- negative integers down to ⁻10;
- fractional values.

Know some properties of mapping diagrams.
For example:

- Functions of the form $x \to x + c$ produce sets of parallel lines.
- Mapping arrows for multiples, if projected backwards, meet at a point on the zero line, e.g. $x \to 2x$:

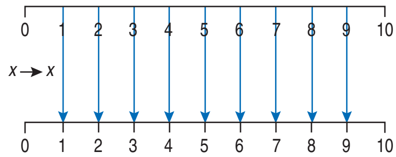


Link to enlargement by a whole-number scale factor (pages 212–13).

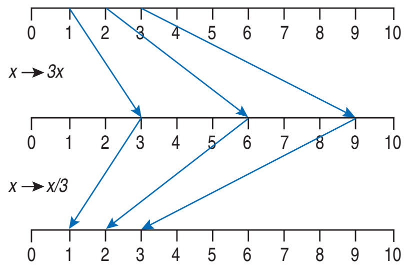
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
identity function, inverse function, quadratic function… inverse mapping… self-inverse…

Know that $x \to x$ is called the identity function, because it maps any number on to itself, i.e. leaves the number unchanged.



Know that every linear function has an inverse function which reverses the direction of the mapping. For example, the inverse of multiplying by 3 is dividing by 3, and this can be expressed in symbols: the inverse of $x \to 3x$ is $x \to x/3$.

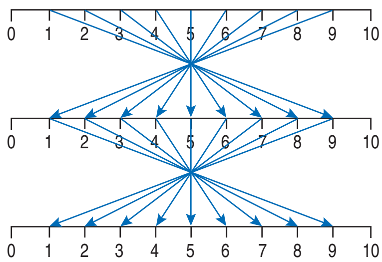


Find the inverse of a linear function such as:

- $x \to 3x + 1$
- $x \to 5x - 4$
- $x \to 2(x - 7)$
- $x \to \frac{x + 8}{10}$
- $x \to \frac{1}{4}x - 5$
- $x \to \frac{1}{2}x + 20$

Know that functions of the form $x \to c - x$ are self-inverse. For example:

- The inverse of $x \to 10 - x$ is $x \to 10 - x$.



Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

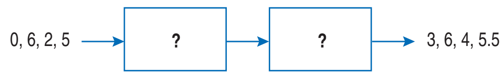
Express functions and represent mappings (continued)

Given inputs and outputs, find the function.
For example:

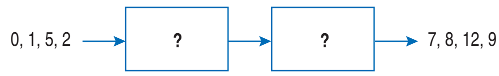
- Find the rule (single machine):

Multiply the input by 4 or, in symbols, $x \rightarrow 4x$.

- Find the rule (double machine):



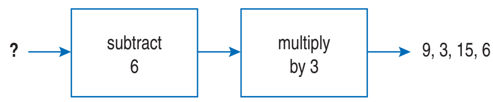
Divide by 2 and add 3, or $x \rightarrow x/2 + 3$.



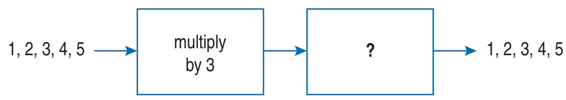
Different solutions are possible – and the two functions can be replaced by a single function.

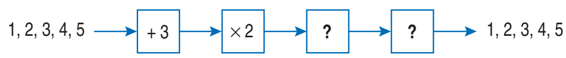
Explore inverse operations to find the input given the output.

- Given the output, find the input for a particular machine:



- Define the other machine(s):





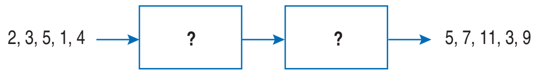
Begin to recognise some properties of simple functions.

- A function can sometimes be expressed in more than one way,
  e.g. red number → (red number – 1) × 2
  or red number → red number × 2 – 2
- A function can sometimes be expressed more simply, e.g.
  red number → red number × 3 × 5
  can be simplified to
  red number → red number × 15
- A function can often be inverted, e.g.
  if (red number – 1) × 2 = green number
  then green number ÷ 2 + 1 = red number

Link to inverse operations, equations and formulae (pages 114–15).

As outcomes, Year 8 pupils should, for example:

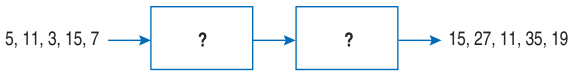
Given inputs and outputs, find the function. Given a linear function, put random data in order and use difference patterns to help find the function.
For example, find the rule:

- 

  Reorganise the data:

Input (x)	1	2	3	4	5
Output (y)	3	5	7	9	11
Difference	2	2	2	2	

  Recognise differences of 2. Try $x \to 2x + c$.
  From the first entry, find that c = 1.
  Check other values.

- 

  Reorganise the data:

Input (x)	3	5	7	11	15
Output (y)	11	15	19	27	35
Difference		4	4	8	8

  Recognise that the first two differences are 4, where x is increasing by 2 each time.
  Try $x \to 2x + c$. From the first entry, find that c = 5.
  Check other values.

Link linear functions to linear sequences, particularly difference patterns (pages 148–51).

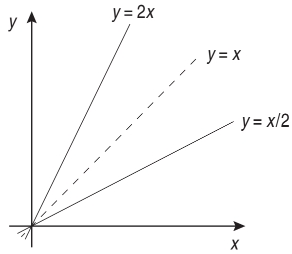
Know some properties of functions produced by combining number operations. For example:

- Two additions, two subtractions, or an addition with a subtraction, will simplify to a single addition or subtraction.
- Two multiplications, two divisions, or a multiplication with a division, will simplify to a single multiplication or division.
- A function may often be expressed in more than one way, e.g.
  $x \to 2x - 2$ is equivalent to $x \to 2(x - 1)$.
- Changing the order of two operations will often change the function, e.g.
  $x \to 3x - 4$ is different from $x \to 3(x - 4)$.
- The inverse of two combined operations is found by inverting the operations and reversing the order, e.g.
  the inverse of $x \to 2(x - 1)$ is $x \to x/2 + 1$.

Link to inverse operations, equations and formulae (pages 114–15).

As outcomes, Year 9 pupils should, for example:

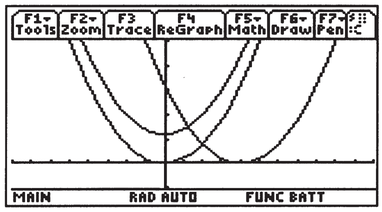
Plot the graph of a linear function, together with its inverse, on paper or using **ICT**. For example:



Observe the relationship between the two graphs: each is the reflection of the other in the line y = x.

Know some properties of quadratic functions and features of their graphs. For example:

- The graph is a curve, symmetrical about the vertical line through its turning point.
- The value of the y-coordinate at the turning point is either a maximum or a minimum value of the function.



Link to properties of quadratic sequences (pages 152–3), and plotting graphs of simple quadratic and cubic functions (pages 170–1).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Generate points and plot graphs of functions

Use, read and write, spelling correctly:
coordinates, coordinate pair/point, x-coordinate…
grid, origin, axis, axes, x-axis…
variable, straight-line graph, equation (of a graph)…

Generate and plot pairs of coordinates that satisfy a simple linear relationship. For example:

- $y = x + 1$
  (0, 1), (1, 2), (2, 3), (3, 4), (4, 5), …
- $y = 2x$
  (0, 0), (1, 2), (2, 4), ($^-1$, $^-2$), ($^-2$, $^-4$), …
- $y = 10 - x$
  (0, 10), (1, 9), (2, 8), …

Complete a table of values, e.g. to satisfy the rule $y = x + 2$:

x	$^-3$	$^-2$	$^-1$	0	1	2	3
$y = x + 2$	$^-1$	0	1	2	3	4	5

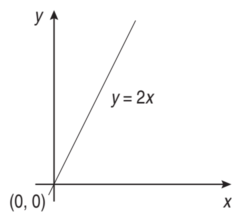
Plot the points on a coordinate grid. Draw a line through the plotted points and extend the line. Then:
- choose an intermediate point, on the line but not one of those plotted;
- read off the coordinate pair for the chosen point and check that it also fits the rule;
- do the same for other points, including some fraction and negative values.

Try this for other graphs.

Recognise that all points on a line will fit the rule.

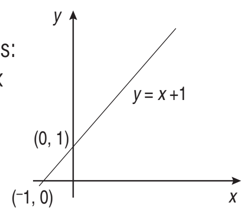
Begin to consider the features of graphs of simple linear functions, where y is given explicitly in terms of x. For example, construct tables of values then use paper or a **graph plotter** to:

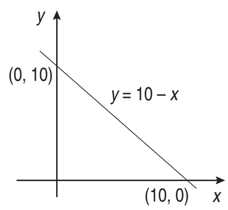
- Plot and interpret graphs such as:
  $y = x$, $y = 2x$, $y = 3x$, $y = 4x$, $y = 5x$



Note that graphs of the form $y = mx$:
- are all straight lines which pass through the origin;
- vary in steepness, depending on the function;
- match the graphs of multiples, but are continuous lines rather than discrete points.

- Plot graphs such as:
  $y = x + 1$, $y = 10 - x$





Note the positive or negative slope of the graph and the intercept points with the axes. Make connections with the value of the constant term.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
linear relationship…
intercept, steepness, slope, gradient…

Generate coordinate pairs and plot graphs of simple linear functions, using all four quadrants. For example:

- $y = 2x - 3$
  $({}^-3, {}^-9)$, $({}^-2, {}^-7)$, $({}^-1, {}^-5)$, $(0, {}^-3)$, $(1, {}^-1)$, $(2, 1)$, …

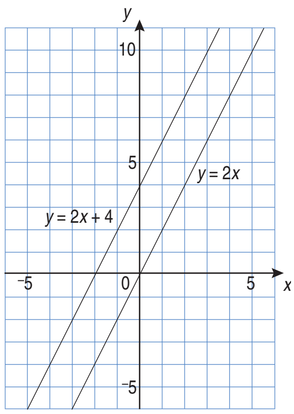
- $y = 5 - 4x$
  $({}^-2, 13)$, $({}^-1, 9)$, $(0, 5)$, $(1, 1)$, $(2, {}^-3)$, …

Plot the points. Observe that the points lie in a straight line and draw the line. Read other coordinate pairs from the line (including fractional values) and confirm that they also fit the function.

Recognise that a graph of the form $y = mx + c$:
- corresponds to a straight line, whereas the graph of a linear sequence consists of set of discrete points lying on an 'imagined straight line';
- represents an infinite set of points, and that:
  - the values of the coordinates of each point satisfy the equation represented by the graph;
  - any coordinate pair which represents a point not on the graph does not satisfy the equation.

Plot the graphs of linear functions in the form $y = mx + c$, on paper and using ICT, and consider their features. For example:

- Construct tables of values.
  Plot and interpret graphs such as:
  $y = 2x$, $y = 2x + 1$, $y = 2x + 4$, $y = 2x - 2$, $y = 2x - 5$



  Describe similarities and differences.
  Notice that:
  - the lines are all parallel to $y = 2x$;
  - the lines all have the same gradient;
  - the number (constant) tells you where the line cuts the y-axis (the intercept).

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
quadratic function, cubic function…

Plot the graphs of linear functions in the form $ay + bx + c = 0$, on paper and using ICT, and consider their features. For example:

Recognise that linear functions can be rearranged to give y explicitly in terms of x. For example:

- Rearrange $y + 2x - 3 = 0$ in the form $y = 3 - 2x$.
  Rearrange $y/4 - x = 0$ in the form $y = 4x$.
  Rearrange $2y + 3x = 12$ in the form $y = \frac{12 - 3x}{2}$.
- Construct tables of values.
  Plot the graphs on paper and using **ICT**.
  Describe similarities and differences.
- Without drawing the graphs, compare and contrast features of graphs such as:
  $y = 3x$ $y = 3x + 4$ $y = x + 4$
  $y = x - 2$ $y = 3x - 2$ $y = {}^-3x + 4$

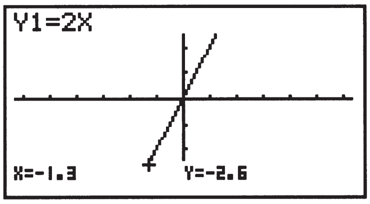
Pupils should be taught to:

Generate points and plot graphs of functions (continued)

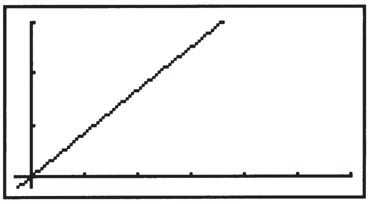
As outcomes, Year 7 pupils should, for example:

Recognise that equations of the form y = mx correspond to straight-line graphs through the origin.

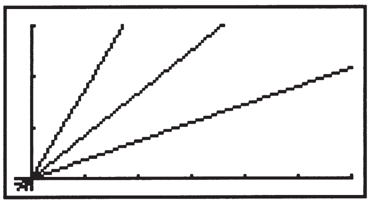
- Use a **graphical calculator** to plot a straight-line graph through the origin, trace along it, and read off the coordinates. Describe the relationship between the values for x and the values for y.



- Draw the graph of y = x.



Draw the graph of a line that is steeper.
Draw the graph of a line that is less steep.



Recognise that equations of the form y = c, where c is constant, correspond to straight-line graphs parallel to the x-axis, and that equations of the form x = c correspond to straight-line graphs parallel to the y-axis.

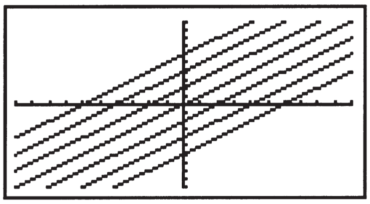
As outcomes, Year 8 pupils should, for example:

Recognise that equations of the form y = mx + c correspond to straight-line graphs.

Use a **graphical calculator** to investigate the family of straight lines y = mx + c.

- Draw the graphs of:
  - y = x + 1   y = x + 2   y = x + 3
  - y = x – 1   y = x – 2   y = x – 3

  Describe what the value of m represents.
  Describe what the value of c represents.



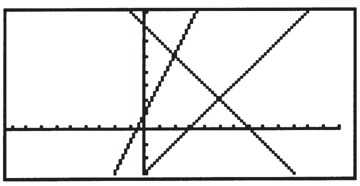
- Use a **graphical calculator** and knowledge of the graph of y = mx + c to explore drawing lines through:
  - (0, 5)
  - (⁻7, ⁻7)
  - (2, 6)
  - (⁻7, 0) and (0, 7)
  - (⁻3, 0) and (0, 6)
  - (0, ⁻8) and (8, 0)

Know and explain the reasons for these properties of functions of the form y = mx + c:

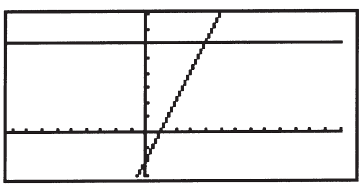
- they are all straight lines;
- for a given value of c, all lines pass through the point (0, c) on the y-axis;
- all lines with the same given value of m are parallel.

Use knowledge of these properties to find the equations of straight-line graphs.
For example, use a **graphical calculator** to:

- Find the equations of these straight-line graphs.



- Find some more straight lines that pass through the point (4, 6).



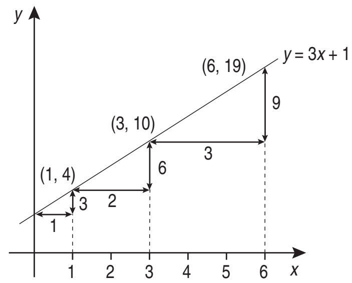
As outcomes, Year 9 pupils should, for example:

Given values for m and c, find the gradient of lines given by equations of the form y = mx + c.

Compare changes in y with corresponding changes in x, and relate the changes to a graph of the function.
For example:

- y = 3x + 1

x	0		1		2		3		4		5
y	1		4		7		10		13		16
Difference		3		3		3		3		3	



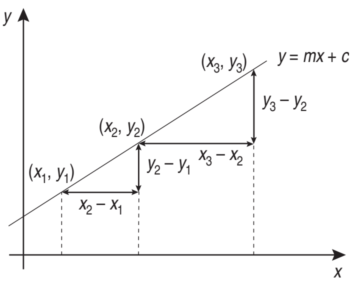
$$\frac{\text{change in y}}{\text{change in x}} = \frac{4-1}{1-0} = \frac{10-4}{3-1} = \frac{19-10}{6-3} = 3$$

Recognise that:

- the change in y is proportional to the change in x;
- the constant of proportionality is 3;
- triangles in the diagram are mathematically similar, i.e. enlargements of a basic triangle.

Know that for any linear function, the change in y is proportional to the corresponding change in x. For example, if A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) are any three points on the line y = mx + c, then

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2} = m$$



Know that for the straight line y = mx + c:

- $m = \frac{\text{change in y}}{\text{change in x}}$;
- m is called the gradient of the line and is a measure of the steepness of the line;
- if y decreases as x increases, m will be negative;
- lines parallel to the x-axis, e.g. y = 3, have gradient 0, and for lines parallel to the y-axis, e.g. x = 7, it is not possible to specify a gradient.

Link to properties of linear sequences (pages 148–9), proportionality (pages 78–81), enlargements (pages 212–15), and trigonometry (pages 242–7).

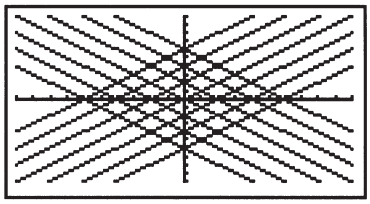
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Generate points and plot graphs of functions (continued)	

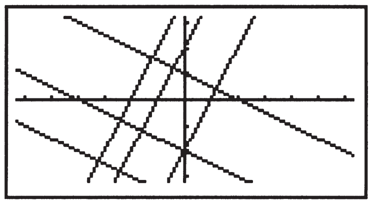
As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

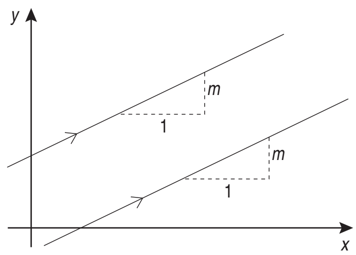
Investigate the gradients of parallel lines and lines perpendicular to the lines. For example:

- Look at pairs of lines $y = m_1x + c_1$ and $y = m_2x + c_2$. Use a **graphical calculator** to investigate the relationship between m_1 and m_2 when the two lines are parallel and the two lines are perpendicular.

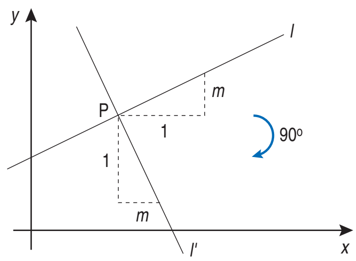




Recognise that any line parallel to a given line will have the same gradient:

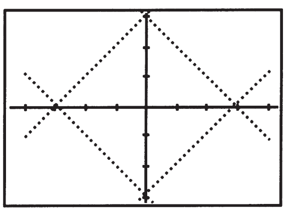


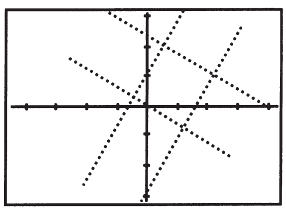
Let l be a line of gradient m, where l is not parallel to either axis. Let l′ be a line perpendicular to l. Explain why the gradient of l′ is $^1/_m$.



Use a **graphical calculator** to solve problems such as:

- Draw these squares.





Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Generate points and plot graphs of functions (continued)	

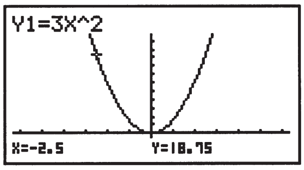
As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Generate points and plot the graphs of simple quadratic or cubic functions, on paper or using **ICT**. For example:

Construct tables of values, including negative values of x, and plot the graphs of these functions:
- $y = x^2$
- $y = 3x^2 + 4$
- $y = x^3$

Use a **graphical calculator** to plot the graph of, for example, $y = 3x^2$. Trace along it. Read coordinates. Describe the relationship between the values for x and the values for y.



Use a **graphical calculator** to explore the effect of changing the values of the parameters a and c in the following functions:
- $y = x^2 + c$
- $y = x^3 + c$
- $y = ax^2$

Construct a table of values and plot the graph of a general quadratic function. For example:
- $y = 2x^2 - 3x + 4$

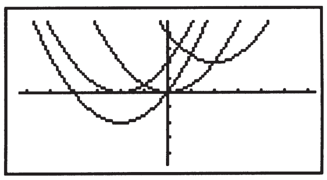
x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$2x^2$	8	2	0	2	8	18	32
$-3x$	6	3	0	-3	-6	-9	-12
+4	4	4	4	4	4	4	4
y	18	9	4	3	6	13	24

Recognise that (1, 3) is not the lowest point on the graph. Identify the axis of symmetry.

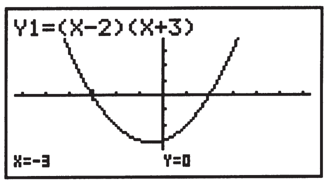
Use a **graphical calculator** to investigate graphs of functions of the form $y = ax^2 + bx + c$, for different values of a, b and c.

- Investigate families of curves such as:

  $y = ax^2$
  $y = (x + b)^2$
  $y = x^2 + c$
  $y = (x + b)^2 + c$



  $y = (x + a)(x + b)$



Link to properties of quadratic functions (pages 162–3).

Pupils should be taught to:

Construct functions arising from real-life problems, and plot and interpret their corresponding graphs

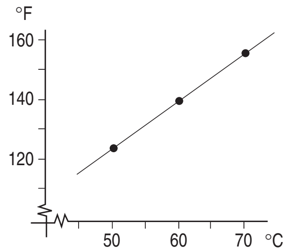
As outcomes, Year 7 pupils should, for example:

Begin to plot the graphs of simple linear functions arising from real-life problems.

In plotting such graphs:

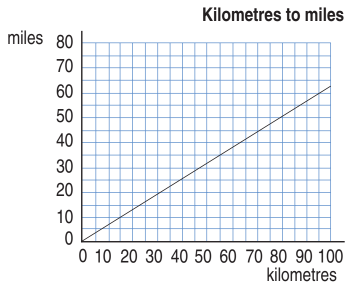
- know that scales are usually marked with multiples of the value (or variable) at equal spaces along the axis and often, but not always, start at zero;
- suggest suitable scales for the axes, based on the range of values to be graphed;
- decide how many points need to be plotted in order to draw an accurate graph;
- when appropriate, construct a table of values;
- know conventions for giving a title and labelling the axes;

- know the conventions for marking axes when scale(s) do not start from 0, e.g.

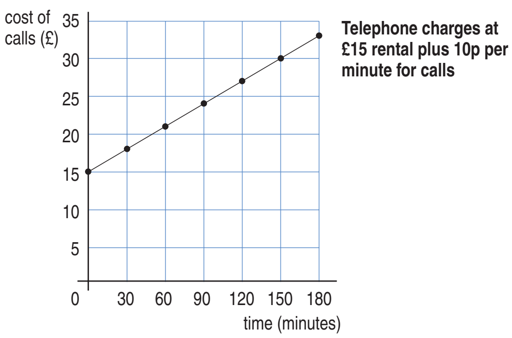


For example:

- Plot a conversion graph, e.g. converting from metric to imperial units or from degrees Fahrenheit to degrees Celsius.



- Plot a graph of charges, e.g. for fuel or mobile telephone calls, based on a fixed charge and a charge per unit consumed.



Use **ICT** to generate graphs of real data.

As outcomes, Year 8 pupils should, for example:

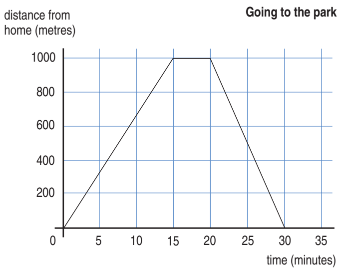
Construct linear functions arising from real-life problems and plot their corresponding graphs.

In plotting such graphs:

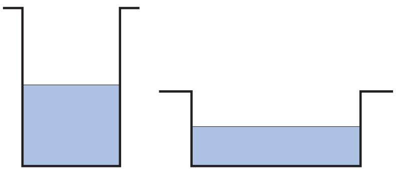
- write the appropriate formula;
- decide how many points to plot;
- construct a table of values;
- choose suitable scales for the axes;
- draw the graph with suitable accuracy;
- provide a title and label the axes.

For example:

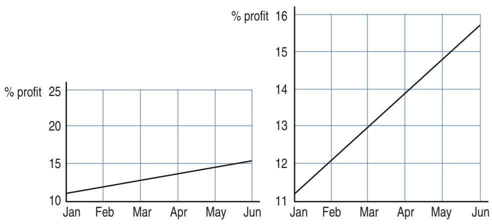
- Plot a simple distance–time graph.



- Sketch a line graph to show the depth of water against time when water runs steadily from a tap into these jars.

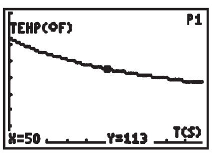


Begin to recognise that the choice of different scales and starting points can have a significant effect on the appearance of a graph, and can mislead or leave data open to misinterpretation. For example:



Use **ICT** to generate graphs of real data.
For example:

- Use a **temperature probe** and **graphical calculator** to plot a cooling curve.



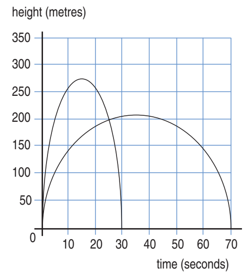
Link to line graphs (pages 264–5).

As outcomes, Year 9 pupils should, for example:

Construct functions arising from real-life problems and plot their corresponding graphs.

Draw and use graphs to solve distance–time problems. For example:

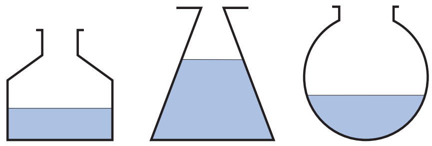
- This graph shows how high two rockets went during a flight. Rocket A reached a greater height than rocket B.



  Estimate how much higher rocket A reached than rocket B.
  Estimate the time after the start when the two rockets were at the same height.
  Estimate the number of seconds that rocket A was more than 200 m above the ground.

Sketch a line graph for the approximate relationship between two variables, relating to a familiar situation. For example:

- Sketch a graph of the depth of water against time when water drips steadily from a tap into these bottles.

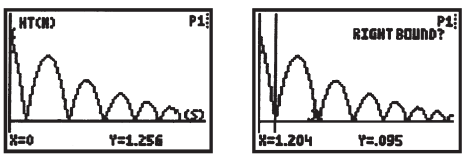


  Sketch graphs for other shapes of bottle.
  Predict the bottle shape from the shape of a graph.

- Sketch a graph of the number of hours of daylight at different times of the year.

Use **ICT** to generate graphs of real data.
For example:

- Use a **motion detector** and **graphical calculator** to plot the distance–time graph of a bouncing ball.



Link to line graphs (pages 264–5).

Pupils should be taught to:

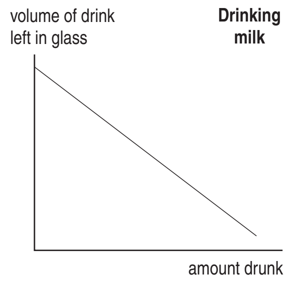
Construct linear functions arising from real-life problems, and plot and interpret their corresponding graphs (continued)

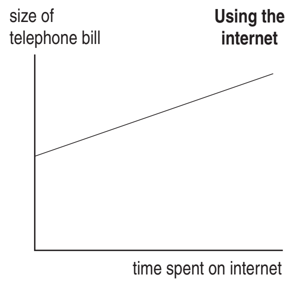
As outcomes, Year 7 pupils should, for example:

Discuss and begin to interpret graphs of linear functions, including some drawn by themselves and some gathered from other sources, such as a newspaper or the **Internet**.

For example:

Explain graphs such as:





In interpreting the graphs of functions:

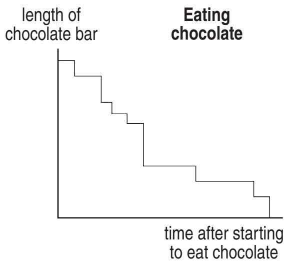
- read values from a graph;
- say whether intermediate points have any practical significance;
- say how the variables are related, e.g. they increase together.

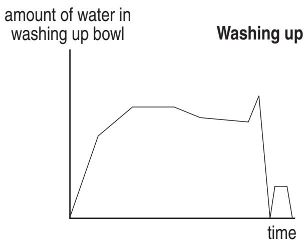
As outcomes, Year 8 pupils should, for example:

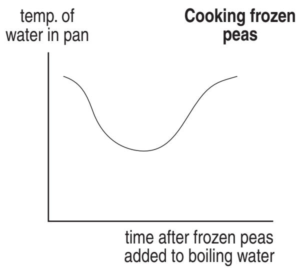
Discuss and interpret graphs of functions from a range of sources.

For example:

Give plausible explanations for the shape of graphs such as:







In interpreting the graphs of functions:

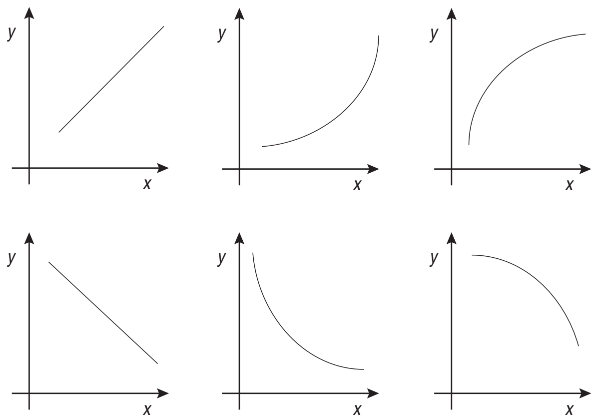
- read values from a graph;
- discuss trends, the shape of the graph and how it is related to the variables and the context represented.

As outcomes, Year 9 pupils should, for example:

Discuss and interpret a range of graphs arising from real situations.

For example:

For each of the situations below, suggest which sketch graph has a shape that most accurately describes it:



- the distance (y) travelled by a car moving at constant speed on a motorway, plotted against time (x);
- the number (y) of litres of fuel left in the tank of a car moving at constant speed, plotted against time (x);
- the distance (y) travelled by an accelerating racing car, plotted against time (x);
- the number (y) of dollars you can purchase for a given amount in pounds sterling (x);
- the temperature (y) of a cup of tea left to cool to room temperature, plotted against time (x);
- the distance (y) you run, plotted against time (x), if you start by running flat out, gradually slowing down until you collapse from exhaustion;
- the amount (y) of an infection left in the body as it responds to treatment, slowly at first, then more rapidly, plotted against time (x).

Choose phrases from these lists to describe graphs such as those above.

a. When x is large:
y is large;
y is small;
y becomes zero.

b. When x is small:
y is large;
y is small;
y becomes zero.

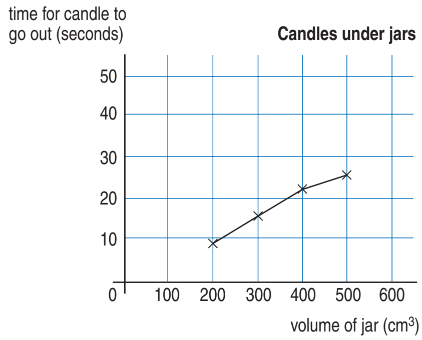
c. As x increases by equal amounts:
y increases by equal amounts;
y increases by increasing amounts;
y increases by decreasing amounts;
y decreases by equal amounts;
y decreases by increasing amounts;
y decreases by decreasing amounts.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Construct linear functions arising from real-life problems, and plot and interpret their corresponding graphs (continued)

Discuss and interpret straight-line graphs from science or geography. For example:

- Some pupils put a lighted candle under jars of different sizes. The jars varied from 200 cm^3 to 500 cm^3 in volume. They timed how long the candle took to go out.



Discuss features of the graph. For example:

Can we join the points up?
How many points (experiments) are needed to give an accurate picture?
Should the points be joined by straight lines?

How long do you think the candle would take to go out under a jar of 450 cm^3? Of 600 cm^3?
Suppose you wanted the candle to burn for 20 seconds. Under which jar would you put it?

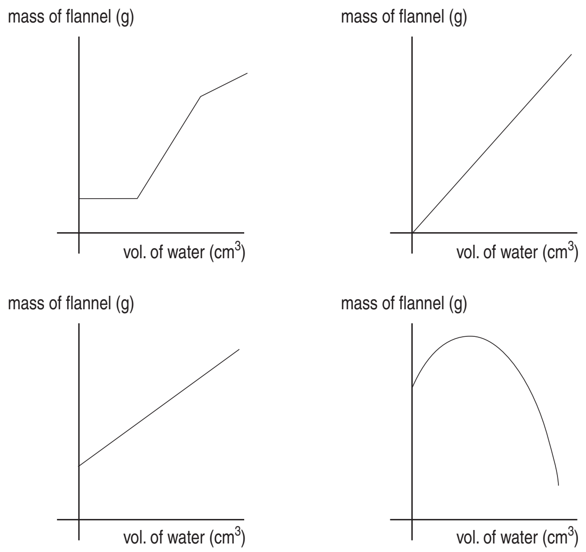
What kind of shape is the graph?
Which of the four sentences below best describes the relationship between the volume of the jar and the time it takes for the candle to go out?

A. The greater the volume, the shorter the time for the candle to go out.
B. The biggest jar kept the candle going longest.
C. As the volume of the jar increases, so the time gets longer.
D. The candle went out most quickly under the smallest jar.

As outcomes, Year 8 pupils should, for example:

Discuss and interpret line graphs from other subjects. For example:

- Some pupils poured different volumes of water on to a small towelling flannel. Each time they found its mass.
  The water was always completely absorbed or soaked up by the flannel.
  Which would be the most likely line for their graph?



- Some very hot water is placed in three test tubes and its temperature recorded over time.
  The first tube has no wrapping.
  The second tube has a wrapping of ice.
  The third tube has a wrapping of plastic foam.
  Sketch the temperature graph for each tube.

Draw and discuss graphs with discontinuities: for example, postal charges for packets of different weights.

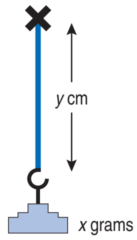
Link to interpreting and discussing results (pages 268–71).

As outcomes, Year 9 pupils should, for example:

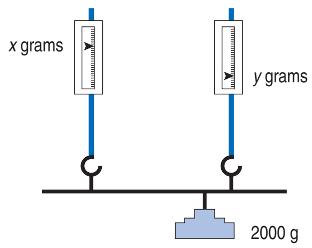
Discuss and interpret linear and non-linear graphs from other subjects. For example:

Think about how y will vary with x in these situations, and describe and sketch a graph to show each relationship.

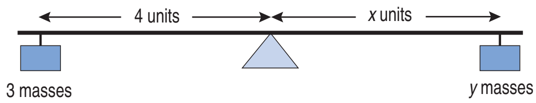
- A mass of x grams is suspended from a piece of elastic which stretches to a length of y cm.



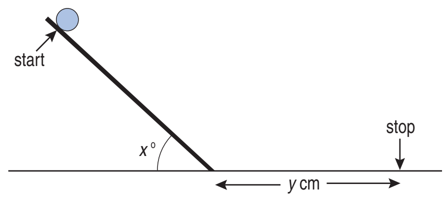
- A thin rod is hung from two spring balances.
  A 2 kg mass is hung from the rod and can be slid along in either direction. The reading on the left-hand balance is x grams and on the right-hand balance is y grams.



- A balance is arranged so that 3 equal masses are placed on the left-hand side at a distance of 4 units from the pivot. On the right-hand side y masses are placed x units from the pivot.



- A ball bearing is rolled down a plane inclined at an angle of x°. It comes to rest at a horizontal distance of y cm from the bottom of the plane.



Draw and discuss graphs with discontinuities: for example, a graph of (x), the greatest integer less than or equal to x.

Link to interpreting and discussing results (pages 268–71).

Pupils should be taught to:

Use accurately the vocabulary, notation and labelling conventions for lines, angles and shapes; distinguish between conventions, facts, definitions and derived properties

As outcomes, Year 7 pupils should, for example:

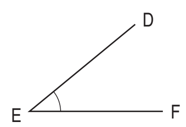
Use, read and write, spelling correctly:
line segment, line… parallel, perpendicular… plane…
horizontal, vertical, diagonal… adjacent, opposite…
point, intersect, intersection… vertex, vertices… side…
angle, degree (°)… acute, obtuse, reflex…
vertically opposite angles… base angles…

Use accurately the notation and labelling conventions for lines, angles and shapes.

Understand that a straight line can be considered to have infinite length and no measureable width, and that a line segment is of finite length, e.g. line segment AB has end-points A and B.

Know that:

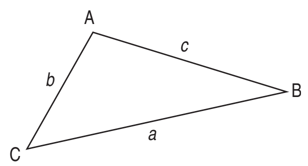
- Two straight lines in a plane (a flat surface) can cross once or are parallel; if they cross, they are said to intersect, and the point at which they cross is an intersection.
- When two line segments meet at a point, the angle formed is the measure of rotation of one of the line segments to the other. The angle can be described as ∠DEF or DÊF or ∠E.



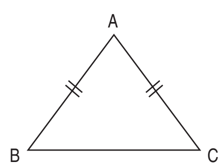
- A polygon is a 2-D or plane shape constructed from line segments enclosing a region. The line segments are called sides; the end points are called vertices. The polygon is named according to the number of its sides, vertices or angles: triangle, quadrilateral, pentagon…

Know the labelling convention for:

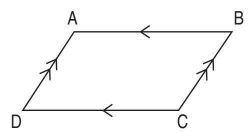
- triangles – capital letters for the vertices (going round in order, clockwise or anticlockwise) and corresponding lower-case letters for each opposite side, the triangle then being described as ΔABC;



- equal sides and parallel sides in diagrams.



AB = AC



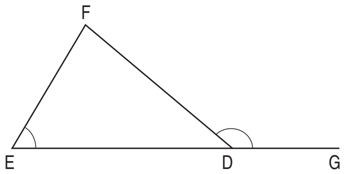
AB is parallel to DC, or AB//DC.
AD is parallel to BC, or AD//BC.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
corresponding angles, alternate angles…
supplementary, complementary…
interior angle, exterior angle… equidistant…
prove, proof…

Continue to use accurately the notation and labelling conventions for lines, angles and shapes.

Know that ∠DEF is an interior angle of ΔDEF and that ∠GDF is an exterior angle of ΔDEF.



Know that:
- A pair of complementary angles have a sum of 90°.
- A pair of supplementary angles have a sum of 180°.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
convention, definition, derived property...

Distinguish between conventions, definitions and derived properties.

A convention is an agreed way of illustrating, notating or describing a situation. Conventions are arbitrary – alternatives could have been chosen. Examples of geometrical conventions are:
- the ways in which letters are used to label the angles and sides of a polygon;
- the use of arrows to show parallel lines;
- the agreement that anticlockwise is taken as the positive direction of rotation.

A definition is a minimum set of conditions needed to specify a geometrical term, such as the name of a shape or a transformation. Examples are:
- A polygon is a closed shape with straight sides.
- A square is a quadrilateral with all sides and all angles equal.
- A degree is a unit for measuring angles, in which one complete rotation is divided into 360 degrees.
- A reflection in 2-D is a transformation in which points (P) are mapped to images (P′), such that PP′ is at right angles to a fixed line (called the mirror line, or line of reflection), and P and P′ are equidistant from the line.

A derived property is not essential to a definition, but consequent upon it. Examples are:
- The angles of a triangle add up to 180°.
- A square has diagonals that are equal in length and that bisect each other at right angles.
- The opposite sides of a parallelogram are equal in length.
- Points on a mirror line reflect on to themselves.

Distinguish between a practical demonstration and a proof. For example, appreciate that the angle sum property of a triangle can be demonstrated practically by folding the corners of a triangular sheet of paper to a common point on the base and observing the result. A proof requires deductive argument, based on properties of angles and parallels, that is valid for all triangles.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

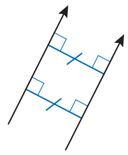
Identify properties of angles and parallel and perpendicular lines, and use these properties to solve problems

Identify parallel and perpendicular lines.

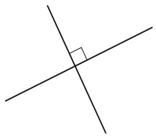
Recognise parallel and perpendicular lines in the environment, and in 2-D and 3-D shapes: for example, rail tracks, side edges of doors, ruled lines on a page, double yellow lines…

Use **dynamic geometry software**, acetate overlays or film to explore and explain relationships between parallel and intersecting lines, such as:

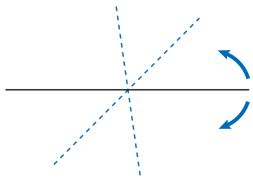
- parallel lines, which are always equidistant;



- perpendicular lines, which intersect at right angles;



- lines which intersect at different angles.
  For example, as one line rotates about the point of intersection, explain how the angles at the point of intersection are related.



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Use ruler and set square to draw parallel and perpendicular lines.

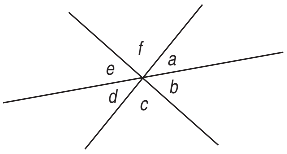
Link with constructions (page 220–3).

As outcomes, Year 8 pupils should, for example:

Identify alternate and corresponding angles.

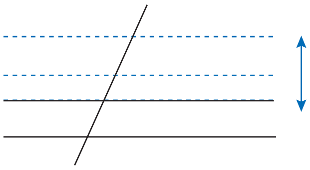
Use **dynamic geometry software** or acetate overlays to explore and explain relationships between lines in the plane, such as:

- three lines that intersect in one point;

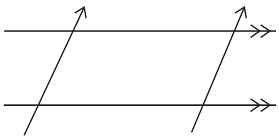


a + b + c = 180°
a = d, b = e, c = f

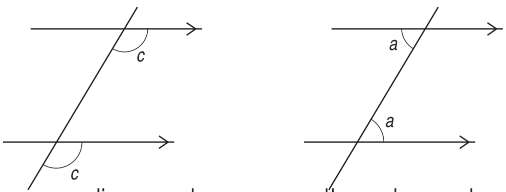
- given two intersecting lines and a third that moves but remains parallel to one of them, explain which angles remain equal;



- two pairs of parallel lines, forming a parallelogram.



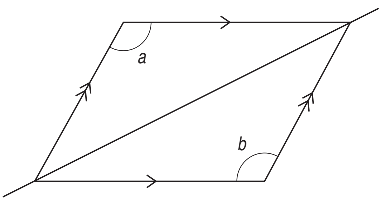
Understand and use the terms corresponding angles and alternate angles.



corresponding angles alternate angles

Use alternate angles to prove that opposite angles of a parallelogram are equal:

a = b

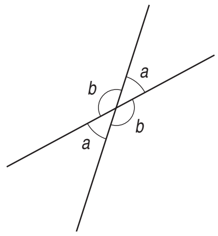


As outcomes, Year 9 pupils should, for example:

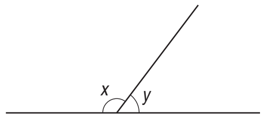
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Identify properties of angles and parallel and perpendicular lines, and use these properties to solve problems (continued)

Know the sum of angles at a point, on a straight line and in a triangle, and recognise vertically opposite angles and angles on a straight line.



vertically opposite angles



$x + y = 180°$

angles on a straight line

Link with rotation (pages 208–12).

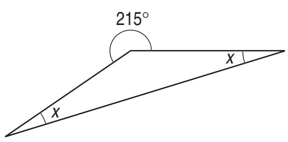
Recognise from practical work such as measuring and paper folding that the three angles of a triangle add up to 180°.

Given sufficient information, calculate:

- angles in a straight line and at a point;
- the third angle of a triangle;
- the base angles of an isosceles triangle.

For example:

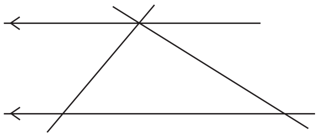
- Calculate the angles marked by letters.



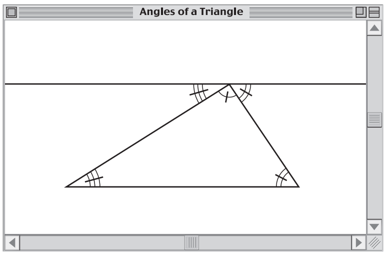
As outcomes, Year 8 pupils should, for example:

Understand a proof that the sum of the angles of a triangle is 180° and of a quadrilateral is 360°, and that the exterior angle of a triangle equals the sum of the two interior opposite angles.

Consider relationships between three lines meeting at a point and a fourth line parallel to one of them.



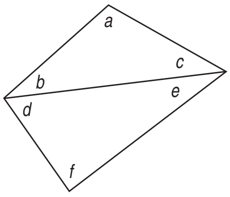
Use **dynamic geometry software** to construct a triangle with a line through one vertex parallel to the opposite side. Observe the angles as the triangle is changed by dragging any of its vertices.



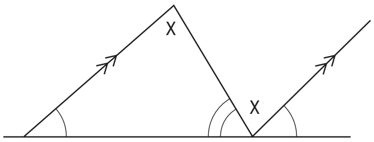
Use this construction, or a similar one, to explain using diagrams a proof that the sum of the three angles of a triangle is 180°.

Use the angle sum of a triangle to prove that the angle sum of a quadrilateral is 360°.

(a + b + c) + (d + e + f)
= 180° + 180° = 360°

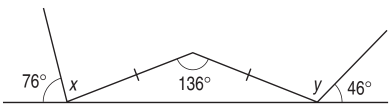


Explain a proof that the exterior angle of a triangle equals the sum of the two interior opposite angles, using this or another construction.



Given sufficient information, calculate:
- interior and exterior angles of triangles;
- interior angles of quadrilaterals.

For example:
- Calculate the angles marked by letters.

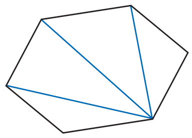


As outcomes, Year 9 pupils should, for example:

Explain how to find, calculate and use properties of the interior and exterior angles of regular and irregular polygons.

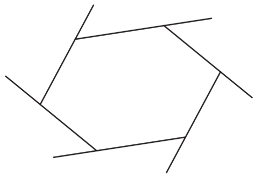
Explain how to find the interior angle sum and the exterior angle sum in (irregular) quadrilaterals, pentagons and hexagons. For example:

- A polygon with n sides can be split into n – 2 triangles, each with an angle sum of 180°.



So the interior angle sum is (n – 2) × 180°, giving 360° for a quadrilateral, 540° for a pentagon and 720° for a hexagon.

At each vertex, the sum of the interior and exterior angles is 180°.

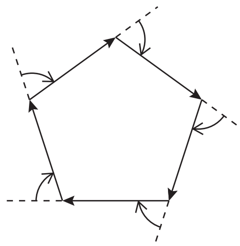


For n vertices, the sum of n interior and n exterior angles is n × 180°.
But the sum of the interior angles is (n – 2) × 180°, so the sum of the exterior angles is always 2 × 180° = 360°.

Find, calculate and use the interior and exterior angles of a regular polygon with n sides.
For example:

- The interior angle sum S for a polygon with n sides is S = (n – 2) × 180°.
  In a regular polygon all the angles are equal, so each interior angle equals S divided by n.
  Since the interior and exterior angles are on a straight line, the exterior angle can be found by subtracting the interior angle from 180°.

- From experience of using **Logo**, explain how a complete traverse of the sides of a polygon involves a total turn of 360° and why this is equal to the sum of the exterior angles.



  Deduce interior angle properties from this result.

Recall that the interior angles of an equilateral triangle, a square and a regular hexagon are 60°, 90° and 120° respectively.

Pupils should be taught to:

Identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems; explain and justify inferences and deductions using mathematical reasoning

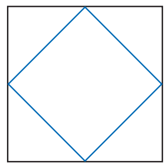
As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
polygon, regular, irregular, convex, concave…
circle, triangle, isosceles, equilateral, scalene, right-angled,
quadrilateral, square, rectangle, parallelogram, rhombus,
trapezium, kite, delta… and names of other polygons.

Visualise and sketch 2-D shapes in different orientations, or draw them using **dynamic geometry software**. Describe what happens and use the properties of shapes to explain why.

For example:

- Imagine a square with its diagonals drawn in. Remove one of the triangles. What shape is left? How do you know?
- Imagine a rectangle with both diagonals drawn. Remove a triangle. What sort of triangle is it? Why?
- Imagine joining adjacent mid-points of the sides of a square. What shape is formed by the new lines? Explain why.



- Imagine a square with one of its corners cut off. What different shapes could you have left?
- Imagine an isosceles triangle. Fold along the line of symmetry. What angles can you see in the folded shape? Explain why.
- Imagine a square sheet of paper. Fold it in half and then in half again, to get another smaller square. Which vertex of the smaller square is the centre of the original square? Imagine a small triangle cut off this corner. Then imagine the paper opened out. What shape will the hole be? Explain your reasoning.

  Imagine what other shapes you can get by folding a square of paper in different ways and cutting off different shapes.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
bisect, bisector, mid-point…
congruent… tessellate, tessellation…

Visualise and sketch 2-D shapes in different orientations, or draw them using **dynamic geometry software**. Describe what happens and use the properties of shapes to explain why.

For example:

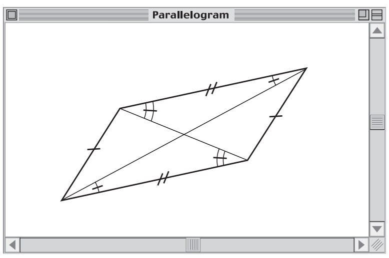
- Imagine a rectangular sheet of paper.
  Cut along the diagonal to make two triangles.
  Place the diagonals together in a different way.
  What shape is formed?

- Imagine two equilateral triangles, placed together, edge to edge.
  What shape is formed? Why?
  Add a third equilateral triangle… a fourth…
  What shapes are formed? Sketch some diagrams and explain what can be seen.

- Imagine two congruent isosceles triangles. Put sides of equal length together. Describe the resulting shape. Is it the only possibility?

- Imagine a quadrilateral with two lines of symmetry.
  What could it be?
  Suppose it also has rotation symmetry of order 2.
  What could it be now?

- Construct a parallelogram by drawing two line segments from a common end-point.
  Draw parallel lines to form the other two sides.
  Draw the two diagonals.



  Observe the sides, angles and diagonals as the parallelogram is changed by dragging its vertices.

- Describe tilings and other geometrical patterns in pictures and posters. Suggest reasons why objects in the environment (natural or constructed) take particular shapes.

- Explore tessellations using plastic or card polygon shapes and/or **computer tiling software**, and explain why certain shapes tessellate.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
similar, similarity…
hypotenuse, Pythagoras' theorem…

Visualise and sketch 2-D shapes or draw them using **dynamic geometry software** as they go through a sequence of changes. Describe what happens and use the properties of shapes to explain why.

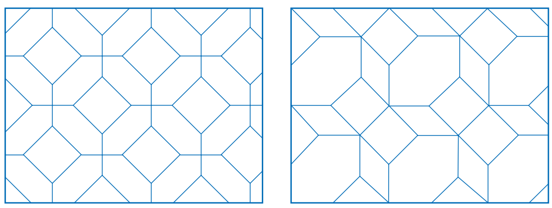
For example:

- Imagine starting with an equilateral triangle with one side horizontal – call it the base.
  Imagine this base is fixed.
  The opposite vertex of the triangle moves slowly in a straight line, perpendicular to the base.
  What happens to the triangle?
  Now imagine that the opposite vertex moves parallel to the base. What happens?
  Can you get a right-angled triangle, or an obtuse-angled triangle?

- Imagine a square sheet of paper.
  Imagine making a straight cut symmetrically across one corner. What shape is left?
  Imagine making a series of straight cuts, always parallel to the first cut. Describe what happens to the original square.

- Imagine two sheets of acetate, each marked with a set of parallel lines, spaced 2 cm apart.
  Imagine one sheet placed on top of the other, so that the two sets of lines are perpendicular.
  What shapes do you see?
  What happens to the pattern as the top sheet slowly rotates about a fixed point (the intersection of two lines)? What if the lines were 1 cm apart on one sheet and 2cm apart on the other?

- Overlay tessellations in various ways, such as octagons and squares on octagons and squares.
  Describe the outcomes.



  Explore how regular polygons which do not tessellate (e.g. nonagons) can be used to cover the plane by leaving holes in a regular pattern.
  Describe the outcomes.

Pupils should be taught to:

Identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems; explain and justify inferences and deductions using

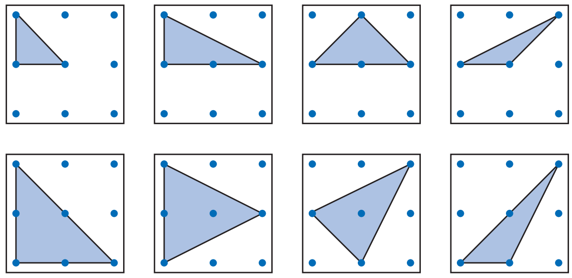
As outcomes, Year 7 pupils should, for example:

Triangles, quadrilaterals and other polygons

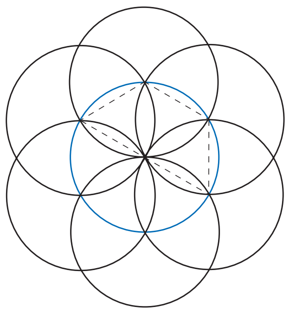
Review the properties of triangles and quadrilaterals
(see Y456 examples, pages 102–3).

For example:

- Using a 3 by 3 array on a pinboard, identify the eight distinct triangles that can be constructed (eliminating reflections, rotations or translations). Classify the triangles according to their side, angle and symmetry properties.



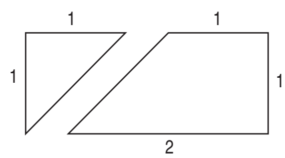
- In this pattern of seven circles, how many different triangles and quadrilaterals can you find by joining three or four points of intersection?



  What are the names of the shapes and what can you find out about their angles?

Begin to identify and use angle, side and symmetry properties of triangles, quadrilaterals and other polygons. For example:

- Start with a 2 by 1 rectangle. Make these two shapes. What new shapes can you make with them? Name them and discuss their properties.



As outcomes, Year 8 pupils should, for example:

Know and use side, angle and symmetry properties of equilateral, isosceles and right-angled triangles.

For example:

- Discuss whether it is possible to draw or construct on a 3 by 3 pinboard:
  a. a triangle with a reflex angle;
  b. an isosceles trapezium;
  c. an equilateral triangle or a (non-square) rhombus.
  If not, explain why not.

Classify quadrilaterals by their geometric properties (equal and/or parallel sides, equal angles, right angles, diagonals bisected and/or at right angles, reflection and rotation symmetry…).

Know properties such as:

- An isosceles trapezium is a trapezium in which the two opposite non-parallel sides are the same length. It has one line of symmetry and both diagonals are the same length.
- A parallelogram has its opposite sides equal and parallel. Its diagonals bisect each other. It has rotation symmetry of order 2.
- A rhombus is a parallelogram with four equal sides. Its diagonals bisect each other at right angles. Both diagonals are lines of symmetry. It has rotation symmetry of order 2.
- A kite is a quadrilateral that has two pairs of adjacent sides of equal length, and no interior angle larger than 180°. It has one line of symmetry and its diagonals cross at right angles.
- An arrowhead or delta has two pairs of adjacent edges of equal length and one interior angle larger than 180°. It has one line of symmetry. Its diagonals cross at right angles outside the shape.

Provide a convincing argument to explain, for example, that a rhombus is a parallelogram but a parallelogram is not necessarily a rhombus.

Devise questions for a tree classification diagram to sort a given set of quadrilaterals.

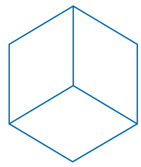
- Identify then classify the 16 distinct quadrilaterals that can be constructed on a 3 by 3 pinboard.

Link to standard constructions (pages 220–3).

As outcomes, Year 9 pupils should, for example:

Know and use angle and symmetry properties of polygons, and angle properties of parallel and intersecting lines, to solve problems and explain reasoning. For example:

- Deduce the angles of the rhombus in this arrangement of three identical tiles.



  What can you deduce about the shape formed by the outline?

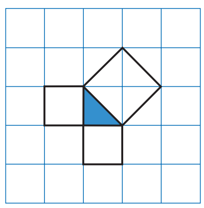
- Explain why:
  – Equilateral triangles, squares and regular hexagons will tessellate on their own but other regular polygons will not.
  – Squares and regular octagons will tessellate together.

Know and use properties of triangles, including Pythagoras' theorem.

Know that:

- In any triangle, the largest angle is opposite the longest side and the smallest angle is opposite the shortest side.
- In a right-angled triangle, the side opposite the right angle is the longest and is called the hypotenuse.

Understand, recall and use Pythagoras' theorem.
Explain special cases of Pythagoras' theorem in geometrical arrangements such as:



Pupils should be taught to:

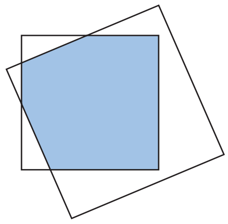
Identify and use the geometric properties of triangles, quadrilaterals and other polygons to solve problems; explain and justify inferences and deductions using mathematical reasoning (continued)

As outcomes, Year 7 pupils should, for example:

Use the properties of angles at a point and on a straight line, and the angle sum of a triangle, to solve simple problems. Explain reasoning.

For example:

- What different shapes can you make by overlapping two squares?



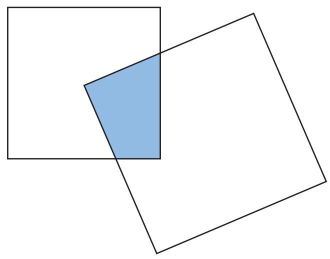
  Can any of these shapes be made?

rectangle	pentagon	decagon
rhombus	hexagon	kite
isosceles triangle	octagon	trapezium

  If a shape cannot be made, explain why.

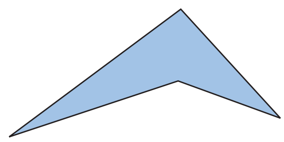
  What shapes can you make by overlapping three squares?

- Two squares overlap like this. The larger square has one of its vertices at the centre of the smaller square.



  Explain why the shaded area is one quarter of the area of the smaller square.

- Explain why a triangle can never have a reflex angle but a quadrilateral can.



- Provide a convincing argument to explain why it is always possible to make an isosceles triangle from two identical right-angled triangles.

- Use **Logo** to write instructions to draw a parallelogram.

Link to problems involving shape and space (pages 14–17).

As outcomes, Year 8 pupils should, for example:

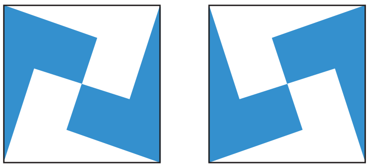
Use angle and side properties of triangles, and angle properties of parallel and intersecting lines, to solve problems. Explain reasoning. For example:

- Use alternate and corresponding angles to explain why any scalene triangle will tessellate.

- This tiling pattern can be found in the Alhambra Palace in Granada, Spain.

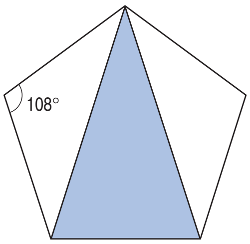
  How would you describe the pattern over the telephone to someone who has never seen it?

  The pattern can be made by using these two tiles.



  Suggest how to construct them.
  What other patterns can you make with these two tiles?
  Reproduce the tiling pattern using **computer tiling software**.

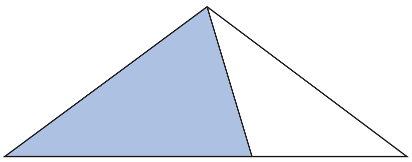
- The angle at the vertex of a regular pentagon is 108°.



  Two diagonals are drawn to the same vertex to make three triangles.

  Calculate the sizes of the angles in each triangle.

  The middle triangle and one of the other triangles are placed together like this.



  Explain why the triangles fit together to make a new triangle. What are its angles?

Link to problems involving shape and space (pages 14–17).

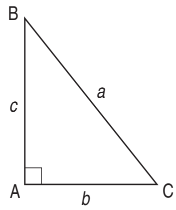
As outcomes, Year 9 pupils should, for example:

Understand and recall Pythagoras' theorem:

- as a property of areas:
  in a right-angled triangle, the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides.

- as a property of lengths:

  $a^2 = b^2 + c^2$



  Appreciate that:

  If $a^2 > b^2 + c^2$, then A is an obtuse angle.
  If $a^2 < b^2 + c^2$, then A is an acute angle.

Know the Pythagorean triples (3, 4, 5) and (5, 12,13).
Explore others using a **spreadsheet** or **calculator**.
Recognise that multiples of a Pythagorean triple are also Pythagorean triples and produce similar triangles.

Link to problem solving – 'Hexagons' (pages 34–5), algebra (pages 120–1), trigonometry (pages 242–7), and coordinates (pages 218–19).

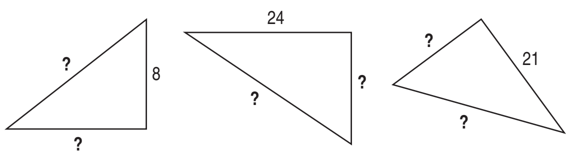
Use Pythagoras' theorem to solve simple problems in two dimensions. For example:

- You walk due north for 5 miles, then due east for 3 miles. What is the shortest distance you are from your starting point?

- A 5 m ladder leans against a wall with its foot 1.5m away from the wall. How far up the wall does the ladder reach?

- The sides of some triangles are:
  a. 5, 12, 13
  b. 6, 7, 8
  c. 5, 8, 11
  d. 16, 30, 34
  e. 13, 15, 23
  Without drawing the triangles, classify them according to whether they are acute-angled, right-angled or obtuse-angled.

- Find whole-number lengths that will satisfy these right-angled triangles. There may be more than one answer.



Link to problems involving shape and space (pages 14–17).

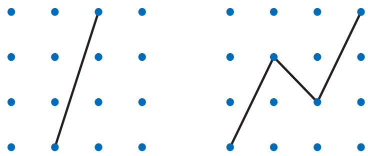
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Understand congruence and similarity	

As outcomes, Year 8 pupils should, for example:

Congruence

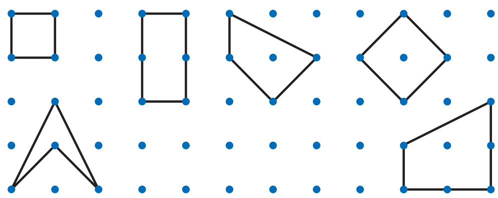
Know that if two 2-D shapes are congruent, they have the same shape and size, and corresponding sides and angles are equal. For example:

- From a collection of different triangles or quadrilaterals, identify those that are congruent to each other by placing one on top of the other. Realise that corresponding sides and angles are equal.
- Divide a 4 by 4 pinboard into two congruent halves. How many different ways of doing this can you find?



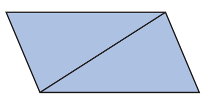
  Divide the pinboard into four congruent quarters.

- Divide a 5 by 5 pinboard into two non-congruent halves.
- Using a 3 by 3 pinboard, make some different triangles or quadrilaterals. For each shape, investigate whether you can produce one or more identical shapes in different positions or orientations on the board. Describe the transformation(s) you use to do this.



  Extend to 3 by 4 and larger grids.

- Two congruent scalene triangles without right angles are joined with two equal edges fitted together.



  What shapes can result?
  What if the two triangles are right-angled, isosceles or equilateral?
  In each case, explain how you know what the resultant shapes are.

As outcomes, Year 9 pupils should, for example:

Congruence

Appreciate that when two shapes are congruent, one can be mapped on to the other by a translation, reflection or rotation, or some combination of these transformations.

See Year 8 for examples.

Link to transformations (pages 202–17).

Know from experience of constructing them that triangles satisfying SSS, SAS, ASA or RHS are unique, but that triangles satisfying SSA or AAA are not.

Link to constructions (pages 220–3).

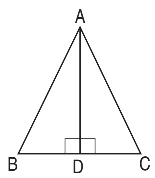
Appreciate that two triangles will be congruent if they satisfy the same conditions:

- three sides are equal (SSS);
- two sides and the included angle are equal (SAS);
- two angles and a corresponding side are equal (ASA);
- a right angle, hypotenuse and side are equal (RHS).

Use these conditions to deduce properties of triangles and quadrilaterals.

For example:

- Draw triangle ABC, with AB = AC. Draw the perpendicular from A to BC to meet BC at point D.



  Show that triangles ABD and ACD are congruent. Hence show that the two base angles of an isosceles triangle are equal.

- Use congruence to prove that the diagonals of a rhombus bisect each other at right angles.
- By drawing a diagonal and using the alternate angle property, use congruence to prove that the opposite sides of a parallelogram are equal.

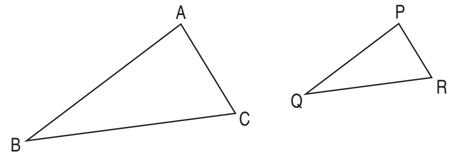
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Understand congruence and similarity (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Similarity

Know that the term 'similar' has a precise meaning in geometry. Objects, either plane or solid figures, are similar if they have the same shape. That is, corresponding angles contained in similar figures are equal, and corresponding sides are in the same ratio.



∠BAC = ∠QPR, ∠ACB = ∠PRQ, ∠ABC = ∠PQR
and AB : PQ = BC:QR = CA:RP

Recognise and use characteristics of similar shapes.

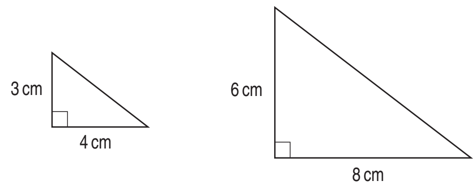
- If two shapes are similar, one can be considered to be an enlargement of the other.
  Link to enlargement (pages 212–15).

- Any two regular polygons with the same number of sides are mathematically similar (e.g. any two squares, any two equilateral triangles, any two regular hexagons).

- All circles are similar. Use this knowledge when considering metric properties of the circle, such as the relationship between circumference and diameter.
  Link to circumference of a circle (pages 234–5).

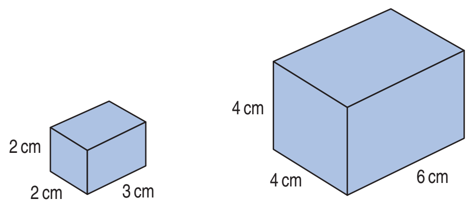
Solve problems involving similarity. For example:

- These two triangles are similar.



  a. Find the perimeter and area of each triangle.
  b. What is the ratio of the two perimeters?
  c. What is the ratio of the two areas?

- Explain why these two cuboids are similar.



  What is the ratio of their surface areas?
  What is the ratio of their volumes?

Link to ratio and proportion (pages 78–81).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Identify and use the properties of circles	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

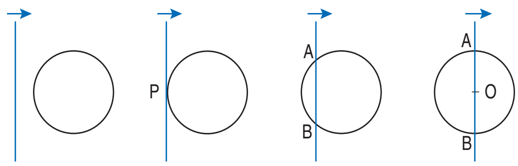
Circles

Know the parts of a circle, including:
centre, radius, diameter, circumference, chord, arc, segment, sector, tangent… and terms such as: circumcircle, circumscribed, inscribed…

Know that:

- A circle is a set of points equidistant from its centre.
- The circumference is the distance round the circle.
- The radius is the distance from the centre to the circumference.
- An arc is part of the circumference; a sector is the region bounded by an arc and two radii.

Use **dynamic geometry software** to show a line and a circle moving towards each other.

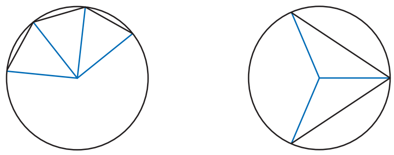


Know that when the line:

- touches the circle at a point P, it is called a tangent to the circle at that point;
- intersects the circle at two points A and B, the line segment AB is called a chord of the circle, which divides the area enclosed by the circle into two regions called segments;
- passes through the centre of the circle, the line segment AB becomes a diameter, which is twice the radius and divides the area enclosed by the circle into two semicircles.

Explain why inscribed regular polygons can be constructed by equal divisions of a circle.

- Appreciate that a chord of a circle, together with the radii through the end points, form an isosceles triangle. If further chords of the same length are drawn, the triangles are congruent and angles between successive chords are equal.



  Hence, if the succession of equal chords divides the circle without remainder, a regular polygon is inscribed in the circle.

- If chords of length equal to the radius are marked on the circumference of a circle, explain why the resultant shape is a regular hexagon.

  Use this to construct a hexagon of a given side.

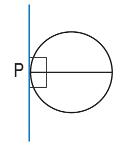
Link to circumference and area of a circle (pages 234–7).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Identify and use the properties of circles (continued)	

As outcomes, Year 8 pupils should, for example:

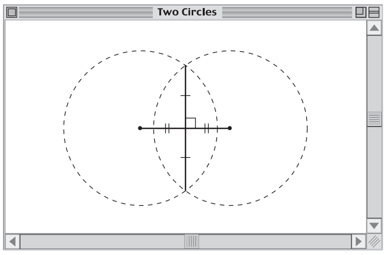
As outcomes, Year 9 pupils should, for example:

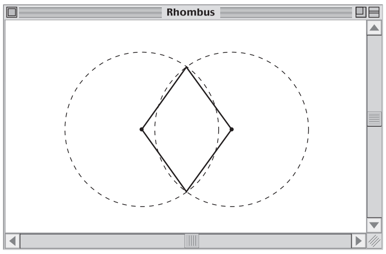
Circles (continued)
Recognise that a tangent is perpendicular to the radius at the point of contact P.



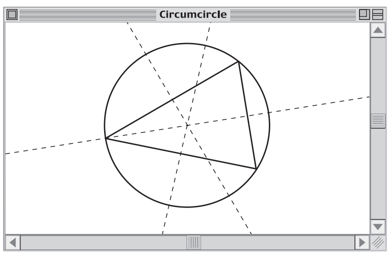
Use **dynamic geometry software** to explore properties of lines and circles. For example:

- Make two circles, of equal radius, touch then intersect. What happens to the common chord and the line joining the centres, or to the rhombus formed by joining the radii to the points of intersection?





- Construct a triangle and the perpendicular bisectors of its three sides. Draw the circumcircle (the circle through the three vertices). What happens when the vertices of the triangle move? Observe in particular the position of the centre of the circumcircle.



Link to standard constructions (pages 220–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

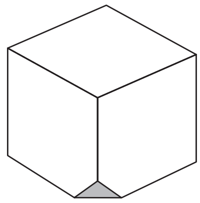
Use 2-D representations, including plans and elevations, to visualise 3-D shapes and deduce some of their properties

Use, read and write, spelling correctly:
2-D, 3-D, cube, cuboid, pyramid, tetrahedron, prism, cylinder, sphere, hemisphere…
face, vertex, vertices, edge… net…

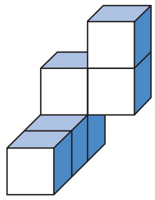
Use 2-D representations and oral descriptions to visualise 3-D shapes and deduce some of their properties.
For example:

- Imagine you have two identical cubes.
  Place them together, matching face to face.
  Name and describe the new solid.
  How many faces, edges, vertices…?

- Imagine cutting off a corner of a cube.
  Describe the new face created.
  Describe the new solid.
  How many faces, edges, vertices...?



- Sit back to back with a partner.
  Look at the picture of the model.
  Don't show it to your partner.
  Tell your partner how to build the model.



- Join in a 'guess the shape' activity.
  A solid made from centimetre cubes is placed in a bag.
  a. Take turns to describe one element of the shape by feeling it. Others try to make the same shape.
  b. Take turns to describe the shape while others try to guess what it is.
  c. Guess a hidden solid shape by asking questions about it to which only yes/no answers can be given.

- On a six-sided dice, the faces are numbered from 1 to 6, and opposite faces should add up to 7.

  Draw a net for a cube.
  Choose a face and write 5 on it.

  Now write numbers on the other faces so that when the cube is folded up, opposite faces add up to 7.

- Imagine you are visiting the pyramids in Egypt.
  You are standing on the ground, looking at one pyramid.
  What is the maximum number of faces you could see?
  What if you were flying overhead?

- Imagine you are looking at a large cardboard box in the shape of a cube.
  Can you stand so that you can see just one face?
  Sketch an outline of what you would see.
  Can you stand so that you can see just 2 faces, 3 faces, 4 faces? Sketch outlines of what you would see.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
view, plan, elevation… isometric…

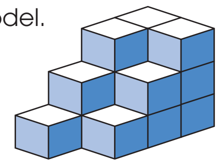
Know and use geometric properties of cuboids and shapes made from cuboids; begin to use plans and elevations. For example:

- Describe 3-D shapes which can be visualised from a wall poster or a photograph.

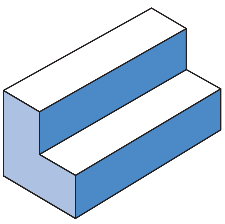
- Visualise and describe relationships between the edges of a cube, e.g. identify edges which:
  - meet at a point;
  - are parallel;
  - are perpendicular;
  - are neither parallel nor intersect each other.

- Imagine a cereal packet standing on a table.
  Paint the front and the back of the packet red.
  Paint the top and bottom red and the other two faces blue.
  Now study the packet carefully.
  How many edges has it?
  How many edges are where a red face meets a blue face?
  How many edges are where a red face meets another red face?
  How many edges are where a blue face meets another blue face?

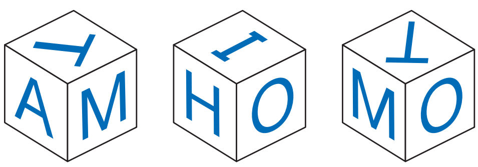
- Sit back to back with a partner.
  Look at the picture of the model.
  Don't show it to your partner.
  Tell your partner how to build the model.



- Sketch a net to make this model.
  Construct the shape.



- Here are three views of the same cube.
  Which letters are opposite each other?



As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
cross-section, projection…
plane…

Analyse 3-D shapes through 2-D projections and cross-sections, including plans and elevations.
For example:

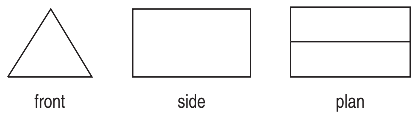
Visualise solids from an oral description. For example:

- In each case, identify the solid shape.
  a. The front and side elevations are both triangles and the plan is a square.
  b. The front and side elevations are both rectangles and the plan is a circle.
  c. The front elevation is a rectangle, the side elevation is a triangle and the plan is a rectangle.
  d. The front and side elevations and the plan are all circles.

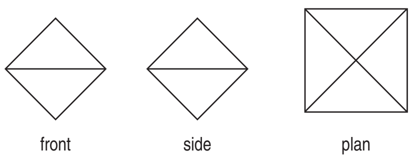
- The following are shadows of solids.
  Describe the possible solids for each shadow (there may be several solutions).

- In each case, identify the solid shape.
  Draw the net of the solid.

a.



b.

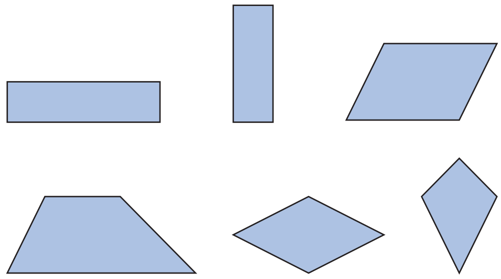


- Write the names of the polyhedra that could have an isosceles or equilateral triangle as a front elevation.

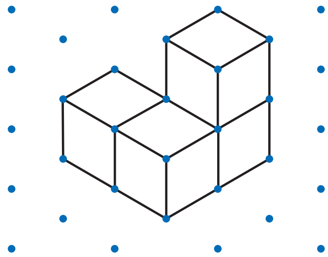
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Use 2-D representations, including plans and elevations, to visualise 3-D shapes and deduce some of their properties (continued)

- A square piece of card is viewed from different angles against the light. Which of the following are possible views of the square card? Which are impossible?



- Find all possible solids that can be made from four cubes. Record the solids using isometric paper.



- Investigate the number of different ways that a 2 by 2 by 2 cube can be split into two pieces:
  a. of the same shape and size;
  b. of different shapes and sizes.

See Y456 examples (pages 104–5).

As outcomes, Year 8 pupils should, for example:

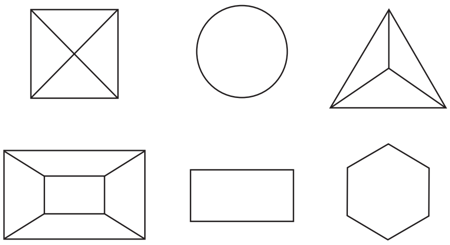
Orient an isometric grid and use conventions for constructing isometric drawings:

- Vertical edges are drawn as vertical lines.
- Horizontal edges are drawn at 30°.

Identify the position of hidden lines in an isometric drawing.

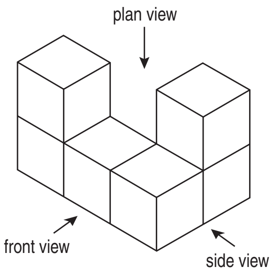
Begin to use plans and elevations. For example:

- The diagrams below are of solids when observed directly from above.

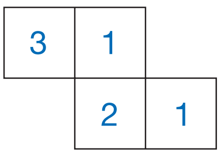


  Describe what the solids could be and explain why.

- Draw the front elevation, side elevation and plan of this shape.

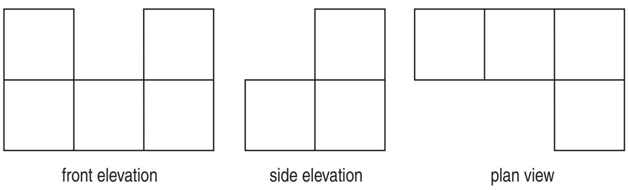


- This diagram represents a plan of a solid made from cubes, the number in each square indicating how many cubes are on that base.



  Make an isometric drawing of the solid from a chosen viewpoint.

- Construct this solid, given the front elevation, side elevation and plan.

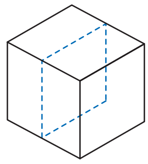


As outcomes, Year 9 pupils should, for example:

Visualise and describe sections obtained by slicing in different planes.
For example:

- Compare horizontal cross-sections of a square-based right pyramid at different heights.
  Repeat for vertical cross-sections at different points.

- This cube has been sliced to give a square cross-section.



  Is it possible to slice a cube so that the cross-section is:
  a. a rectangle?
  b. a triangle?
  c. a pentagon?
  d. a hexagon?
  If so, describe how it can be done.

- For eight linked cubes, find the solids with the smallest and the largest surface area.
  Draw the shapes on isometric paper.
  Extend to 12 cubes.

- Imagine you have a cube.
  Put a dot in the centre of each face.
  Join the dots on adjacent sides by straight lines.
  What shape is generated by these lines?

- Visualise an octahedron.
  Put a dot in the centre of each face.
  Join the dots on adjacent sides by straight lines.
  What shape is generated by these lines?

- Imagine a slice cut symmetrically off each corner of a cube. Describe the solid which remains.
  Is there more than one possibility?
  Repeat for a tetrahedron or octahedron.

- Triangles are made by joining three of the vertices of a cube. How many different-shaped triangles can you make like this?
  Draw sketches of them.

Link to plane symmetry (pages 206–7).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Understand and use the language and notation associated with reflections, translations and rotations

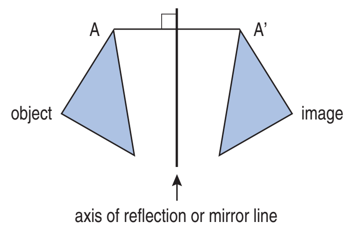
Use, read and write, spelling correctly:
transformation… image, object, congruent…
reflection, mirror line, line of symmetry, line symmetry, reflection symmetry, symmetrical…
translation… rotate, rotation, rotation symmetry, order of rotation symmetry, centre of rotation…

Recognise and visualise transformations and symmetries of shapes

Reflection

Understand reflection in two dimensions as a transformation of a plane in which points are mapped to images in a mirror line or axis of reflection, such that:

- the mirror line is the perpendicular bisector of the line joining point A to image A′;
- the image is the same distance behind the mirror as the original is in front of it.



Know that a reflection has these properties:

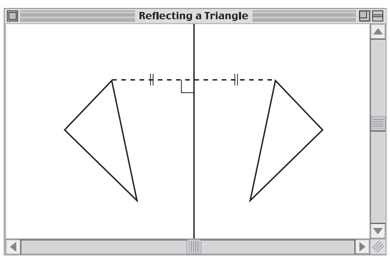
- Points on the mirror line do not change their position after the reflection, i.e. they map to themselves.
- A reflection which maps A to A′ also maps A′ to A, i.e. reflection is a self-inverse transformation.

Relate reflection to the operation of folding. For example:

- Draw a mirror line on a piece of paper. Mark point P on one side of the line. Fold the paper along the line and prick through the paper at point P. Label the new point P′. Open out the paper and join P to P′ by a straight line. Check that PP′ is at right angles to the mirror line and that P and P′ are the same distance from it. Repeat for other points.

Explore reflection using **dynamic geometry software**.
For example:

- Construct a triangle and a line to act as a mirror line. Construct the image of one vertex by drawing a perpendicular to the mirror line and finding a point at an equal distance on the opposite side. Repeat for the other vertices and draw the image triangle.
  Observe the effect of dragging vertices of the original triangle. What happens when the triangle crosses the mirror line?



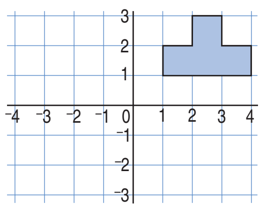
As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year.

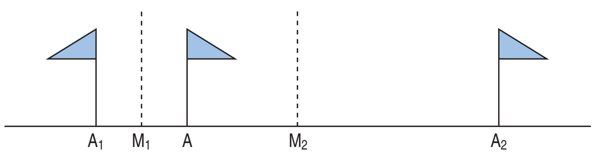
Combinations of two transformations

Transform 2-D shapes by repeated reflections, rotations or translations. Explore the effect of repeated reflections in parallel or perpendicular lines. For example:

- Reflect a shape in one coordinate axis and then the other. For example, reflect the shape below first in the x-axis and then in the y-axis. What happens? What is the equivalent transformation? Now reflect it first in the y-axis and then in the x-axis. What happens? What is the equivalent transformation?



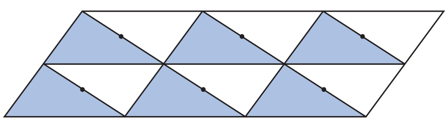
- Investigate reflection in two parallel lines. For example, find and explain the relationship between the lengths AA_2 and M_1M_2.



- Investigate how repeated reflections can be used to generate a tessellation of rectangles.

Explore the effect of repeated rotations, such as half turns about different points. For example:

- Generate a tessellation of scalene triangles (or quadrilaterals) using half-turn rotations about the mid-points of sides.



Explain how the angle properties of a triangle (or quadrilateral) relate to the angles at any vertex of the tessellation.

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
plane symmetry, plane of symmetry…
axis of rotation symmetry…

Combinations of transformations

Transform 2-D shapes by combining translations, rotations and reflections, on paper and using **ICT**.

Know that reflections, rotations and translations preserve length and angle, and map objects on to congruent images.

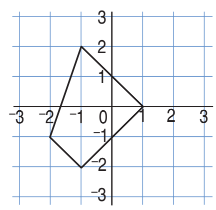
Link to congruence (pages 190–1).

Use mental imagery to consider a combination of transformations and relate the results to symmetry and other properties of the shapes. For example:

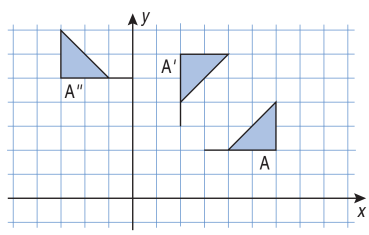
- Say what shape the combined object and image(s) form when:
  a. a right-angled triangle is reflected along its hypotenuse;
  b. a square is rotated three times through a quarter turn about a corner;
  c. a scalene triangle is rotated through 180° about the mid-point of one of its sides.

Working practically when appropriate, solve problems such as:

- Reflect this quadrilateral in the y-axis. Then reflect both shapes in the x-axis. In the resulting pattern, which lines and which angles are equal in size?



- Flag A is reflected in the line y = x to give A′. A′ is then rotated through 90° centre (0, 3) to give A″.



Show that A could also be transformed to A″ by a combination of a reflection and a translation. Describe other ways of transforming A to A″.

Pupils should be taught to:

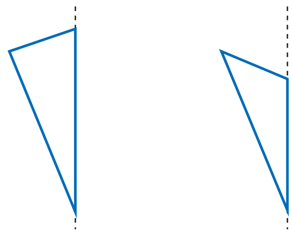
Recognise and visualise transformations and symmetries of 2-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

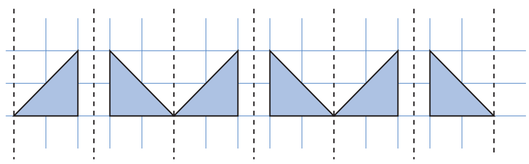
Reflection (continued)

Reflect a shape in a line along one side. For example:

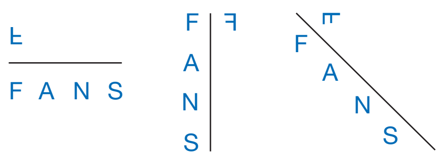
- Reflect each shape in the dotted line.
  What is the name of the resulting quadrilateral?
  Which angles and which sides are equal?
  Explain why.



Reflect a shape in parallel mirrors and describe what is happening. Relate this to frieze patterns created by reflection.

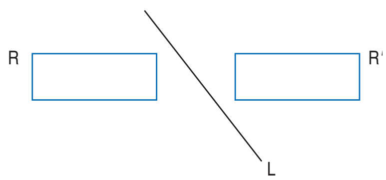


Construct the reflections of shapes in mirror lines placed at different angles relative to the shape. For example:



- Which shapes appear not to have changed after a reflection? What do they have in common?

- In this diagram, explain why rectangle R′ is not the reflection of rectangle R in line L.



As outcomes, Year 8 pupils should, for example:

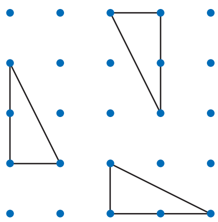
Combinations of transformations (continued)

Understand and demonstrate some general results about repeated transformations. For example:

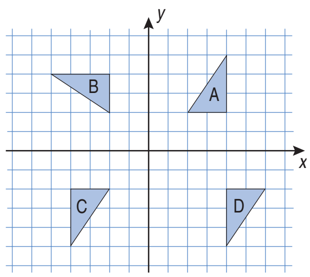
- Reflection in two parallel lines is equivalent to a translation.
- Reflection in two perpendicular lines is equivalent to a half-turn rotation.
- Two rotations about the same centre are equivalent to a single rotation.
- Two translations are equivalent to a single translation.

Explore the effect of combining transformations.

- Draw a 1 by 2 right-angled triangle in different positions and orientations on 5 by 5 spotty paper. Choose one of the triangles to be your original. Describe the transformations from your original to the other triangles drawn. Can any be done in more than one way?



- Triangles A, B, C and D are drawn on a grid.



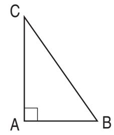
a. Find a single transformation that will map:
   i. A on to C; ii. C on to D.
b. Find a combination of two transformations that will map: i. B on to C; ii. C on to D.
c. Find other examples of combined transformations, such as:
   A to C: with centre (0, 0), rotation of 90°, followed by a further rotation of 90°;
   A to C: reflection in the y-axis followed by reflection in the x-axis;
   B to C: rotation of 90° centre ($^-2$, 2), followed by translation (0, $^-4$);
   C to D: reflection in the y-axis followed by reflection in the line x = 4;
   C to D: rotation of 270° centre (0, 0), followed by a rotation of 90° centre (4, 4).

Use **ICT**, or plastic or card shapes, to generate tessellations using a combination of reflections, rotations and translations of a simple shape.

As outcomes, Year 9 pupils should, for example:

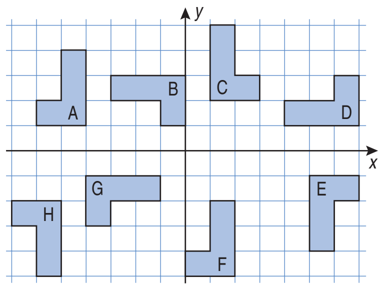
Combinations of transformations (continued)

- ABC is a right-angled triangle. ABC is reflected in the line AB and the image is then reflected in the line CA extended. State, with reasons, what shape is formed by the combined object and images.



- Two transformations are defined as follows:
  - Transformation A is a reflection in the x-axis.
  - Transformation C is a rotation of 90° centre (0, 0).

  Does the order in which these transformations are applied to a given shape matter?
- Some congruent L-shapes are placed on a grid in this formation.



Describe transformations from shape C to each of the other shapes.

- Some transformations are defined as follows:
  - P is a reflection in the x-axis.
  - Q is a reflection in the y-axis.
  - R is a rotation of 90° centre (0, 0).
  - S is a rotation of 180° centre (0, 0).
  - T is a rotation of 270° centre (0, 0).
  - I is the identity transformation.

  Investigate the effect of pairs of transformations and find which ones are commutative.
- Investigate the effect of a combination of reflections in non-perpendicular intersecting mirror lines, linking to rotation symmetry, the kaleidoscope effect and the natural world.

Use **dynamic geometry software** to explore equivalences of combinations of transformations, for example:

- to demonstrate that only an even number of reflections can be equivalent to a rotation;
- to demonstrate that two half turns about centres C_1 and C_2 are equivalent to a translation in a direction parallel to C_1C_2 and of twice the distance C_1C_2.

Pupils should be taught to:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

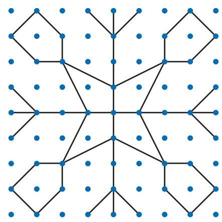
Reflection symmetry

Know that if a line can be found such that one half of a shape reflects to the other, then the shape has reflection symmetry, and the line is called a line of symmetry. Recognise:

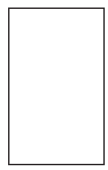
- reflection symmetry in familiar shapes such as an isosceles triangle, a rectangle, a rhombus, a regular hexagon…
- lines of symmetry in 2-D shapes;
- shapes with no lines of symmetry.

For example:

- Identify the lines of symmetry in this pattern.

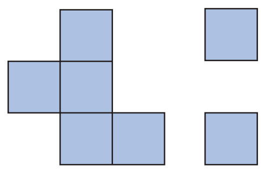


- Imagine that this is how a piece of paper looks after it has been folded twice, each time along a line of symmetry.
  What shape might the original piece of paper have been?
  How many possibilities are there?

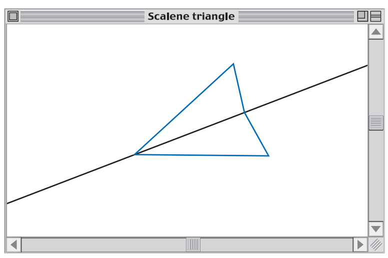


- Combine these shapes to make a shape with reflection symmetry.

  How many different solutions are there? (Five.)



- Use **dynamic geometry software** to reflect a triangle in any one of its sides. What shape do the combined object and image form?



  Is this always the case?

See Y456 examples (pages 106–7).

Link to properties of 2-D shapes (pages 186–9).

As outcomes, Year 8 pupils should, for example:

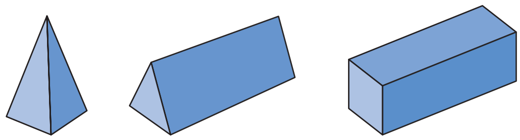
As outcomes, Year 9 pupils should, for example:

Reflection symmetry

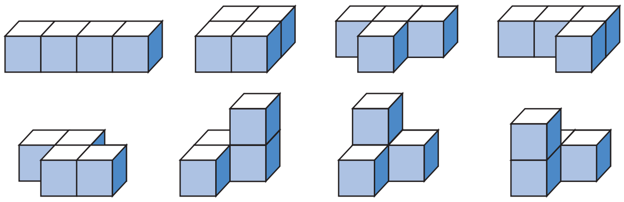
Understand reflection symmetry in three dimensions and identify planes of symmetry.

For example:

- Discuss symmetry in 3-D objects, such as a pair of semi-detached houses…
- Visualise and describe all the planes of symmetry of familiar solids, such as a cube, cuboid, triangular prism, square-based pyramid, regular tetrahedron, regular octahedron.



- Use four cubes to make as many different shapes as possible.



Prove that no other shapes are possible.

For each shape, identify any planes of symmetry. Investigate asymmetrical shapes, looking for any which together form a symmetrical pair.

- Investigate axes of rotation symmetry for a cuboid.

Link to properties of 3-D shapes (pages 198–201).

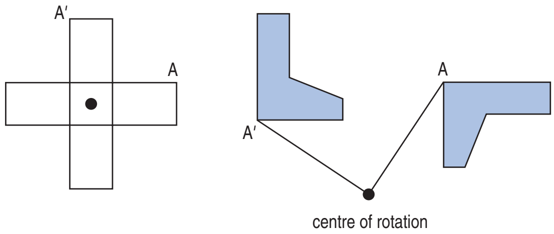
Pupils should be taught to:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

Rotation

Understand rotation in two dimensions as a transformation of a plane in which points (such as A) are mapped to images (A′) by turning about a fixed point in the plane, called the centre of rotation.

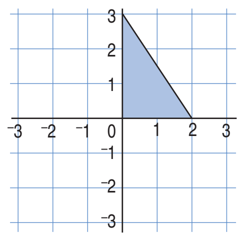


A rotation is specified by a centre of rotation and an (anticlockwise) angle of rotation.

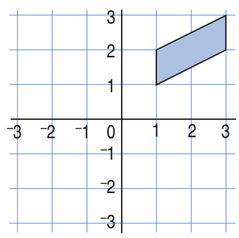
Know that a rotation has these properties:

- The centre of rotation can be inside or outside the shape and its position remains fixed throughout the rotation.
- The inverse of any rotation is either:
  - a. an equal rotation about the same point in the opposite direction, or
  - b. a rotation about the same point in the same direction, such that the two rotations have a sum of 360°.

Rotate shapes anticlockwise about (0, 0) through right angles and simple fractions of a turn.



Rotate shapes about points other than (0, 0).

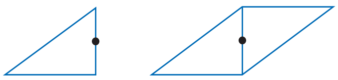


As outcomes, Year 8 pupils should, for example:

Rotation

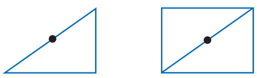
Rotate shapes, and deduce properties of the new shapes formed, using knowledge that the images are congruent to the original and identifying equal angles and equal sides. For example:

- Rotate a right-angled triangle through 180° about the mid-point of its shortest side.



  Name the shape formed by the object and image.
  Identify the equal angles and equal sides.
  Explain why they are equal.

  What happens when you rotate the triangle about the mid-point of its longest side?



As outcomes, Year 9 pupils should, for example:

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

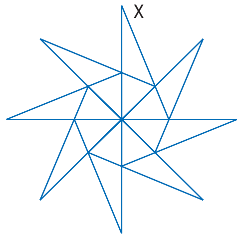
Rotation symmetry

Know that:

- A 2-D shape has rotation symmetry of order n when n is the largest positive integer for which a rotation of 360° ÷ n produces an identical looking shape in the same position.
- The order of rotation symmetry is the number of ways the shape will map on to itself in a rotation of 360°.

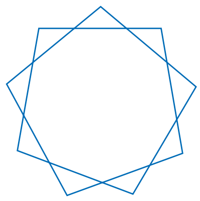
For example:

- This shape has rotation symmetry of order 8 because it maps on to itself in eight distinct positions under rotations of 45° about the centre point.

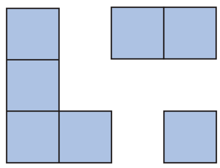


Solve problems such as:

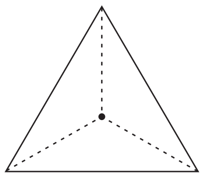
- Prove that the central shape in the above diagram is a regular octagon.
  What shape is traced by point X as it moves through one complete revolution?
- Recognise the rotation symmetry of familiar shapes, such as parallelograms and regular polygons.
- Use **Logo** or other **ICT resource** to produce shapes with a specified order of rotation symmetry, e.g. 9.



- Combine these shapes to make a single shape with rotation symmetry of order 2.



- Identify the centre of rotation in a shape with rotation symmetry, such as an equilateral triangle.
  Justify the choice.



Link to properties of 2-D shapes (pages 186–9).

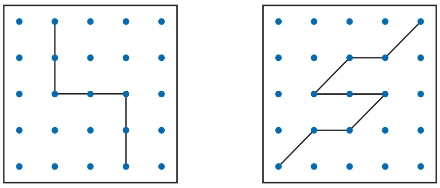
As outcomes, Year 8 pupils should, for example:

Reflection symmetry and rotation symmetry

Recognise all the symmetries of 2-D shapes.

For example:

- Make polygons on a 3 by 3 pinboard. Explore:
  - the maximum number of sides;
  - whether any of the polygons are regular;
  - symmetry properties of the polygons…
- Recognise the rotation symmetry in congruent divisions of a 5 by 5 pinboard.



Identify and describe the reflection and rotation symmetries of:

- regular polygons (including equilateral triangles and squares);
- isosceles triangles;
- parallelograms, rhombuses, rectangles, trapeziums and kites.

Devise a tree diagram to sort quadrilaterals, based on questions relating to their symmetries. Compare and evaluate different solutions.

Relate symmetries of triangles and quadrilaterals to their side, angle and diagonal properties.
For example:

- An isosceles triangle has reflection symmetry.
  Use this to confirm known properties, such as:
  - The line of symmetry passes through the vertex which is the intersection of the two equal sides, and is the perpendicular bisector of the third side.
  - The base angles on the unequal side are equal.
- A parallelogram has rotation symmetry of order 2, and the centre of rotation is the intersection of the diagonals.
  Use this to confirm known properties, such as:
  - The diagonals bisect each other.
  - Opposite sides and opposite angles are equal.
  - The angles between a pair of opposite sides and a diagonal are equal.

Link to properties of 2-D shapes (pages 186–91).

As outcomes, Year 9 pupils should, for example:

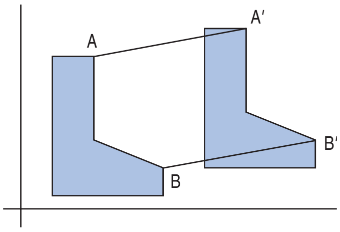
Pupils should be taught to:

Recognise and visualise transformations and symmetries of 2-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

Translation

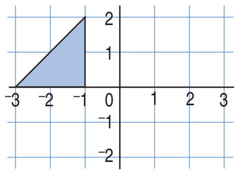
Understand translation as a transformation of a plane in which points (such as A and B) are mapped on to images (A′ and B′) by moving a specified distance in a specified direction.



Know that when describing a translation, it is essential to state either the direction and distance or, with reference to a coordinate grid, the moves parallel to the x-axis and parallel to the y-axis.

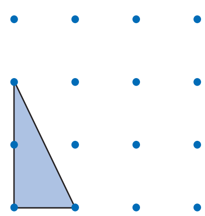
Know that a translation has these properties:
- The orientations of the original and the image are the same.
- The inverse of any translation is an equal move in the opposite direction.

Translate shapes on a coordinate grid, e.g. 4 units to the right, 2 units down, then 3 units to the left. Determine which two instructions are equivalent to the three used.



Investigate translations. For example:

- What are the possible translations of a 1 by 2 right-angled triangle on 3 by 3 pinboard?
  What about a 4 by 4 pinboard, then a 5 by 5 pinboard?



- Examine a tessellating pattern (e.g. of equilateral triangles, squares or regular hexagons).
  Start from one particular shape in the middle.
  Identify translations of that shape, and the direction of the translation.

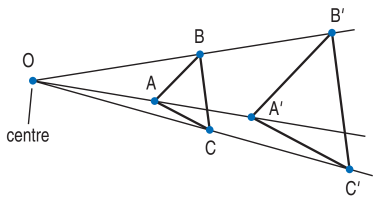
See Y456 examples (pages 106–7).

As outcomes, Year 8 pupils should, for example:

Enlargement

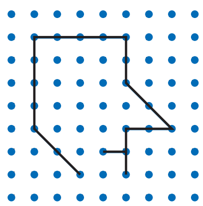
Use, read and write, spelling correctly:
enlarge, enlargement, centre of enlargement…
scale, scale factor, ratio…
scale drawing, map, plan…

Understand enlargement as a transformation of a plane in which points (such as A, B and C) are mapped on to images (A′, B′ and C′) by multiplying their distances from a fixed centre of enlargement by the same scale factor. In this example, triangle ABC maps to A′B′C′:



$$\text{scale factor} = \frac{OA'}{OA} = \frac{OB'}{OB} = \frac{OC'}{OC}$$

- Draw a simple shape on a 1 cm spotty grid, e.g. a 'standard angular person'.



Choosing a suitable centre, enlarge the shape by different positive scale factors, such as ×2, ×3, ×4, (double person, treble person, quadruple person). Construct a table of measurements.

Scale factor	×1	×2	×3	×4
Width of head (cm)	4			
Width of neck (cm)	2			
Full height of head (cm)	6			
Width of mouth (cm)	1			
Diagonal length of nose (cm)	2.8			

Check that the ratio of corresponding linear measurements is always equal to the scale factor.

Experiment with different centres.

Begin to understand the property that the ratios of corresponding lengths in the image and in the object are equal to the scale factor, and to recognise this as a constant proportion:

$$\text{scale factor} = \frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$$

As outcomes, Year 9 pupils should, for example:

Enlargement

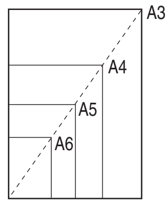
Use vocabulary from previous year and extend to:
similar…

Understand and use the definition of enlargement from a centre. Recognise that:
- The object and its image are similar.
- The ratio of any two corresponding line segments is equal to the scale factor.

- Extend the 'standard angular person' activity to enlargements by a fractional scale factor, such as ½ or ¼.

- Investigate the proportions of metric paper sizes, A6 to A1. For example, start with a sheet of A3 paper and, with successive folds, produce A4, A5 and A6.

  Demonstrate practically that the different sizes of paper can be aligned, corner to corner, with a centre of enlargement.



  Confirm by measurement and calculation that the scale factor of enlargement is approximately 0.7.

  Follow an explanation that, if the metric paper has dimensions h and w, then h : w = w:h/2.
  Deduce that $h = \sqrt{2}w$.

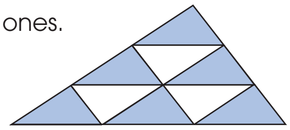
- Compare a simple shape with enlargements and reductions of it made on a photocopier.
  Estimate the scale factors of the enlargements as accurately as possible.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Recognise and visualise transformations and symmetries of 2-D shapes (continued)	

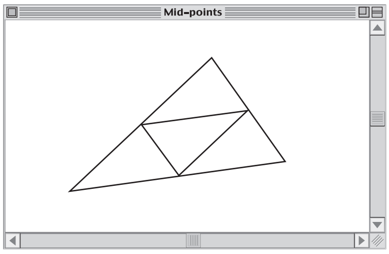
As outcomes, Year 8 pupils should, for example:

Explore some practical activities leading to consideration of enlargement. For example:

- Build triangles into bigger ones.



- Use **dynamic geometry software** to draw a triangle. Join the mid-points of the sides. Observe the effect as the vertices of the original triangle are dragged. Describe the resulting triangles.



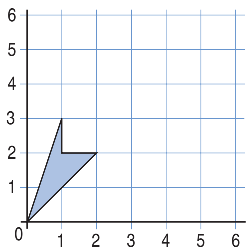
From practical work, appreciate that an enlargement has these properties:
- An enlargement preserves angles but not lengths.
- The centre of enlargement, which can be anywhere inside or outside the figure, is the only point that does not change its position after the enlargement.

Discuss common examples of enlargement, such as photographs, images projected from slide or film, images from binoculars, telescopes or microscopes.

Know that when describing an enlargement the centre of enlargement and the scale factor must be stated.

Enlarge 2-D shapes, given a centre of enlargement and a positive whole-number scale factor. For example:

- Draw a simple shape on a coordinate grid. Take the origin as the centre of enlargement. Enlarge the shape by a whole-number scale factor.



Relate the coordinates of the enlargement to those of the original.

Link to ratio and proportion (pages 78–81).

As outcomes, Year 9 pupils should, for example:

Describe and classify some common examples of reductions, e.g. maps, scale drawings and models (scale factors less than 1).

Recognise how enlargement by a scale factor relates to multiplication:
- Enlargement with scale factor k relates to multiplication by k.
- The inverse transformation has scale factor $\frac{1}{k}$ and relates to multiplication by $\frac{1}{k}$.
- The terms 'multiplication' and 'enlargement' are still used, even when the multiplier or scale factor is less than 1.
- Two successive enlargements with scale factors k_1 and k_2 are equivalent to a single enlargement with scale factor k_1k_2.

Understand that enlargements meet the necessary conditions for two shapes to be mathematically similar, i.e. corresponding angles are equal and corresponding sides are in the same ratio.

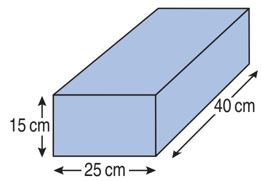
Understand the implications of enlargement for area and volume.
Know that if a shape is enlarged by a scale factor k, then its area (or surface area) is enlarged by scale factor k^2 and its volume by scale factor k^3.

For example:

- Find the area covered by the standard angular person's head (see page 213), and the areas covered by enlargements of it. Tabulate results and compare scale factors for length and for area. Confirm that the scale factor for area is equal to the square of the scale factor for length.

- Start with a unit cube or a simple cuboid, enlarge it by a chosen scale factor and compare with the scale factors for surface area and volume.

- Find the surface area and volume of this cuboid.



  Find the surface area and volume after you have:
  a. doubled its length;
  b. doubled both its length and its width;
  c. doubled each of its length, width and height.

  What are the relationships between the original and the enlarged surface area and volume?

Appreciate some of the practical implications of enlargement, e.g. why a giant would tend to overheat and find standing upright rather painful.

Link to ratio and proportion (pages 78–81), and similarity (pages 192–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Use and interpret maps and scale drawings	

As outcomes, Year 8 pupils should, for example:

Use scales and make simple scale drawings.

For example:

- Design a layout for a bedroom, drawing the room to scale and using cut-outs to represent furniture.
- Estimate from a photograph the height of a tall tree or building by comparison with a person standing alongside it.
- On a sunny day, estimate the height of a telegraph pole or tall tree by using shadows.
- Find the scale of each of these:
  a. the plan of a room with 2 cm representing 1m;
  b. the plan of the school field with 1 inch representing 50 yards;
  c. a map of the area surrounding the school, with 4cm representing 1km.

Link to measuring lengths and conversion of one unit of measurement to another (pages 228–31), and to ratio and proportion (pages 78–81).

As outcomes, Year 9 pupils should, for example:

Use and interpret maps and scale drawings in geography, design and technology and other subjects as well as in mathematics.

Understand different ways in which the scale of a map can be represented and convert between them, e.g. 1 : 50000 or 2cm to 1km.

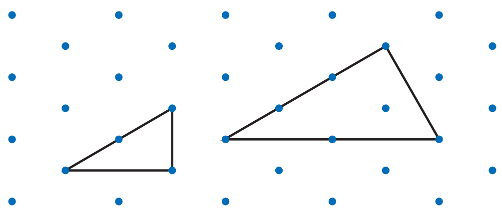
Understand that in a scale drawing:
- linear dimensions remain in proportion, e.g. actual length of object : actual width of object = scaled length : scaled width;
- angles remain the same, e.g. the slope of the floor in a cross-sectional drawing of a swimming pool will be the same as it is in reality.

Measure from a real map or scale drawing.
For example:
- Use the scale of a map to convert a measured map distance to an actual distance 'on the ground'.
- Measure dimensions in a scale drawing and convert them into actual dimensions.

Understand that maps, plans and scale drawings are examples of enlargement by a fractional scale factor.

Understand the implications of enlargement for the area of a scale drawing. For example:

- On a map of scale 1 : 25000, a given distance is represented by a line **twice** the length of the corresponding line on a 1:50000 map. Show that this requires a sheet **four** times the area to cover the same ground.
- Show that these two triangles are similar.
  Find the ratio of the areas of the two triangles.



Link to measuring lengths and conversion of one unit of measurement to another (pages 228–31), ratio and proportion (pages 78–81), and area and volume (pages 234–41).

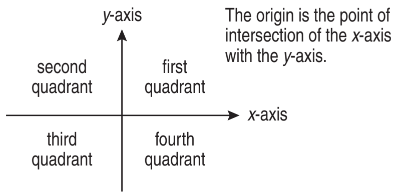
Pupils should be taught to:

Use coordinates in all four quadrants

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
row, column, coordinates, origin, x-axis, y-axis…
position, direction… intersecting, intersection…

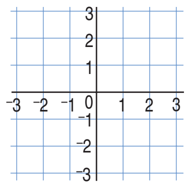
Read and plot points using coordinates in all four quadrants.



- Given an outline shape drawn with straight lines on a coordinate grid (all four quadrants), state the points for a partner to connect in order to replicate the shape.

Plot points determined by geometric information.
For example:

- On this grid, players take turns to name and then mark a point in their own colour. Each point can be used only once.



  Game 1
  The loser is the first to have 3 points in their own colour in a straight line in any direction.

  Game 2
  Players take turns to mark points in their own colour until the grid is full. Each player then identifies and records 4 points in their own colour forming the four corners of a square. The winner is the player who identifies the greatest number of different squares.

- The points (⁻3, 1) and (2, 1) are two points of the four vertices of a rectangle.
  Suggest coordinates of the other two vertices.
  Find the perimeter and area of the rectangle.

- Plot these three points: (1, 3), (⁻2, 2), (⁻1, 4).
  What fourth point will make:
  a. a kite? b. a parallelogram? c. an arrowhead?
  Justify your decisions.
  Is it possible to make a rectangle? Explain why or why not.

Use 'plot' and 'line' on a **graphical calculator** to draw shapes.

- Draw your initials on the screen.

- Draw a shape with reflection symmetry around the y-axis.

See Y456 examples (pages 108–9).

In geography, interpret and use grid references, drawing on knowledge of coordinates.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
mid-point…

Read and plot points in all four quadrants.

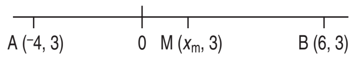
For example:

- The points $(^-5, {}^-3)$, $(^-1, 2)$ and $(3, {}^-1)$ are the vertices of a triangle. Identify where the vertices lie after:
  a. translation of 3 units parallel to the x-axis;
  b. reflection in the x-axis;
  c. rotation of 180° about the origin.

Link to transformations (pages 202–15).

Given the coordinates of points A and B, find the mid-point of the line segment AB.

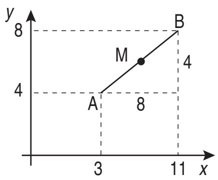
- Two points A and B have the same y-coordinate. Find the mid-point of the line segment AB, e.g.



$x_m = {}^-4 + \frac{1}{2} \times 10 = 1$

Generalise the result: the x-coordinate of the mid-point of AB, where A is the point (x_1, y_1) and B is the point (x_2, y_2), is $x_1 + \frac{1}{2}(x_2 - x_1) = \frac{1}{2}(x_1 + x_2)$.

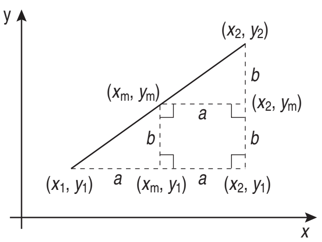
- A is the point (3, 4) and B is the point (11, 8). Find the mid-point (x_m, y_m) of AB.



Complete a right-angled triangle with AB as hypotenuse.
Use the properties of a rectangle to deduce that:
$x_m = 3 + \frac{1}{2} \times 8 = 7$, $y_m = 4 + \frac{1}{2} \times 4 = 6$

Use the properties of a rectangle to generalise the result: the mid-point of the line segment joining A (x_1, y_1) to B(x_2, y_2) is given by:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



Note that $\frac{x_1 + x_2}{2}$ is the mean of the x-coordinates.

As outcomes, Year 9 pupils should, for example:

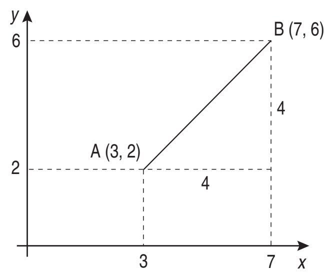
Use vocabulary from previous years and extend to:
Pythagoras' theorem…

Find points that divide a line in a given ratio, using the properties of similar triangles.

Given the coordinates of points A and B, calculate the length of AB.

- A is the point (3, 2) and B is the point (7, 6). Find the length of AB.

  Complete a right-angled triangle with AB as the hypotenuse. Use Pythagoras' theorem to calculate AB.



Use Pythagoras' theorem to deduce the general result: the distance, d, between points A (x_1, y_1) and B (x_2, y_2) is given by the formula:

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

- The coordinates of point A are (3, 2).
  The x-coordinate of point B is 11.
  Line AB is 10 units long.
  Find the coordinates of the mid-point of AB.

Link to Pythagoras' theorem (pages 186–9).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

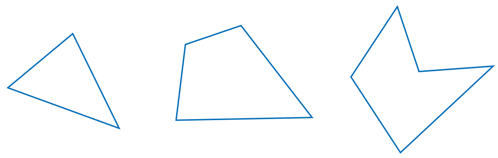
Construct lines, angles and shapes

Use, read and write, spelling correctly:
construct, draw, sketch, measure… perpendicular, distance…
ruler, protractor (angle measurer), set square…

Use ruler and protractor to measure and draw lines to the nearest millimetre and angles, including reflex angles, to the nearest degree.

For example:

- Measure the sides and interior angles of these shapes.



See Y456 examples (pages 92–5).

Link to angle measure (pages 232–3).

As outcomes, Year 8 pupils should, for example:

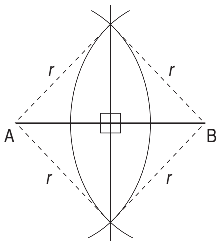
Use vocabulary from previous year and extend to:
bisect, bisector, mid-point… equidistant…
straight edge, compasses… locus, loci…

In work on construction and loci, know that the shortest distance from point P to a given line is taken as the distance from P to the nearest point N on the line, so that PN is perpendicular to the given line.

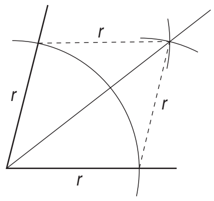
Use straight edge and compasses for constructions.

Recall that the diagonals of a rhombus bisect each other at right angles and also bisect the angles of the rhombus. Recognise how these properties, and the properties of isosceles triangles, are used in standard constructions.

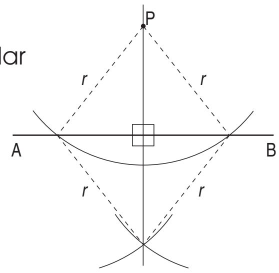
- Construct the mid-point and perpendicular bisector of a line segment AB.



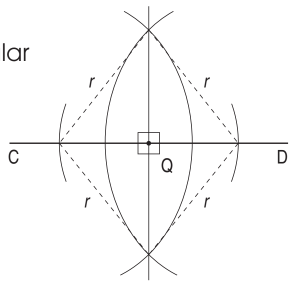
- Construct the bisector of an angle.



- Construct the perpendicular from a point P to a line segment AB.



- Construct the perpendicular from a point Q on a line segment CD.



Know that:
- The perpendicular bisector of a line segment is the locus of points that are equidistant from the two end points of the line segment.
- The bisector of an angle is the locus of points that are equidistant from the two lines.

Link to loci (pages 224–7) and properties of a rhombus (pages 186–7), and to work in design and technology.

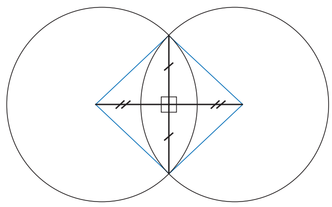
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
circumcircle, circumcentre, inscribed circle…

Use straight edge and compasses for constructions.

Understand how standard constructions using straight edge and compasses relate to the properties of two intersecting circles with equal radii:

- The common chord and the line joining the two centres bisect each other at right angles.
- The radii joining the centres to the points of intersection form two isosceles triangles or a rhombus.

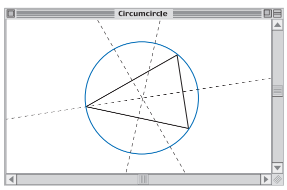


Use congruence to prove that the standard constructions are exact.

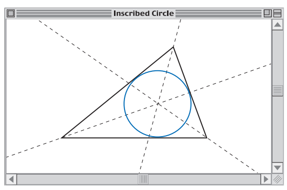
Use construction methods to investigate what happens to the angle bisectors of any triangle, or the perpendicular bisectors of the sides. For example:

- Observe the position of the centres of these circles as the vertices of the triangles are moved.

Construct a triangle and the perpendicular bisectors of the sides. Draw the circumcircle.



Construct a triangle and the angle bisectors. Draw the inscribed circle.



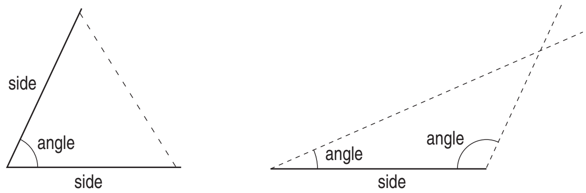
Link to properties of a circle (pages 194–7), and to work in design and technology.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Construct lines, angles and shapes (continued)

Construct triangles.
Use ruler and protractor to construct triangles:

- given two sides and the included angle (SAS);
- given two angles and the included side (ASA).



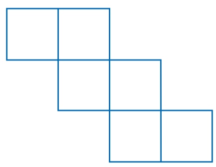
For example:

- Construct ΔABC with ∠A = 36°, ∠B = 58° and AB = 7 cm.
- Construct a rhombus, given the length of a side and one of the angles.

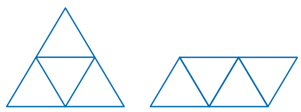
See Y456 examples (pages 102–3).

Construct solid shapes. Use ruler and protractor to construct simple nets. For example:

- Look at this net of a cube.
  When you fold it up, which edge will meet the edge marked A?
  Mark it with an arrow.



- Imagine two identical square-based pyramids.
  Stick their square faces together.
  How many faces does your new shape have?
- Construct on plain paper a net for a cuboid with dimensions 2 cm, 3cm, 4cm.
- Construct the two possible nets of a regular tetrahedron, given the length of an edge.



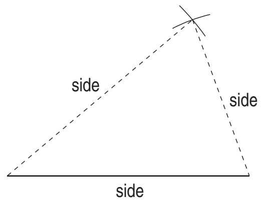
As outcomes, Year 8 pupils should, for example:

Construct triangles.
Construct triangles to scale using ruler and protractor, given two sides and the included angle (SAS) or two angles and the included side (ASA).

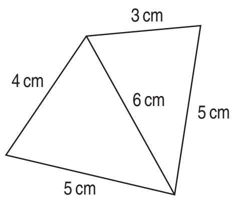
- A tower is 30 metres high.
  It casts a shadow of 10 metres on the ground.
  Construct a triangle to scale to represent this.
  Using a protractor, measure the angle that the light from the sun makes with the ground.

Extend to constructions with straight edge and compasses. For example:

- Construct a triangle given three sides (SSS).



- Construct this quadrilateral.



Link to scale drawings (pages 216–17).

Construct nets of solid shapes. For example:

- Construct a net for a square-based pyramid given that the side of the base is 3 cm and each sloping edge is 5cm.

As outcomes, Year 9 pupils should, for example:

Construct triangles.
Use the method for constructing a perpendicular from a point on a line to construct triangles, given right angle, hypotenuse and side (RHS). For example:

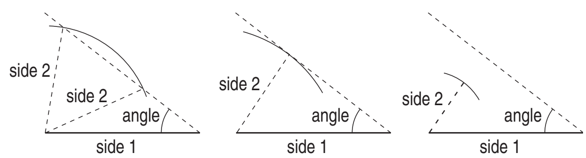
- A 10 metre ladder rests against a wall with its foot 3 metres away from the wall.
  Construct a diagram to scale. Then use a ruler and protractor to measure as accurately as possible:
  a. how far up the wall the ladder reaches;
  b. the angle between the ladder and the ground.

Review methods for constructing triangles given different information. For example:

- Is it possible to construct triangle ABC such that:
  a. A = 60°, B = 60°, C = 60°
  b. BC = 6 cm, AC = 4cm, AB = 3cm
  c. BC = 7cm, AC = 3cm, AB = 2cm
  d. A = 40°, B = 60°, AB = 5cm
  e. A = 30°, B = 45°, AC = 6cm
  f. BC = 8cm, AC = 6cm, C = 50°
  g. BC = 7cm, AC = 5.5cm, B = 45°
  h. BC = 7cm, AC = 4.95cm, B = 45°
  i. BC = 7cm, AC = 4cm, B = 45°
  j. BC= 6cm, AC = 10cm, B = 90°?

Know from experience of constructing them that triangles given SSS, SAS, ASA and RHS are unique but that triangles given SSA or AAA are not.

To specify a triangle three items of data about sides and angles are required. In particular:
- Given three angles (AAA) there is no unique triangle, but an infinite set of similar triangles.
- Given three sides (SSS) a unique triangle can be drawn, provided that the sum of the two shorter sides is greater than the longest side.
- Given two angles and any side (AAS), a unique triangle can be drawn.
- Given two sides and an included angle (SAS), a unique triangle can be drawn.
- If the angle is not included between the sides (SSA), there are three cases to consider:
  i. the arc, of radius equal to side 2, cuts side 3 in two places, giving two possible triangles, one acute-angled and the other obtuse-angled;
  ii. the arc touches side 3, giving one right-angled triangle (RHS);
  iii. the arc does not reach side 3 so no triangle is possible.



Link to congruence and similarity (pages 190–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Find simple loci, both by reasoning and by using ICT	

As outcomes, Year 8 pupils should, for example:

Find simple loci, both by reasoning and by using ICT to produce shapes and paths.

Describe familiar routes. For example:

- Rajshree walked to the youth club.
  She turned left out of her front gate.
  She turned right at the telephone box.
  She went straight on at the crossroads.
  At the traffic lights she turned right then left.
  She turned left at the station.
  The youth club is on the left-hand side of the road.

  Describe her route home from the youth club.

Give practical examples of paths such as:
- the trail left on the ground by a snail;
- the vapour trail of an aircraft;
- the path traced out by a conker on a string;
- the path of a ball thrown into the air;
- the path you follow on a fairground ride;
- the path of the tip of a windscreen wiper.

Visualise a simple path. For example:

- Imagine a robot moving so that it is always the same distance from a fixed point. Describe the shape of the path that the robot makes.
  (A circle.)

- Imagine two trees. Imagine walking so that you are always an equal distance from each tree. Describe the shape of the path you would walk.
  (The perpendicular bisector of the line segment joining the two trees.)

Understand locus as a set of points that satisfy a given set of conditions or constraints. Place counters on a table according to a given rule and determine the locus of their centres. For example:

- Place a red counter in the middle of the table. Place white counters so that their centres are all the same distance from the centre of the red counter. (Centres lie on a circle.)

- Place a red counter and a green counter some distance apart from each other. Place white counters so that their centres are always an equal distance from the centres of the red and green counters.
  (Centres lie on the perpendicular bisector of RG.)

- Place white counters so that their centres are the same distance from two adjacent edges of the table. (Centres lie on the bisector of the angle at the corner of the table.)

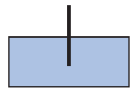
Link to construction (pages 220–3).

As outcomes, Year 9 pupils should, for example:

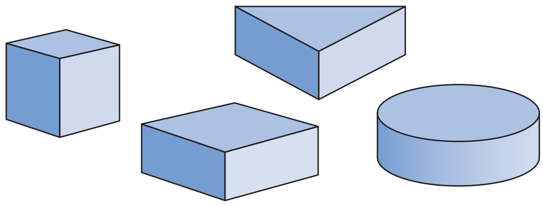
Find loci, both by reasoning and by using ICT to produce shapes and paths.

Visualise paths and loci in two or three dimensions. For example:

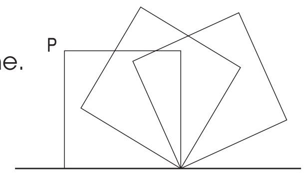
- Imagine the black line is a stick stuck flat to a rectangular card so that it lies in the same plane.

  Hold the card and stick upright and spin it as fast as you can.
  Which of these shapes would you seem to see?

- Imagine a square being rolled along a straight line. What path would the point P trace?

- A spider is dangling motionless on a single web. I move a finger so that its tip is always 10 cm from the spider. What is the locus of my finger tip?
  (The surface of a sphere.)

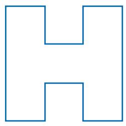
- I hold a ruler in my left hand, then move the tip of my right forefinger so that it is always 8 cm from the ruler. What is the locus of my fingertip?
  (The surface of a cylinder with a hemisphere on each end.)

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Find simple loci, both by reasoning and by using ICT (continued)	

As outcomes, Year 8 pupils should, for example:

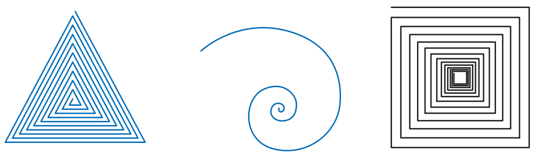
Use ICT to generate shapes and paths.
For example, generate using **Logo**:

- rectilinear shapes



- regular polygons

- equi-angular spirals

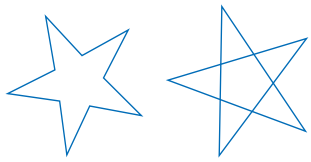


Link to properties of triangles, quadrilaterals and polygons (pages 184–9).

As outcomes, Year 9 pupils should, for example:

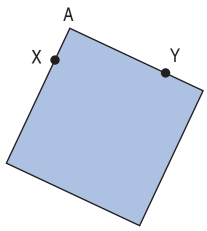
Use ICT to investigate paths.
For example:

- Use **Logo** to produce a five-pointed star.

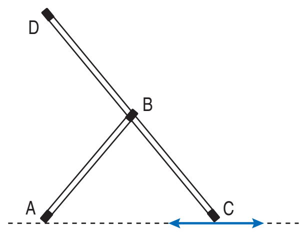


Investigate problems involving loci and simple constructions. For example:

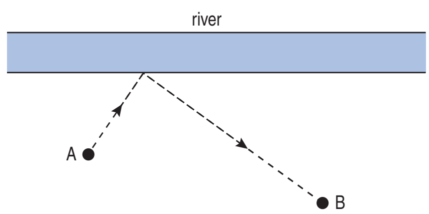
- Two points X and Y are 10 cm apart.
  Two adjacent sides of a square pass through points X and Y.
  What is the locus of vertex A of the square?



- In a design for the mechanism of a shower door, AB is a bracket, fixed at A and joined by a pivot to the middle of the door frame at B. One end of the door frame, C, moves along a groove shown by the dotted line. What is the locus of point D on the other edge of the door?



- A man has to run from point A to point B, collecting a bucket of water from the river on his way. What point on the river bank should he aim for, in order to keep his path from A to B as short as possible?



Link to properties of circles (pages 194–7).

Pupils should be taught to:

Use units of measurement to measure, estimate, calculate and solve problems in a range of contexts; convert between metric units and know rough metric equivalents of common imperial measures

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly, names and abbreviations of:

Standard metric units

- millimetre (mm), centimetre (cm), metre (m), kilometre (km)
- gram (g), kilogram (kg)
- millilitre (ml), centilitre (cl), litre (l)
- square millimetre (mm^2), square centimetre (cm^2), square metre (m^2), square kilometre (km^2)

Units of temperature, time, angle

- degree Celsius (°C)
- second (s), minute (min), hour (h), day, week, month, year, decade, century, millennium
- degree (°)

Know relationships between units of a particular measure, e.g.

- 1 kg = 1000g

See Y456 examples (pages 90–1).

Convert between one metric unit and another.

Know the relationship between metric units in common use and how they are derived from the decimal system. For example:

1000	100	10	1	0.1	0.01	0.001
km	–	–	m	–	cm	mm
8	0	0	0			
			4			
				3	7	
				2	3	0

8000 m = 8 km
4 m = 400 cm = 4000 mm
37 cm = 0.37 m
230 mm = 0.23 m

Understand that for the same measurement in two different units:

- if the unit is smaller, the number of units will be greater;
- if the unit is bigger, the number of units will be smaller.

Change a larger unit to a smaller one. For example:

- Change 36 centilitres into millilitres.
- Change 0.89 km into metres.
- Change 0.56 litres into millilitres.

Change a smaller unit to a larger one. For example:

- Change 750 g into kilograms.
- Change 237ml into litres.
- Change 3cm into metres.
- Change 4mm into centimetres.

Begin to know and use rough metric equivalents of imperial measures in daily use. For example, know that:

- 1 gallon ≈ 4.5 litres
- 1 pint is just over half a litre.

For example:

- A litre of petrol costs 89.9p.
  Approximately, how much would 1 gallon cost?

See Y456 examples (pages 90–1).

Link to mental recall of measurement facts (pages 90–1).

As outcomes, Year 8 pupils should, for example:

Continue to use familiar units of measurement from previous year and extend to:

Standard metric units

- tonne (not usually abbreviated)
- hectare (ha)
- cubic millimetre (mm^3), cubic centimetre (cm^3), cubic metre (m^3)

Commonly used imperial units

- ounce (oz), pound (lb), foot (ft), mile, pint, gallon

Know the relationships between the units of a particular measure, e.g.

- 1 hectare = 10 000m^2
- 1 tonne = 1000kg

and extend to:

- 1 litre = 1000 cm^3
- 1 millilitre =1cm^3
- 1000 litres = 1m^3

Convert between one unit and another.

Convert between area measures in simple cases. For example:

- A rectangular field measures 250 m by 200m. What is its area in hectares (ha)?
- Each side of a square tablecloth measures 120cm. Write its area in square metres (m^2).

Convert between units of time. For example:

- At what time of what day of what year will it be:
  a. 2000 minutes
  b. 2000 weeks
  after the start of the year 2000?
- How many seconds will pass before your next birthday?

Consolidate changing a smaller unit to a larger one. For example:

- Change 760 g into kilograms.
- Change 400ml into litres.

Know rough metric equivalents of imperial measures in daily use (feet, miles, pounds, pints, gallons) and convert one to the other. For example, know that:

- 1 m ≈ 3ft
- 8 km ≈ 5 miles
- 1kg ≈ 2.2lb and 1 ounce ≈ 30g
- 1 litre is just less than 2 pints.

Link to mental recall of measurement facts (pages 90–1).

As outcomes, Year 9 pupils should, for example:

Continue to use familiar units of measurement from previous years and extend to:

Compound measures

- average speed (distance/time)
- density (mass/volume)
- pressure (force/area)

Convert between metric units, including area, volume and capacity measures.

For example:

- Change 45 000 square centimetres (cm^2) into square metres (m^2).
- Change 150000 square metres (m^2) into hectares;
- Change 5.5 cubic centimetres (cm^3) into cubic millimetres (mm^3);
- Change 3.5 litres into cubic centimetres (cm^3).

Link to mental recall of measurement facts (pages 90–1).

Convert between currencies. For example:

- Use £1 = 10.6 rands to work out how much 45p is in rands.
- Use 565 drachmae = £1 to work out how much 1000 drachmae is in pounds.

Convert one rate to another. For example:

- Convert 30 mph to metres per second.

Link to direct proportion (pages 78–9) and conversion graphs (pages 172–3).

Pupils should be taught to:

Use units of measurement to measure, estimate, calculate and solve problems in a range of contexts; convert between metric units and know rough metric equivalents of common imperial measures (continued)

As outcomes, Year 7 pupils should, for example:

Use opportunities in science, design and technology, geography and other subjects to estimate and measure using a range of measuring instruments.

Suggest appropriate units and methods to estimate or measure length, area, capacity, mass and time. For example, estimate or suggest units to measure:

- the length of a football field… the thickness of a hair… the diameter of a CD…
- the area of the school hall… of a postage stamp… of the school grounds… the surface area of a matchbox…
- the mass of a coin… of a van…
- the time to run the length of a football field… to boil an egg… to mature a cheese… to travel to the moon…

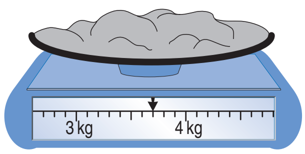
Give a suitable range for an estimated measurement.
For example, estimate that:

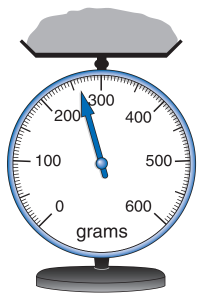
- 3 m < width of classroom window < 4m
- 0.5 litre < capacity of a tankard < 1 litre
- 50g < mass of a golf ball < 1000g

Check by measuring as precisely as possible whether the measurement lies within the estimated range.

Read and interpret scales on a range of measuring instruments, with appropriate accuracy, including:

- vertical scales, e.g. thermometer, tape measure, ruler, measuring cylinder…
- scales around a circle or semicircle, e.g. for measuring time, mass, angle…





See Y456 examples (pages 94–5).

Link to rounding (pages 42–5).

Solve problems involving length, area, capacity, mass, time and angle, rounding measurements to an appropriate degree of accuracy.

See Y456 examples (pages 86–93).

Link to problem solving (pages 18–21) and area (pages 234–7).

As outcomes, Year 8 pupils should, for example:

Use opportunities in other subjects to estimate and measure using a range of measuring instruments, particularly opportunities to measure volume and bearings.

Suggest appropriate units and methods to estimate or measure volume. For example, estimate or suggest units to measure:

- the volume of a matchbox, of a telephone box, of the school hall…

Estimate measures within a given range. Suggest approximate measures of objects or events to use as reference points or benchmarks for comparison.
For example, the approximate:

- height of a door is 2 m;
- height of an average two-storey house is 10m;
- mass of a large bag of sugar is 1kg;
- mass of a small family car is about 1000kg;
- capacity of a small tumbler is about 250ml;
- area of a football pitch is $7500m^2$;
- area of a postcard is $100\ cm^2$;
- time to walk one mile is about 20 minutes.

Suggest and justify an appropriate degree of accuracy for a measurement. For example:

- John says he lives 400 metres from school.
  Do you think this measurement is correct to:
  A. the nearest centimetre,
  B. the nearest metre,
  C. the nearest 10 metres, or
  D. the nearest 100 metres?

  It takes John 7.5 minutes to walk to the school.
  Do you think this measurement is correct to:
  P. the nearest second,
  Q. the nearest 30 seconds, or
  R. the nearest minute?

Solve problems involving length, area, volume, capacity, mass, time, angle and bearings, rounding measurements to an appropriate degree of accuracy.

Link to problem solving (pages 18–21), area (pages 234–7), volume (pages 238–9), and bearings (pages 232–3).

As outcomes, Year 9 pupils should, for example:

Suggest appropriate units to estimate or measure speed. For example, estimate or suggest units to measure the average speed of:

- an aeroplane flying to New York from London, a mountain climber, a rambler on a country walk, a swimmer in a race, a snail in motion, a jaguar chasing prey…

Know that no measurement can be made exactly so it is conventional to give any measured value to the nearest whole unit or decimal place (e.g. the nearest mark on a scale). A measurement may be in error by up to half a unit in either direction.
For example:

- A length d m is given as 36 m. It is presumed to be to the nearest metre so $35.5 \le d < 36.5$.
- A volume V cm^3 is given as 240 cm^3. It is presumed to be to the nearest 10 cm^3 so $235 \le V < 245$.
- A mass m kg is given as 2.3 kg. It is presumed to be to the nearest 0.1kg so $2.25 \le m < 2.35$.

Suggest a range for measurements such as:

- 123 mm   1860mm   3.54kg $6800m^2$

Solve problems such as:

- The dimensions of a rectangular floor, measured to the nearest metre, are given as 28 m by 16m. What range must the area of the floor lie within? Suggest a sensible answer for the area, given the degree of accuracy of the data.

Link to rounding and approximation (pages 42–7).

Solve problems involving length, area, volume, capacity, mass, time, speed, angle and bearings, rounding measurements to an appropriate degree of accuracy.

Link to problem solving (pages 18–21), area (pages 234–7), volume (pages 238–9), bearings (pages 232–3), and speed (pages 232–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Extend the range of measures used to angle measureandbearings,andcompoundmeasures

Angle measure

Use, read and write, spelling correctly:
angle, degree (°)… protractor (angle measurer), set square…
right angle, acute angle, obtuse angle, reflex angle…

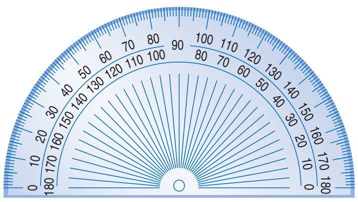
Use angle measure; distinguish between and estimate the size of acute, obtuse and reflex angles.

Discuss 'rays' of lines emanating from a point, using angles in degrees as a measure of turn from one ray to another.

Know that:

- An angle less than 90° is an acute angle.
- An angle between 90° and 180° is an obtuse angle.
- An angle between 180° and 360° is a reflex angle.
- An angle greater than 360° involves at least one complete turn.

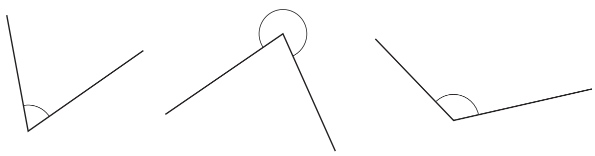
Use a 180° or 360° protractor to measure and draw angles, including reflex angles, to the nearest degree. Recognise that an angle can be measured as a clockwise or anticlockwise rotation and that the direction chosen determines which will be the zero line and whether the inner or outer scale is to be used.



- Draw angles of 36°, 162° and 245°.

Estimate acute, obtuse and reflex angles. For example:

- Decide whether these angles are acute, obtuse or reflex, estimate their size, then measure each of them to the nearest degree.



- Imagine a semicircle cut out of paper.
  Imagine folding it in half along its line of symmetry.
  Fold it in half again, and then once more.
  How many degrees is the angle at the corner of the shape now?

Link to angles and lines (pages 178–83), and construction (pages 220–3).

As outcomes, Year 8 pupils should, for example:

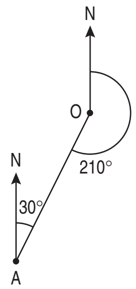
Bearings

Use, read and write, spelling correctly:
bearing, three-figure bearing…
and compass directions.

Use bearings to specify direction and solve problems, including making simple scale drawings.

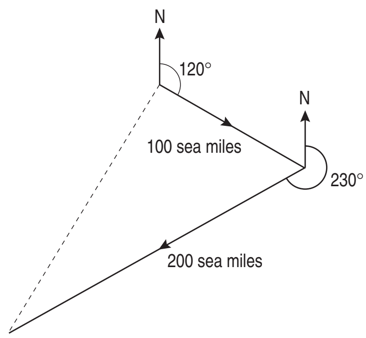
Know that the bearing of a point A from an observer O is the angle between the line OA and the north line through O, measured in a clockwise direction.

In the diagram the bearing of A from O is 210°.
The three-figure bearing of O from A is 030°.



For example:

- If the bearing of P from Q is 045°, what is the bearing of Q from P?
- If the bearing of X from Y is 120°, what is the bearing of Y from X?
- A ship travels on a bearing of 120° for 100 sea miles, then on a bearing of 230° for a further 200 sea miles. Represent this with a scale drawing. What is the ship's distance and bearing from the starting point?



Link to angles and lines (pages 178–83), and scale drawings (pages 216–17).

As outcomes, Year 9 pupils should, for example:

Compound measures

Use, read and write, spelling correctly:
speed, density, pressure… and units such as:
miles per hour (mph), metres per second (m/s).

Understand that:

- **Rate** is a way of comparing how one quantity changes with another, e.g. a car's fuel consumption measured in miles per gallon or litres per 100 km.
- The two quantities are usually measured in different units, and 'per', the abbreviation 'p' or an oblique '/' is used to mean 'for every' or 'in every'.

Know that if a rate is **constant** (uniform), then the two variables are in direct proportion and are connected by a simple formula. For example:

- **speed** $= \frac{\text{distance travelled}}{\text{time taken}}$
- **density** $= \frac{\text{mass of object}}{\text{volume of object}}$
- **pressure** $= \frac{\text{force on surface}}{\text{surface area}}$

Know that if a rate varies, the same formula can be used to calculate an **average rate**. For example:

- A cyclist travels 36 miles in 3 hours.
  Her **average speed** is 12 mph.

Solve problems involving average rates of change.
For example:

- The distance from London to Leeds is 190 miles. An Intercity train takes about 2¼ hours to travel from London to Leeds. What is its average speed?
- a. A cyclist travels 133 km in 8 hours, including a 1 hour stop. What is her average cycling speed?
  b. She cycles for 3 hours on the flat at 20 km/h and 1½ hours uphill at 12km/h. What is her total journey time?
  c. The cyclist travels 160km at an average speed of 24km/h. How long does the journey take?
- Naismith's rule, used by mountain walkers, says that you should allow 1 hour for every 3 miles travelled and ½ hour for each 1000 ft climbed. At what time would you expect to return from a walk starting at 09:00, if the distance is 14 miles and 5000 feet have to be climbed, allowing an extra 2 hours for stops and possible delays?

Use speed, density and pressure in other subjects, such as science or physical education.

Link to formulae and direct proportion in algebra (pages 136–7), and distance–time graphs (pages 172–7).

Pupils should be taught to:

Deduce and use formulae to calculate lengths, perimeters, areas and volumes in 2-D and 3-D shapes

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
area, surface, surface area, perimeter, distance, edge…
and use the units: square centimetre (cm^2),
square metre (m^2), square millimetre (mm^2)…

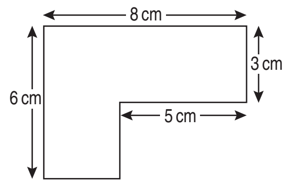
Deduce and use formulae for the perimeter and area of a rectangle.

Derive and use a formula for the area of a right-angled triangle, thinking of it as half a rectangle:

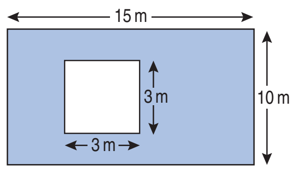
area = ½ × base length × height
area = ½bh

Calculate the perimeter and area of shapes made from rectangles. For example:

- Find the area of this shape.



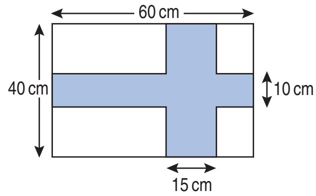
- Find the shaded area.



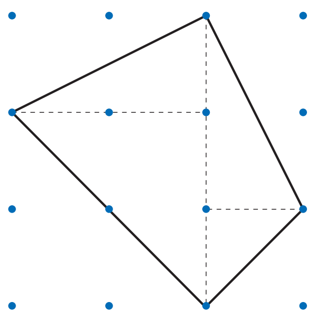
- Find the area and outside perimeter of a path 1 metre wide bordering a 5 metre square lawn.

- Here is a flag.

  Calculate the area of the shaded cross.



- Find the area of this quadrilateral:
  a. by completing the 3 by 3 square and subtracting the pieces outside the quadrilateral;
  b. by dissecting the inside of the quadrilateral into rectangles and/or right-angled triangles.

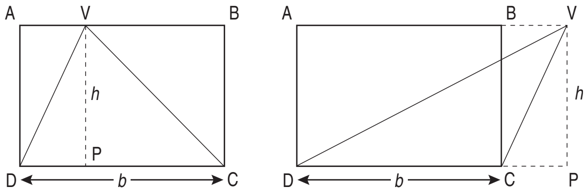


As outcomes, Year 8 pupils should, for example:

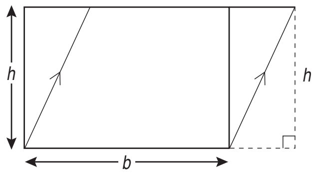
Use vocabulary from previous year and extend to:
volume, space, displacement…
and use the units: hectare (ha), cubic centimetre (cm^3), cubic metre (m^3), cubic millimetre (mm^3)…

Deduce formulae for the area of a parallelogram, triangle and trapezium. For example, explain why:

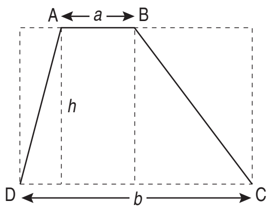
- The area of a triangle is given by A = ½bh, where b is the base and h is the height of the triangle.



- A rectangle and parallelogram on the same base and between the same parallels have the same area, A = bh, where b is the base and h is the perpendicular distance between the parallels.

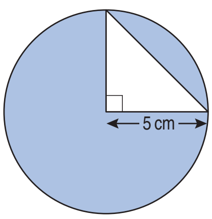


- The area of a trapezium, where h is the perpendicular distance between the parallel sides, is ½ (sum of parallel sides) × h.

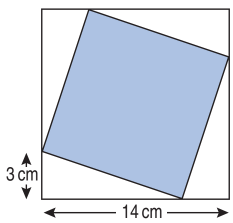


Calculate areas of triangles, parallelograms and trapezia, and of shapes made from rectangles and triangles. For example:

- A right-angled triangle lies inside a circle.
  The circle has a radius of 5 cm.
  Calculate the area of the triangle.



- The diagram shows a shaded square inside a larger square.
  Calculate the area of the shaded square.



As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
circumference, π…
and names of the parts of a circle.
Link to circles (pages 194–7).

Know and use the formula for the circumference of a circle. For example:

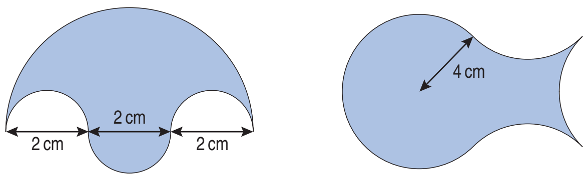
Know that the formula for the circumference of a circle is C = πd, or C = 2πr, and that different approximations to π are 3, 22/7, or 3.14 correct to 2 d.p.

Use the π key on a **calculator**.

Calculate the circumference of circles and arcs of circles. For example:

- A circle has a circumference of 120 cm.
  What is the radius of the circle?
- The diameter of King Arthur's Round Table is 5.5 m.
  A book claims that 50 people sat round the table.
  Assume each person needs 45 cm round the circumference of the table.
  Is it possible for 50 people to sit around it?
- The large wheel on Wyn's wheelchair has a diameter of 60 cm.
  Wyn pushes the wheel round exactly once.
  Calculate how far Wyn has moved.

  The large wheel on Jay's wheelchair has a diameter of 52 cm.
  Jay moves her wheelchair forward 950cm.
  How many times does the large wheel go round?
- A Ferris wheel has a diameter of 40 metres. How far do you travel in one revolution of the wheel?
- A touring cycle has wheels of diameter 70cm. How many rotations does each wheel make for every 10km travelled?
- All curves in the left-hand figure are semicircles.
  All curves in the right-hand figure are quarter circles or three-quarter circles.
  Calculate the perimeter of each shape.

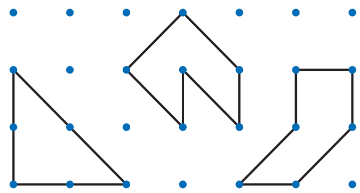


Know that the length of an arc is directly proportional to the size of the angle θ between the two bounding radii, or arc length = 2πr × θ/360, where θ is in degrees and r is the length of the radius.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

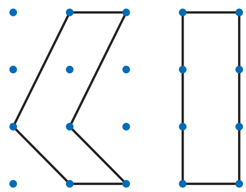
Deduce and use formulae to calculate lengths, perimeters, areas and volumes in 2-D and 3-D shapes (continued)

- Using a pinboard, make different shapes with an area of 2 square units.



Investigate relationships between perimeters and areas of different rectangles. For example:

- Sketch and label some shapes which have an area of 1 m^2. Find the perimeter of each shape.
- A rectangle has a fixed area of 36 cm^2. What could its perimeter be? What shape gives the smallest perimeter?
- A rectangle has a fixed perimeter of 20 cm. What could its area be? What shape encloses the most area?
- Do these shapes have:
  a. the same area?
  b. the same perimeter?



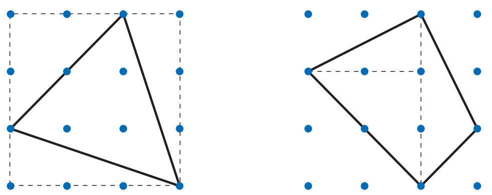
See Y456 examples (pages 96–7).

Link to problem solving (pages 18–19).

As outcomes, Year 8 pupils should, for example:

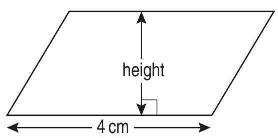
- On a 1 cm grid, draw a triangle with an area of 7.5cm^2 and an obtuse angle.

- Use methods such as dissection or 'boxing' of shapes to calculate areas. For example:
  Draw these shapes on a 1cm spotty grid and use the dashed lines to help find their areas.

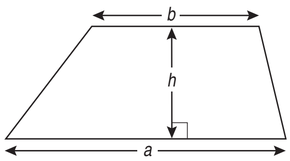


- Make quadrilaterals on a 3 by 3 pinboard.
  Find the area of each quadrilateral.

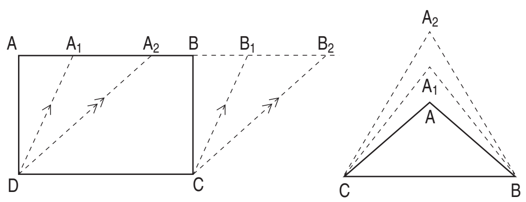
- The area of the parallelogram is 10 cm^2.
  Calculate the height of the parallelogram.



- The area of the trapezium is 10 cm^2.
  What might be the values of h, a and b (a > b)?



- Use a pinboard and spotty paper to investigate simple transformations of the vertices of triangles and parallelograms, and how they affect the area of the shapes. For example:



  Find the area of each parallelogram or triangle and explain what is happening.

Link to problem solving (pages 18–19).

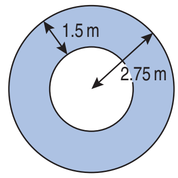
As outcomes, Year 9 pupils should, for example:

Know and use the formula for the area of a circle:
$A = \pi d^2/4$ or $A = \pi r^2$

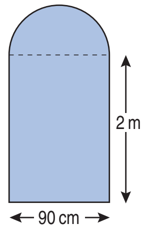
For example:

- A circle has a radius of 15 cm.
  What is its area?

- Calculate the area of the shaded shape.



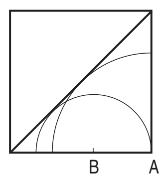
- A church door is in the shape of a rectangle with a semicircular arch.
  The rectangular part is 2 m high and the door is 90cm wide.
  What is the area of the door?



- Napoli Pizzas make two sizes of pizza.
  a. A small pizza has a diameter of 25 cm.
  What is the surface area of the top of the pizza?
  b. A large pizza has twice the surface area of the small one. What is the diameter of the large pizza?
  c. A small boy reckons he could just about manage to eat a 120° wedge (sector) of the large pizza. What is the area covered by topping on this piece?

- The inside lane of a running track is 400 m long, 100m on each straight and 100m on each semicircular end.
  What area in the middle is free for field sports?

- A donkey grazes in a 60 m by 60m square field with a diagonal footpath. The donkey is tethered to a post. It can just reach the path, but not cross it.
  Consider two tethering positions:
  A. corner A, making the largest possible quarter circle (quadrant) for the donkey to graze;
  B. point B, somewhere on the field boundary, making the largest possible semicircle.



  Which tethering position gives the bigger grazing area? Use scale drawing and measurement to help calculate the answer.
  What if the field is rectangular?

Know that the area of a sector of a circle is directly proportional to the size of the angle θ between the two bounding radii, or area of sector = $2\pi r^2 \times \theta/360$, where θ is in degrees and r is the length of the radius.

Link to problem solving (pages 18–19).

Pupils should be taught to:

Deduce and use formulae to calculate lengths, perimeters, areas and volumes in 2-D and 3-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

Find the surface area of cuboids and shapes made from cuboids. Check by measurement and calculation.

Unfold packets in the shape of cuboids and other 3-D shapes to form a net. Relate the surface area to the shape of the net.

Estimate the surface area of everyday objects. For example:

- Estimate the surface area of a house brick, a large cereal packet, a matchbox…
  Check estimates by measurement and calculation.

Derive and use a formula for the **surface area S of a cuboid** with length l, width w and height h:

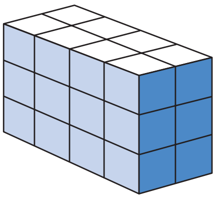
$S = 2(\text{length} \times \text{width}) + 2(\text{length} \times \text{height}) + 2(\text{height} \times \text{width})$
$S = 2lw + 2lh + 2hw$

As outcomes, Year 8 pupils should, for example:

Know the formula for the volume of a cuboid and use it to solve problems involving cuboids.

Understand the formula for the volume of a cuboid by considering how to count unit cubes.

- Suppose the cuboid is l units long, w units wide and h units high.



Then:
area of base = lw square units
volume = area of base × number of layers
= lwh cubic units

Estimate volumes. For example:

- Estimate the volume of everyday objects such as a rectangular chopping board, a bar of soap, a shoe box…
Check estimates by measurement and calculation.

Suggest volumes to be measured in cm^3, m^3.

Volume and displacement
In science, start to appreciate the connection between volume and displacement. For example,

- Make some cubes or cuboids with different numbers of Centicubes.
Put them into a measuring cylinder half filled with water. How many millilitres does the water rise?
What is the connection between the volume of the cube or cuboid and the volume of water displaced?
(1 ml of water has a volume of $1 cm^3$.)

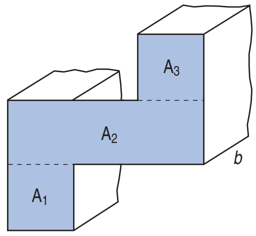
As outcomes, Year 9 pupils should, for example:

Calculate the surface area and volume of a right prism.

Know that a prism is a polyhedron of uniform cross-section throughout its length. A cuboid is a common example.

Use knowledge of prisms made up of cuboids to write an expression for the total volume of such a prism. For example:

- A prism has cross-section areas A_1, A_2, A_3, …, all of length b.



$$V = A_1b + A_2b + A_3b + \dots$$
$$= (A_1 + A_2 + A_3 + \dots)b$$
$$= \text{total area of cross-section} \times \text{length}$$

Surface area and volume of a cylinder
Know that the total surface area A of a cylinder of height h and radius r is given by the formula
$A = 2\pi r^2 + 2\pi rh$
and that the volume V of the cylinder is given by the formula
$V = \pi r^2 h$

- In geography, use a rain gauge, then estimate the volume of water which has fallen on a specified area over a given period.

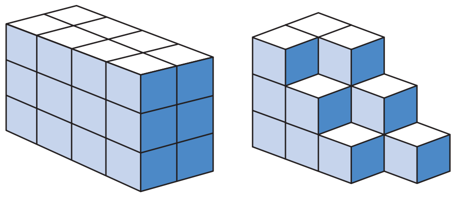
Pupils should be taught to:

Deduce and use formulae to calculate lengths, perimeters, areas and volumes in 2-D and 3-D shapes (continued)

As outcomes, Year 7 pupils should, for example:

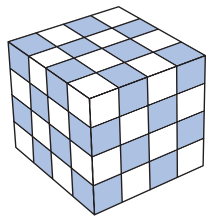
Solve simple problems such as:

- How many unit cubes are there in these shapes?
  What is the surface area of each shape?

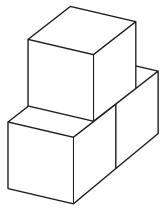


- Investigate the different cuboids you can make with 24 cubes. Do they all have the same surface area?

- This solid cube is made from alternate blue and white centimetre cubes.
  What is its surface area?
  How much of its surface area is blue?



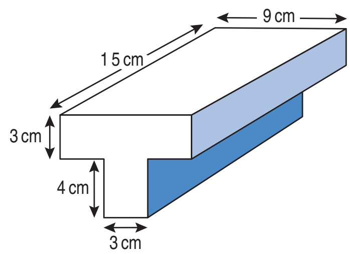
- This shape is made from three identical cubes.
  The top cube is placed centrally over the other two.



  The faces of the shape are to be covered in sticky paper.
  Sketch the different shapes of the pieces of paper required.
  Say how many of each shape are needed.

  If each cube has an edge of 5 cm, what is the surface area of the whole shape?
  Compare different methods of working this out.

- Calculate the surface area in cm^2 of this girder.



- 12 cubes with 1 cm edges are each covered in sticky paper.
  How much paper is needed?

  The 12 cubes are wrapped in a single parcel. What arrangement of the cubes would need the least paper?

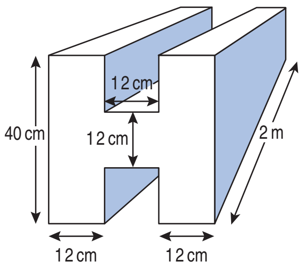
Link to lines, angles and shapes (pages 178–201), and problem solving (pages 18–19).

As outcomes, Year 8 pupils should, for example:

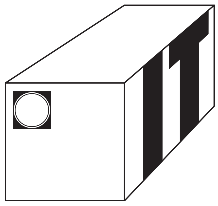
Solve problems such as:

- Find the volume of a 3 cm by 4cm by 5cm box.

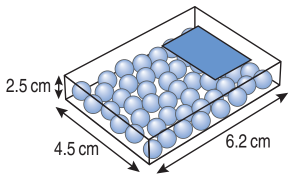
- Find the volume in cm^3 of this H-shaped girder by splitting it into cuboids.



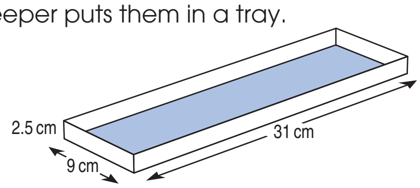
- Containers come in three lengths: 12 m, 9m and 6m.
Each is 2.3m wide and 2.3m tall.
How many crates measuring 1.1m by 1.1m by 2.9m will fit in each of the three containers?



- Boxes measure 2.5 cm by 4.5cm by 6.2cm.

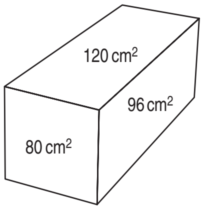


A shopkeeper puts them in a tray.

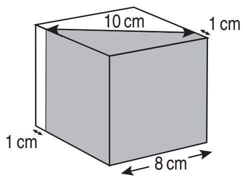


Work out the largest number of boxes that can lie flat in the tray.

- What is the total surface area of this box?
Find the length of each edge.



- This block of cheese is in the shape of a cube.
Each edge is 8 cm long.
It is cut into two pieces with one vertical cut.



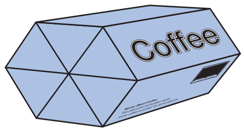
Calculate the volume and surface area of the shaded piece.

Link to lines, angles and shapes (pages 178–201), and problem solving (pages 18–19).

As outcomes, Year 9 pupils should, for example:

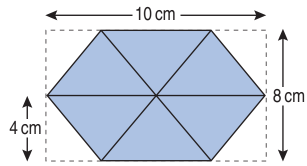
Solve problems such as:

- A box for coffee is in the shape of a hexagonal prism.



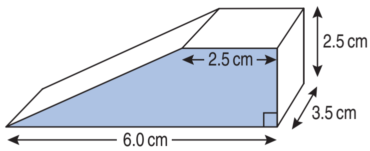
Each of the six triangles in the hexagon has the same dimensions.

Calculate the total area of the hexagon.



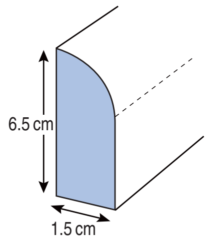
The box is 12 cm long.
After packing, the coffee fills 80% of the box.
How many grams of coffee are in the box?
(The mass of $1 cm^3$ of coffee is 0.5 grams.)

- This door wedge is in the shape of a prism.



The shaded face is a trapezium.
Calculate its area.
Calculate the volume of the door wedge.

- The cross-section of a skirting board is in the shape of a rectangle, with a quadrant (quarter circle) on top. The skirting board is 1.5 cm thick and 6.5cm high. Lengths totalling 120m are ordered. What volume of wood is contained in the order?



- Large wax candles are made in the shape of a cylinder of length 20 cm and diameter 8 cm. They are packed neatly into individual rectangular boxes in which they just fit.
What percentage of the space in each box will be occupied by air?
If the dimensions of the candle and its box are doubled, what effect does this have on the percentage of air space?

- A can in the shape of a cylinder is designed to have a volume of $1000 cm^3$. The amount of metal used is to be a minimum. What should the height of the can and radius of the top be?
Use a **spreadsheet** to help you.

Link to lines, angles and shapes (pages 178–201), and problem solving (pages 18–19).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Begin to use sine, cosine and tangent to solve problems	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

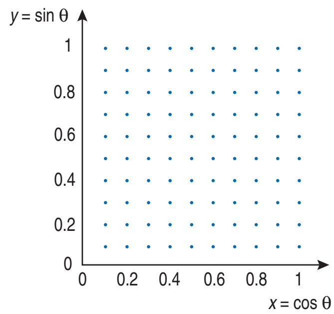
Use, read and write, spelling correctly:
sine (sin), cosine (cos), tangent (tan)…
opposite, adjacent, hypotenuse…
angle of elevation, angle of depression…

Begin to use sine, cosine and tangent to solve problems involving right-angled triangles in two dimensions. For example:

- Use the SIN (sine) key and the COS (cosine) key on a **calculator** to complete this table. Round each value to two decimal places.

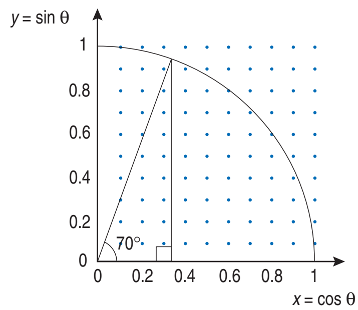
θ	cos θ	sin θ
0°		
10°		
20°		
30°		
40°		
50°		
60°		
70°		
80°		
90°		

  Plot points on a graph, using cos θ for the x-coordinate and sin θ for the y-coordinate.

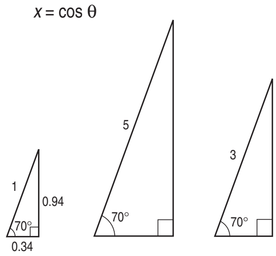


  What do you notice about the graph and table?

- Draw a right-angled triangle with an angle of 70° on the grid. The hypotenuse is of length 1. The lengths of the other two sides are given by cos 70° and sin 70° (or 0.34 and 0.94 respectively, correct to two decimal places).



  Use this information to write the lengths of the sides of the other two triangles.



- Find the lengths of the sides of the triangles if the angle changes to 50°.

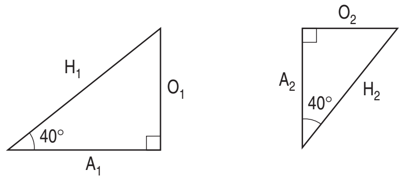
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Begin to use sine, cosine and tangent to solve problems (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Consider sine, cosine and tangent as ratios.
For example:

- Use a ruler and protractor to draw a variety of similar right-angled triangles, e.g. with an angle of 40°.



Measure the hypotenuse H, the side A adjacent to the known angle, and the side O opposite to the known angle.

Use a **spreadsheet** to explore the value of A/H.

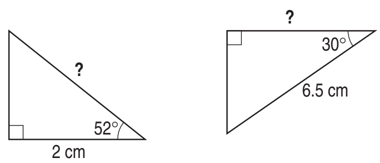
A (cm)	H (cm)	A/H
10.0	13.06	0.766
	4.5	0.766
6.0		0.766
	10.5	0.766
4.42		0.766

Conclude that for each triangle the approximate value of A/H is the same as the **cosine** of the angle, or cos 40°.

Similarly, know that the approximate value O/H is the **sine** of the angle, or sin40°, and that the approximate value O/A is the **tangent** of the angle, or tan40°.

For example:

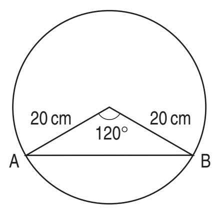
- Find the sides marked with a question mark in triangles such as:



Solve problems such as:

- A girl is flying a kite. The string is 30 m long and is at an angle of 42° to the horizontal.
  How high is the kite above the girl's hand?

- A circle has a radius of 20 cm.
  Calculate the length of the chord AB.

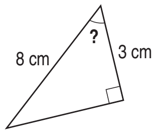


Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Begin to use sine, cosine and tangent to solve problems (continued)	

As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

- Use a **calculator** to find the value of an angle of a right-angled triangle, given two sides.



  For example:

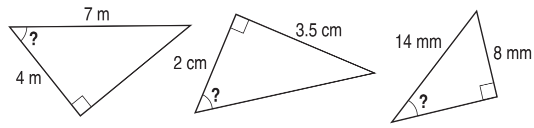
  The adjacent side is 3 cm and the hypotenuse is 8cm. On a **scientific calculator** press, for example:

  [3] [÷] [8] [=] [INV] [COS] [=]

  On a **graphical calculator** press, for example:

  [SHIFT] [COS] [(] [3] [÷] [8] [)] [EXE]

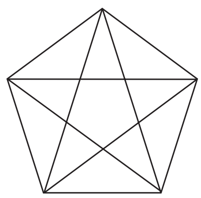
- Find the angle marked with a question mark in triangles such as:



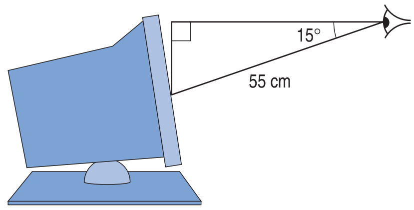
Solve problems such as:

- A boy walks 6 km west and then 8km north. How far is he now from his starting point? The boy wants to get back to his starting point by the shortest route. On what bearing should he walk?

- The sides of a regular pentagon are 12cm long. Find the length of a diagonal.



- The most comfortable viewing distance from the eye direct to the centre of a computer screen is 55 cm, looking down from the horizontal at 15°.



  a. What height should your eyes be above the centre of the screen?
  b. How far away (horizontally) should you sit from the screen?

- What angle does the line y = 2x – 1 make with the x-axis?

- What is the angle of slope of a 1 in 5 (20%) hill?

Link to Pythagoras' theorem (pages 186–9), similarity (pages 192–3), and gradient (pages 166–9).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Discuss a problem that can be addressed by statistical methods, and identify related questions to explore	Use, read and write, spelling correctly: survey, questionnaire, experiment, data, statistics… grouped data, class interval… tally, table, frequency, data collection sheet… database… **Given a problem that can be addressed by statistical methods, suggest possible answers,** in mathematics or other subjects. For example: • Problem What method of transport do pupils use to travel to school, and why? Possible answers Most pupils catch a bus because it's quicker. Few pupils cycle to school because of the busy roads. Pupils who walk to school have less distance to travel. A bus journey is quicker than walking the same distance. Some pupils must leave home before 7:30 a.m. • Problem Do different types of newspaper use words (or sentences) of different lengths? If so, why? Possible answers Tabloid newspapers use shorter words (or sentences) so that they are easier to read and so appeal to a wider audience. There is not likely to be much difference in the use of two- and three-letter words. There may be more difference in sentence length between newspapers of different types than in word length. • Cross-curricular problem with geography How will the population of a typical MEDC (more economically developed country) change over the next 50 years as compared with an LEDC (less economically developed country)? Possible answers The large number of younger people in the LEDC will lead to an explosion in the population in the future. The smaller number of younger people in the MEDC may lead to a population decline. Future changes may be difficult to predict because improvements in factors such as health care and nutrition, for example, are unknown.

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
sample… primary source, secondary source,
data log… two-way table… discrete, continuous…

Discuss a problem that can be solved by statistical methods; identify related questions to explore, in mathematics or other subjects. For example:

- Problem
  At what time during a football match is there most likely to be a goal?
  Related questions
  Where could you find the necessary data?
  Are there differences between football divisions?
  When is the best time to buy a snack if you don't want to miss a goal or to queue at half-time?
  What is the likelihood of missing a goal if you leave 10 minutes early?

- Problem
  A neighbour tells you that the local bus service is not as good as it used to be.
  How could you find out if this is true?
  Related questions
  How can 'good' be defined? Frequency of service, cost of journey, time taken, factors relating to comfort, access…?
  How does the frequency of the bus service vary throughout the day/week?

- Problem
  How much TV do pupils and adults watch?
  Related questions
  What factors affect TV viewing habits? Hours of work per week, hours of sleep per week, weekly travelling distance…?

- Cross-curricular problem with geography
  How do modes of transport to an out-of-town shopping centre compare with those to a town centre?
  Related questions
  Are there variations at different times of the day/week? If so, are they linked to variations in the number of visitors at each location?

- Cross-curricular problem with science
  What are the factors affecting invertebrate communities in freshwater habitats?
  Related questions
  What data could be collected?
  What is the variation in light intensity at different depths of the water?

- Cross-curricular problem with science
  Why do penguins huddle together to keep warm?
  Related questions
  Can the process be modelled by comparing the cooling of a single warm test-tube with that of one surrounded by other similarly warm tubes?

As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
raw data… representative, bias…
census…

Suggest a problem to explore using statistical methods, frame questions and raise conjectures, in mathematics or other subjects. For example:

- Cross-curricular problem with physical education
  How far can people jump from a standing start?
  To what extent does a run-up help?
  Does practice improve the distance?
  Are Year 9 pupils able to jump or throw further than Year 7 pupils of the same height?
  Conjectures
  Your height and the length of your run-up are likely to affect how far you can jump.
  A moderate run-up is useful, but the effect will diminish after a certain point.

- Cross-curricular problem with science
  What effect does engine size have on the acceleration of a car?
  Conjecture
  In general more powerful engines produce the greatest acceleration.

- Cross-curricular problem with science
  What factors affect the distribution of grass and non-grass plants on the school field?
  Conjecture
  The direction that the field faces in relation to the school building will affect the distribution.

- Cross-curricular problem with geography
  Are development indicators, such as GNP, and measures of development, such as educational attainment, telephones per 1000 people, energy consumption per capita, life expectancy… consistent with each other?
  Which are the most closely connected?
  Conjecture
  The measure of development that will show the greatest contrast between MEDCs (more economically developed countries) and LEDCs (less economically developed countries) is energy consumption per capita.

- Cross-curricular problem with PSHE
  How available are fairly-traded goods in local shops? What sort of organisations promote these goods and why?
  Conjecture
  People with experience of or links with LEDCs are more likely to be aware of and to buy fairly-traded goods.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Decide which data to collect and identify possible sources

Decide which data would be relevant to the enquiry and possible sources.

Relevant data might be obtained from:

- a survey of a sample of people;
- an experiment involving observation, counting or measuring;
- secondary sources such as tables, charts or graphs, from reference books, newspapers, **websites**, **CD-ROMs** and so on.

For example:

- How do pupils travel to school?
  Data needed for each individual pupil:
  the method of travel, why that method is used, how long the journey takes, the distance to school.

- Do different types of newspaper use words (or sentences) of different lengths?
  Data needed from each newspaper:
  a count of an agreed number of words (or sentences) from each paper.

- How does the population vary from one country to another?
  Data needed for each country:
  population figures, e.g. from books, **websites** or **CD-ROMs**.

- Can taller people hold their breath longer than shorter people?
  Data needed for each person:
  height, time that they can hold their breath.

Determine the sample size and type, e.g. who to ask, how many to ask, where and when the sample should be taken.

Decide what units to use for measurements such as pupils' heights, distances travelled, times of journeys…

As outcomes, Year 8 pupils should, for example:

Decide which data to collect to answer a question, and the degree of accuracy needed; identify possible sources.

Relevant data might be obtained from:

- a questionnaire or survey of a sample of people;
- an experiment involving the use of hand-held technology such as **data-loggers** with **graphic calculators** or **computers**;
- secondary sources, such as reference materials, including **websites**, **CD-ROMs**, newspapers, directories, historical records…

For example:

- Plan a questionnaire to find out how often and how people travel to shopping centres, or what their TV viewing habits are.
- Plan an experiment using hand-held data-logging equipment to measure light intensity in different parts of a stream, or to measure cooling rates.
- Plan how to research sports results on the **Internet**, including what to look for and what to record.

Recognise that data from primary sources may take more time and resources to collect than from secondary sources but may give more insight and address more precisely the problem being explored.

Determine the sample size and type, e.g. who and how many to ask, how, where and when the sample should be taken. Recognise that too small a sample may give unrepresentative results, while too large a sample may be expensive in resources and time.

As outcomes, Year 9 pupils should, for example:

Discuss how data relate to the enquiry and identify possible sources, including primary and secondary sources.

Relevant data might be obtained from:

- a questionnaire or survey of a sample of people;
- printed tables and lists;
- the **Internet**;
- other **computer databases**.

For example:

- Plan how to conduct a survey into long jumps or throws with different lengths of run-up.
- Identify magazines and books with information on engine sizes of cars and acceleration times for 0–60 mph.
- Determine a range of countries with different sizes of population, development and income. Search the **Internet**, **CD-ROMs** or printed sources of information for relevant information.
- Visit the library to access census data for the local area, relating to a study of housing.
- Construct a questionnaire to explore attitudes to fairly-traded goods and a survey for shops.

Identify possible sources of bias and plan how to minimise it. For example:

- When investigating pupils' aptitudes in PE activities, aim to reduce possible bias due to selection, non-response and timing of the enquiry, by:
  a. choosing pupils from a range of year groups and with a range of heights;
  b. making sure that there are equal numbers of girls and boys;
  c. choosing pupils from the full range of athletic prowess.
- When designing a questionnaire for a survey:
  a. ensure the sample is representative by choosing it randomly and/or selecting people from particular categories;
  b. phrase questions in a neutral manner so that they do not bias results by encouraging a particular response.
- When conducting an experiment, vary one factor at a time, keeping other factors constant.

Pupils should be taught to:

Plan how to collect and organise the data and design suitable data collection sheets and tables

As outcomes, Year 7 pupils should, for example:

Decide how to collect and organise the data needed; design a data collection sheet or questionnaire to use in a simple survey. For example:

- Survey of ways of travelling to school

Name	Class	Age	Form of travel	Distance (m)	Depart time	Arrival time	Duration

- Survey of lengths of words (or sentences) in newspapers
  Decide:
  sample size, where in the newspaper to collect the words, what to do with data such as numbers, hyphenated words, abbreviations and other exceptional data.

- Investigation of populations of different countries
  In geography, design a table to collect population data.

Construct frequency tables for sets of data, grouped where appropriate in equal class intervals. Know that the final or initial interval may be open, e.g. for ages 'over 80'.
For example:

- How do pupils travel to school?
  Intervals of 5 minutes are likely to be more useful than intervals of 1 minute or 1 hour.
  Discuss where to put a journey time such as 15 minutes.

Journey time in minutes	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35+
No. of pupils								

- How easy are newspapers to read?
  Take the first 100 words from a front-page story of a newspaper. Record the number of letters in each word.
  Record the data in intervals of 4 letters.

No. of letters (n)	1–4	5–8	9–12	13–16	17–20
Express					
Mail					
Telegraph					

  Record the data again in intervals of 3 letters.
  Which are more useful: intervals of 3 letters or intervals of 4 letters? Why?

- Which is your better catching hand?
  Place 10 centimetre cubes on the back of your writing hand. Toss them gently upwards, turn your hand round quickly, and catch as many as you can.
  Repeat for the non-writing hand.
  Record the results of the whole class in a frequency table.

Cubes caught (c)	0	1	2	3	4	5	6	7	8	9	10
Writing hand											
Non-writing hand											

As outcomes, Year 8 pupils should, for example:

Decide how to collect the data (e.g. a survey or experiment), including sample size. Plan a simple survey or experiment, and design a suitable data collection sheet. For example:

- Survey of the price of second-hand cars

Vendor	Price band <£2000	£2000–£3999	£4000–£5999	£6000–£7999	£8000–£9999	£10 000+
Private						
Dealer						

- Comparison of invertebrate communities in two contrasting sections of a stream

Invertebrate indicator animals	No. at site A	No. at site B
Bloodworm		
Caddis fly larva		
Freshwater shrimp		
Mayfly nymph		
Rat tailed maggot		
Sludge worm		
Stonefly nymph		
Water louse		

Construct frequency tables for sets of continuous data, with given equal class intervals. Know that class intervals should be continuous with no gaps or overlaps; the last group may be open. For example:

- Timing of goals scored in Premier League matches on one Saturday
  Discuss how and where to record a goal scored after, say, 60 minutes, emphasising that the choice must be consistent for each similar occurrence.

Time (minutes)	Frequency
$0 < T \leq 15$	4
$15 < T \leq 30$	5
$30 < T \leq 45$	6
$45 < T \leq 60$	1
$60 < T \leq 75$	4
$75 < T \leq 90$	7

As outcomes, Year 9 pupils should, for example:

Design a survey or experiment to capture relevant data from one or more sources; determine sample size and degree of accuracy needed; design, trial and if necessary refine data collection sheets. For example:

- Investigation of jumping or throwing distances
  Check that the data collection sheet is designed to record all factors that may have a bearing on the distance jumped or thrown, such as age or height. Decide the degree of accuracy needed for each factor. Recognise that collecting too much information will slow down the experiment; too little may limit its scope.

- Survey of acceleration data for popular cars
  Check that the published data contain what is expected. Round published engine sizes to the nearest 0.1 litre to eliminate unnecessary accuracy in data such as 1428 cc and 1964 cc.

- Study of the distribution of grass
  Use a quadrat or points frame to estimate the numbers of grass and non-grass plants growing in equal areas at regular intervals from a north-facing building. Repeat next to a south-facing building. Increase accuracy by taking two or more independent measurements.

- Questionnaire on attitudes to fairly-traded goods
  Test questions on a small sample before refining them for a larger sample.

Construct tables for large sets of raw data, discrete and continuous, choosing suitable class intervals. Know that to group data loses information but grouping is necessary to ensure that large data sets are manageable. For example:

- Group long jump distances initially in intervals of 5 cm or 10 cm, recognising that the number of groups created may be too few for further analysis, or too many to reveal key features.

When organising large data sets, find suitable class intervals to fit the full range of the data. For example:

- Population distribution data for the UK 1999

Ages	Population
0–9	3 890 782
10–19	4 088 469
20–29	4 172 971
30–39	5 042 082
40–49	4 818 389
50–59	3 454 277
60–69	2 374 917
70–79	1 777 692
80–89	744 780
90+	126 935

Ages	Population
0–14	5 917 365
15–29	6 234 857
30–44	7 608 956
45–59	5 705 792
60–74	3 364 102
75–89	1 533 287
90+	126 935

Ages	Population
0–19	7 979 251
20–39	9 215 053
40–59	8 272 666
60–79	4 152 609
80+	871 715

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Plan how to collect and organise the data and design suitable data collection sheets and tables (continued)	
Collect and record data from primary and secondary sources	Collect and record small sets of data as planned from surveys and experiments or secondary sources. For plans, see pages 248–53.

As outcomes, Year 8 pupils should, for example:

Design and use simple two-way tables. For example, design, read and compare the cells in two-way tables such as:

- Method of transport to different shopping centres

	Car	Bus	Train	Walk	Coach	Other
Local centre						
Town centre						
Out-of-town development						

- Method of transport and distance to school

Distance (km)	Car	Bus	Train	Walk	Coach	Other
0–0.5						
0.5–1						
1–1.5						
1.5–2						
2–2.5						
2.5–3						
3+						

Collect data as planned from secondary sources or by carrying out a survey or experiment, involving observation, data-logging using ICT, or questionnaire.

For plans, see pages 248–53.

As outcomes, Year 9 pupils should, for example:

Design and use two-way tables. For example:

- Use a two-way table to highlight the difference between male and female smoking patterns in the UK in different age groups, and any trends visible over time.

1996

Age	Male %	Female %
16–19	25	32
20–24	43	37
25–34	38	33
35–49	30	30
50–59	27	26
60+	17	18

1986

Age	Male %	Female %
16–19	30	31
20–24	41	38
25–34	37	35
35–49	37	33
50–59	34	34
60+	28	22

1976

Age	Male %	Female %
16–19	38	35
20–24	48	44
25–34	48	41
35–49	49	44
50–59	49	45
60+	40	42

UK smoking and age distribution 1976–1996

- The cost of an old Barbie doll depends on both its condition and whether or not it is in the original box. The table shows what percentage of the original cost the secondhand value retains.

Condition	Boxed	Not boxed
Excellent	100%	65%
Good	80%	32%
Poor	50%	15%

By considering the percentage lost by not having its original box, examine the claim that the importance of the box increases with the doll's condition.

Gather data as planned from specified secondary sources, including printed tables and lists from ICT-based sources. Identify what extra information may be required to pursue a further line of enquiry.

For example:

- As part of a study in science of car engine sizes, discuss whether the published value for bhp or 'torque' may be a better measure of 'power' than engine size.
- When designing a questionnaire about fairly-traded goods, include some questions on social or moral attitudes for later analysis.
- As part of a study of development indicators, discuss whether:
  - features relating to development are particular to the countries selected or are representative of wider trends;
  - some measures appear not to vary as much as others, and may be less useful as indicators of development.

Respond to problems of unavailable data, or data that relate to different dates, or that are organised in different ways (e.g. some appearing in tabular form, some in graphical form, some in summarised form), for example, by estimating the required comparable value.

Pupils should be taught to:

Calculate statistics from data, using ICT as appropriate

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
statistic, interval…
range, mean, median, mode, modal class/group, average…

Know that:
- The mode is the only statistic appropriate for data based on non-numeric categories, e.g. the most common way of travelling to school.
- The mean is often referred to as 'the average'.

Find the mode of a small set of discrete data.

Know that the mode of a set of numbers is the number that occurs most often in the set. For example:

- For 1, 2, 3, 3, 4, 6, 9, the mode is 3.
  For 3, 4, 4, 4, 7, 7, 8, the mode is 4.
  For 2, 2, 3, 5, 6, 9, 9, there are two modes, 2 and 9.

In a grouped frequency distribution, the group that contains the most members is called the modal class or modal group.

Calculate the mean for a small set of discrete data, using a **calculator** for a larger number of items.

The mean of a set of numbers is the sum of all the numbers divided by the number of numbers in the set. For example:

- The mean of 2, 6, 8, 9 and 12 is:

$$\frac{2 + 6 + 8 + 9 + 12}{5} = \frac{37}{5} = 7.4$$

- For this data set for 100 words in a newspaper passage:

No. of letters:	1	2	3	4	5	6	7	8	9	10	11	12	13
No. of words:	5	15	31	12	7	6	14	5	3	0	2	0	0
Total letters:	5	30	93	48	35	36	98	40	27	0	22	0	0

the mean number of letters in a word is:

$$\frac{5 + 30 + 93 + 48 + 35 + 36 + 98 + 40 + 27 + 22}{100} = \frac{434}{100} = 4.34$$

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
distribution…
stem-and-leaf diagram…

Know when it is appropriate to use the mode (or modal class), mean, median and range:

- The median is useful for comparing with a middle value, e.g. half the class swam more than 500 m.
- The range gives a simple measure of spread.
- The mode indicates the item or class that occurs most often and is useful in reporting opinion polls.
- The mean gives an idea of what would happen if there were 'equal shares'.

Find the modal class of a set of continuous data, i.e. the group with the most members. For example:

- London marathon times: top 100 women

Time (hours:minutes)	Frequency
2:20 ≤ T < 2:25	0
2:25 ≤ T < 2:30	7
2:30 ≤ T < 2:35	3
2:35 ≤ T < 2:40	5
2:40 ≤ T < 2:45	8
2:45 ≤ T < 2:50	4
2:50 ≤ T < 2:55	12
2:55 ≤ T < 3:00	10
3:00 ≤ T < 3:05	33
3:05 ≤ T < 3:10	18

Source: www.london-marathon.co.uk

The modal class is a marathon time, T hours:minutes, of 3:00 ≤ T < 3:05.

Calculate the mean for a large set of data, using a **calculator** or **spreadsheet**. For example:

- Calculate the mean score thrown by a dice.

	A	B	C	D	E	F	G	H
1	Score	1	2	3	4	5	6	
2	No. of throws	26	30	28	32	31	29	=SUM(B2:G2)
3	Total	=B1*B2	=C1*C2	=D1*D2	=E1*E2	=F1*F2	=G1*G2	=SUM(B3:G3)/H2

	A	B	C	D	E	F	G	H
1	Score	1	2	3	4	5	6	
2	No. of throws	26	30	28	32	31	29	176
3	Total	26	60	84	128	155	174	3.56

The mean score for 176 throws is 3.56 (to 2 d.p.).

Calculate a mean using an assumed mean.
For example:

- Find the mean of 28.7, 28.4, 29.1, 28.3 and 29.5.

  Use 29.0 as the assumed mean.
  The differences are ⁻0.3, ⁻0.6, 0.1, ⁻0.7 and 0.5, giving a total difference of ⁻1.0.
  The actual mean is 29.0 – (1.0 ÷ 5) = 28.8.

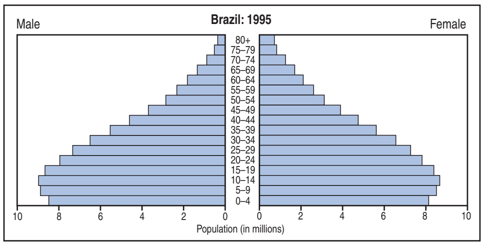
As outcomes, Year 9 pupils should, for example:

Use vocabulary from previous years and extend to:
raw data, estimate of the mean/median, cumulative frequency…

Select statistics most appropriate to the problem.
Decide which statistics are most suitable in a particular case, choosing between the median and the mean partly on the basis of whether extreme or chance values will influence the measure unduly. Be aware that the difference will be most significant in skew distributions, where both may need to be quoted.

Find the modal class of a large set of data.
For example:

- Use a population pyramid to find that there are more teenagers in Brazil than other age groups.



Source: US Census Bureau, International Data Base

Recognise that if the Brazilian population were grouped in 15-year intervals, data would be easier to plot and may show general trends just as clearly.

Calculate an estimate of the mean of a large set of grouped data to a suitable degree of accuracy.

Choose suitable mid-points for class intervals, justifying decisions, e.g. that a suitable mid-interval of the range 10–19 years is 15 years, and of an open interval such as '80+ years' is 90 years. For example:

- Using the data in the table, estimate the mean time spent on homework.

Time spent on homework (minutes)	Frequency
0 ≤ time ≤ 30	6
30 < time ≤ 60	14
60 < time ≤ 90	21
90 < time ≤ 120	9
Total	50

The mean time is approximately:

$$\frac{(15 \times 6) + (45 \times 14) + (75 \times 21) + (105 \times 9)}{50} = 64.8 \text{ min}$$

- Estimate the mean age of a head of household in Brazil from this table, using a **spreadsheet** or the statistical facilities on a **calculator**.

Age group	10–19	20–29	30–39	40–49	50–59	60–69	70–79	80+	Totals
Mid-range (x)	15	25	35	45	55	65	75	90	
Frequency (f)	232 813	5 395 646	8 080 436	6 185 652	4 192 554	2 766 020	1 255 301	335 960	28 444 382
$f \times x$	3 492 195	134 891 150	282 815 260	278 354 340	230 590 470	179 791 300	94 147 575	30 236 400	1 234 318 690

The mean age is approximately:
1 234318690 ÷ 28444382 = 43.3941, or 43.4 years

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Calculate statistics from data, using ICT as appropriate, finding the mode, mean, median and range (continued)

Find and use the range of a small set of discrete data.

The range of a set of values is the difference between the largest and smallest numbers in the set. For example, for 2, 3, 4, 7, 9, 10, 12, 15, the range is 15 – 2 = 13.

Find the median of a small set of discrete data.

The median of a set of numbers is the value of the middle number when they are arranged in ascending order. For example, 2, 5, 8, 3, 1, 7, 6 becomes 1, 2, 3, 5, 6, 7, 8, and the median is 5.

If there is no single middle number, the mean of the two middle numbers is taken. For example, the set 1, 5, 7, 8, 9, 10 has a median of (7 + 8)/2 = 7.5.

As outcomes, Year 8 pupils should, for example:

Find the range of a set of continuous data, calculating this as the highest rounded-off figure minus the lowest rounded-off figure. For example:

- Calculate the range of temperatures recorded at a weather station over a 24-hour period. Compare with the range of data from weather stations at different sites.

Range of temperatures at Snowdon on 10 May 2000

	Min.	Max.	Range
Summit	6.8 °C	13.8 °C	7 degrees
Llanberis	9.3 °C	15.5 °C	6.2 degrees
Clogwyn Station	9.3 °C	22.0 °C	12.7 degrees

Source: Snowdonia Weather Stations Project

Know that it can be helpful to state the range of a set of data as well as the mean, mode or median.

Find the median of a large set of data.
For example, find the median of:

- the time taken to run the London marathon;
- marks in a test taken by Year 8 pupils;
- the cost of a particular chocolate bar from various retailers.

Use a stem-and-leaf diagram to help find the median, range and mode. For example:

- Hours of sunshine for UK weather stations 10/05/00

Hours of sunshine for UK weather stations: 10/05/00

(stem = hours, leaves = tenths)

0	6	9													
1	6	9													
2	2	2	5	6	6	7	9								
3	0	0	0	0	1	2	2	5	5	5	7	8	9		
4	0	1	5	5	6	6	6	7	9						
5	0	1	5	5	6	8	9	9							
6	1	2	2	2	3	6	6	7	8	8	9	9	9	9	9
7	0	0	1	6	7	8	8								
8	0	0													

There are 65 items of data.
The median is the 33rd item, 4.9 hours (4 hours and 54 minutes) of sunshine.
The range is 8.0 – 0.6 = 7.4 hours.
The mode is 6.9 hours.

As outcomes, Year 9 pupils should, for example:

Estimate the range of a large set of grouped data.
For example:

- Estimate the range of the distances jumped by 67 pupils, from the data in this table.

Distance jumped (cm)	No. of pupils
$180 \le d < 190$	2
$190 \le d < 200$	6
$200 \le d < 210$	9
$210 \le d < 220$	7
$220 \le d < 230$	15
$230 \le d < 240$	18
$240 \le d < 250$	8
$250 \le d < 260$	2

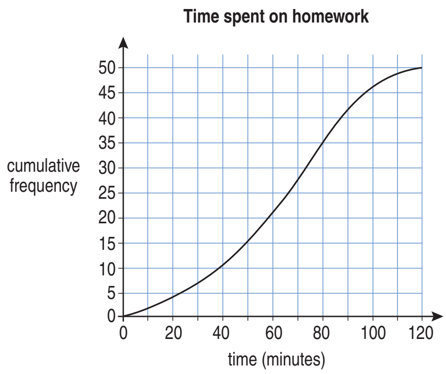
The first class is 180–189 cm, then 190–199cm, and so on. The last class is 250–259cm.
An estimate of the range is calculated by using the lowest class value in the first class (or 179.5cm) and the highest class value in the last class (or 259.5cm), giving an estimate of the range to be 259.5 – 179.5 = 80cm.

Estimate the median and interquatile range of a large set of grouped data, where the original data are not available. For example:

- Estimate the median distance jumped by 67 pupils, from the data in the table above.

  There are 67 pupils; the middle pupil is the 34th. The median must lie in the interval $220 \le d < 230$, representing 15 pupils, from the 25th to 39th pupils. The 34th pupil is estimated to be 10/15 of the way along the interval of 10 cm, so an estimate of the median is 227cm to the nearest centimetre.

- Estimate the median and quartiles from a cumulative frequency diagram, e.g. by reading data for the 25th pupil from a graph showing the time that 50 pupils spent on homework.



Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Calculate statistics from data, using ICT as appropriate, finding the mode, mean, median and range (continued)

Calculate statistics. For example:

- A competition has three different games.
  Jane has played two of the games.

	Game A	Game B	Game C
Score	62	53	

  To win, Jane needs a mean score of 60.
  How many points does she need to score in game C?

- Phil has these four cards.
  The mean is 4.

  1 8 5 2

  Phil takes another card. The mean of the five cards is still 4.
  What number is on his new card?

  1 8 5 2 ?

- Rajshree has six cards.

  10 10 10 10 ? ?

  The six cards have a mean of 10 and a range of 6.
  What are the numbers on the other two cards?

- I can catch either a Direct bus or a Transit bus to go home.
  For my last five journeys on each bus, this is how long I had to wait:

Direct bus	10 min	8 min	5 min	9 min	8 min
Transit bus	16 min	1 min	2 min	15 min	1 min

  Calculate the mean of the waiting time for each bus.
  Decide which bus it would be more sensible to catch.
  Explain why.

- Five careful measurements were made to find the mass of a nugget of gold. The five measurements were:
  2.003 2.012 1.998 2.000 1.989 ounces.
  Find the mean of the five measurements.

See Y456 examples (pages 116–17).

As outcomes, Year 8 pupils should, for example:

Calculate statistics. For example:

- Imran and Nia play three games.
  Their scores have the same mean.
  The range of Imran's scores is twice the range of Nia's scores.
  Write the missing scores in the table below.

Imran's score		40	
Nia's score	35	40	45

- John has three darts scores with a mean of 30 and a range of 20.
  His first dart scored 26.
  What were his other two scores?

- Collect data from weather stations over a 24-hour period.

Wind speed (mph) Snowdon 10/05/00

Location/time	Summit (1085 m)	Clogwyn (770 m)	Llanberis (105 m)
00:00	1	6	2
01:00	2	8	3
02:00	3	7	8
03:00	3	8	9
04:00	5	6	11
05:00	3	7	9
06:00	5	5	21
07:00	8	10	15
08:00	6	12	11
09:00	3	9	10
10:00	3	4	10
11:00	3	5	12
12:00	8	3	18
13:00	8	2	17
14:00	10	2	17
15:00	10	2	24
16:00	12	5	30
17:00	15	9	30
18:00	17	11	33
19:00	20	12	38
20:00	27	12	41
21:00	35	14	47
22:00	36	14	57
23:00	34	13	45

Source: Snowdonia Weather Stations Project

Calculate the mean and median wind speeds and the range.

	Summit	Clogwyn	Llanberis
Mean	11.54	7.75	21.58
Median	8	7.5	17
Range	35	12	55

Which place had the least reliable weather?

As outcomes, Year 9 pupils should, for example:

Calculate statistics. For example:

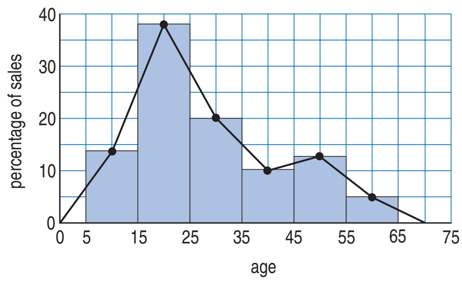
- Three people have a median age of 30 and a mean age of 36. The range of their ages is 20.
  How old is each person?

- Three children have a mean age of 10.
  The range of their ages is 6.
  What is the lowest possible age:
  a. of the youngest child?
  b. of the oldest child?

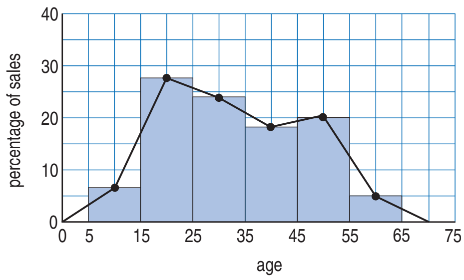
- Amrita has five cards numbered in the range 0 to 20. She says:
  'The range of my cards is 4, the mode is 6 and the mean is 5.'
  Is this possible?

- Look at these two frequency diagrams.

Purchasing of singles by age group



Purchasing of albums by age group



a. Estimate the mean age of people buying singles.
b. Estimate the median age of people buying singles.
c. Estimate the mean age of people buying albums.
d. Estimate the median age of people buying albums.

What conclusions can you draw from your answers?

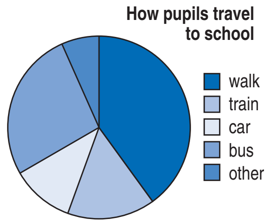
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Construct graphs and diagrams to represent data, on paper and using ICT

Use, read and write, spelling correctly:
frequency diagram, bar chart, bar-line graph, pie chart…

Construct graphs and diagrams to represent data, on paper and using ICT, and identify key features. For example:

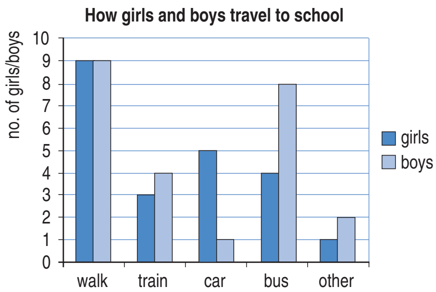
- **Pie charts** generated by **ICT**, for example:



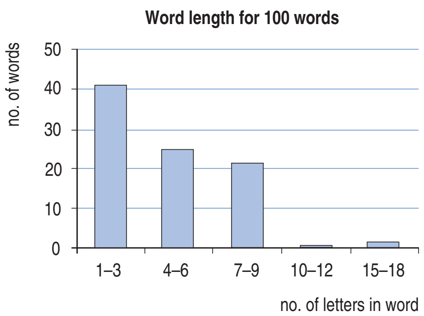
Know that the sizes of sectors of the chart represent the proportions in each category.

Link to percentages (pages 70–7).

- **Bar charts** for categorical data, for example:



- **Bar charts** for grouped discrete data, for example:



Choose suitable class intervals.
Know that the bars may be labelled with the range that they represent, but not the divisions between the bars.

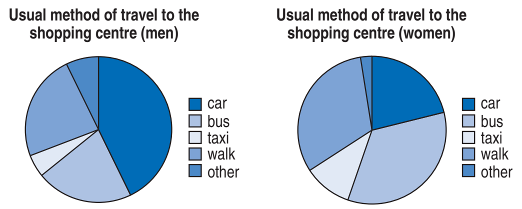
Know the conventions for marking the axes when the scale does not start from zero (see **page 172**).

As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to: population pyramid, scatter graph, distance–time graph, line graph…

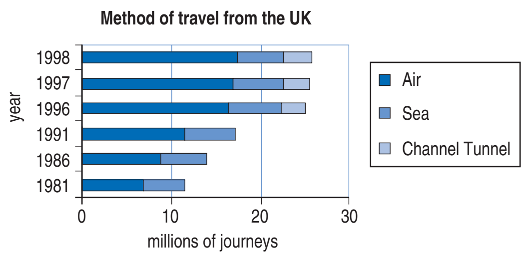
Construct graphs and diagrams to represent data, on paper and using ICT, and identify key features.

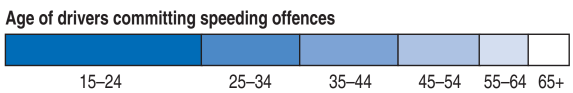
- **Pie charts:** Understand that pie charts are mainly suitable for categorical data. Draw pie charts using **ICT** and by hand, usually using a **calculator** to find the angles. For example, draw these graphs to compare shopping travel habits.



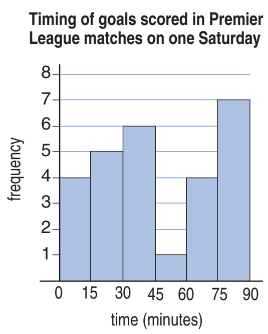
Link to percentages (pages 70–7).

- **Bar charts:** Compound bar charts allow both overall trends and changes in subcategories to be shown, for example:





- **Frequency diagrams** for a continuous variable, for example:



Choose suitable class intervals.
The bars in this graph represent intervals of $0 \leq t < 15$ minutes, $15 \leq t < 30$ minutes, etc.

Know that for continuous data the divisions between the bars should be labelled.

As outcomes, Year 9 pupils should, for example:

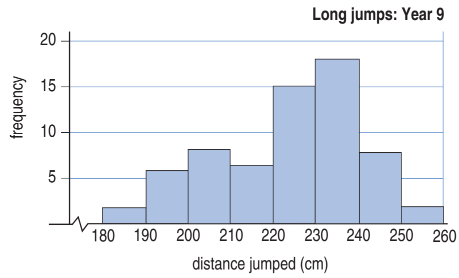
Use vocabulary from previous years and extend to: line of best fit, cumulative frequency graph…

Construct graphs and diagrams to represent data, on paper and using ICT, and identify key features.

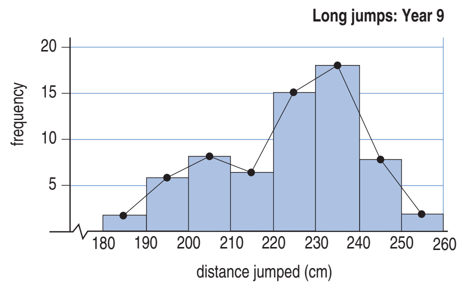
Appreciate that:

- A table usually gives all the data that can be retrieved.
- A graph, chart or diagram representing the data highlights particular features that a table does not.
- Data shown in a graph, chart or diagram are often in an aggregated form that does not allow the original data to be extracted.
- Calculated statistics are representative values of data sets.

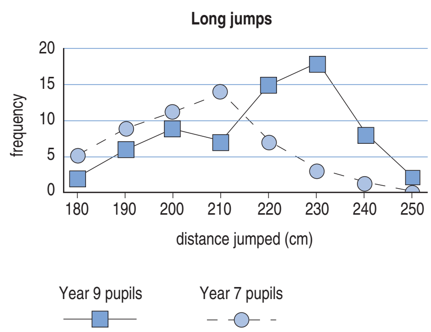
- **Frequency diagrams and polygons,** e.g. in this graph bars represent intervals of $180 \leq d < 190$, etc.



Use frequency polygons, for example:



Use superimposed frequency polygons rather than bar charts to compare results, for example the distances jumped by pupils in Year 7 and pupils in Year 9.

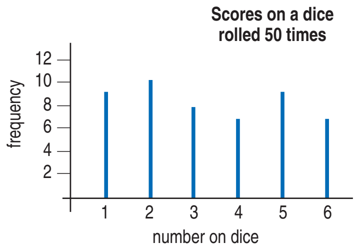


Pupils should be taught to:

Construct graphs and diagrams to represent data, on paper and using ICT (continued)

As outcomes, Year 7 pupils should, for example:

- **Bar-line graphs** for a discrete variable, for example:



Know that:

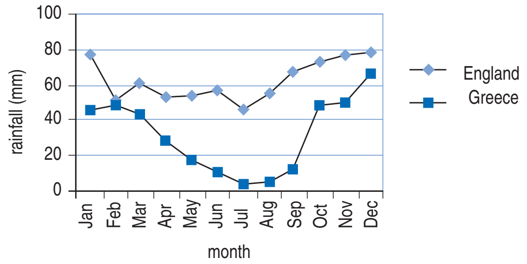
- The length of the bar represents the frequency.
- What is being counted or measured (the independent variable) is placed on the horizontal axis, and the count or measure (the dependent variable) on the vertical axis.
- It is not appropriate to join the tops of the bars.

See Y456 examples (pages 114–17).

As outcomes, Year 8 pupils should, for example:

- **Line graphs** comparing two sets of data, for example:

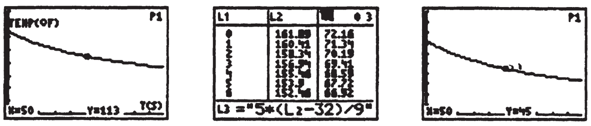
Mean monthly rainfall in two European countries



Know that it can be appropriate to join the points on the graph in order to compare trends over time.

- **Line graphs** comparing continuous data, for example:

Use a **temperature probe** and **graphical calculator** to compare cooling rates, e.g. to model the problem 'Why do penguins huddle together to keep warm?'



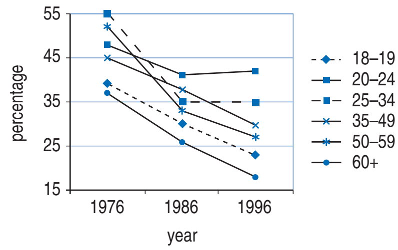
Understand that every point on the cooling curve has a meaning.

Link to graphs of functions, including distance–time graphs (pages 172–7).

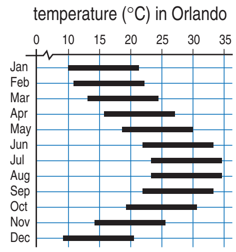
As outcomes, Year 9 pupils should, for example:

- **Line graphs** comparing several sets of data, for example:

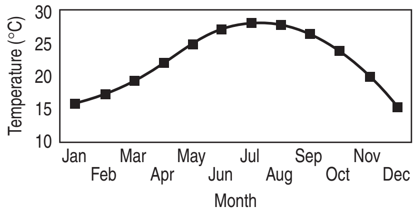
Percentage of each age group smoking
UK 1976, 1986, 1996



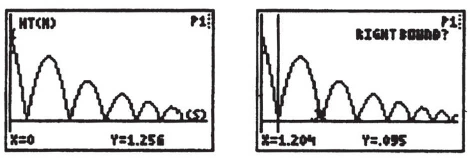
Maximum and minimum temperatures in Orlando



Mean temperatures in Orlando (°C)



Distance–time graph for a bouncing ball recorded on a **graphical calculator** using a **motion detector**

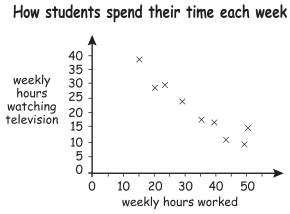


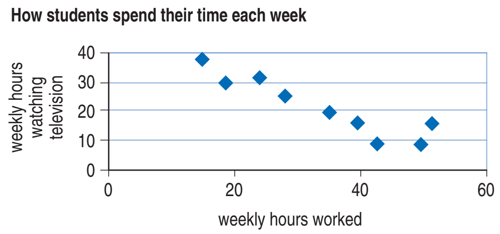
Link to graphs of functions, including distance–time graphs (pages 172–7).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Construct graphs and diagrams to represent data, on paper and using ICT (continued)	

As outcomes, Year 8 pupils should, for example:

- **Scatter graph** for continuous data, two variables, for example to show weekly hours worked against hours of TV watched (plotted by hand and using **ICT**).

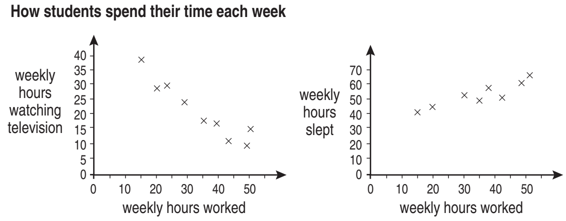




Link to two-way tables (page 254–5).

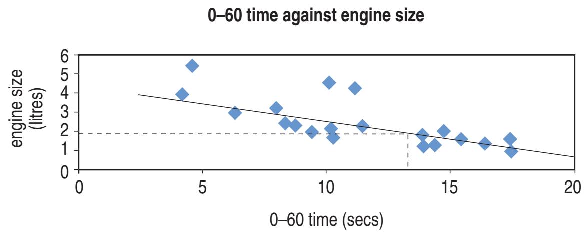
As outcomes, Year 9 pupils should, for example:

- **Scatter graphs**, for example:



Use the two scatter graphs above to suggest a relationship between the amount of TV a pupil watches and the number of hours he or she sleeps.

- **Draw a line of best fit,** by eye or using a **graphical calculator** or **spreadsheet**, e.g. engine size against 0–60 mph acceleration times.



Find the mean point through which the line should pass.

Predict a 0–60 mph time of about 13 seconds for a new car with a 1.8 litre engine.

- This table shows the heights jumped in a field test by different BMX bikes, each carrying the same rider.

Mass (kg)	Height (cm)
8.0	26.8
8.5	26.4
9.0	26.1
9.5	25.7
10.0	25.0
10.5	24.8
11.0	24.3

a. Draw a scatter graph to show the results.
b. Draw a line of best fit.
c. Estimate the height jumped by a bike weighing 9.7 kg.

Understand that:

- A prediction based on a line of best fit is an estimate and may be subject to substantial error.
- A line of best fit indicates an estimated relationship which may not mean anything in practice.

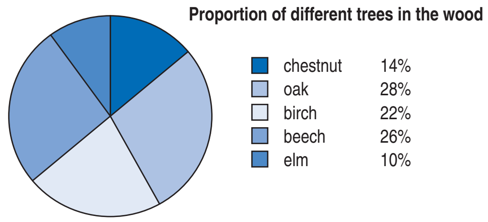
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Interpret diagrams and graphs, and draw inferences

Interpret diagrams, graphs and charts, and draw inferences based on the shape of graphs and simple statistics for a single distribution. Relate these to the initial problem. For example:

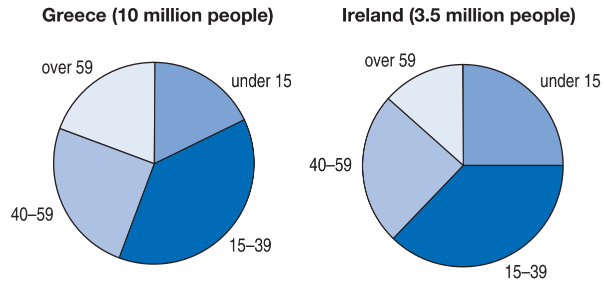
- Interpret data in a pie chart from a newspaper, or generated by a **computer**. For example:

  a. Which species of trees grow best in the local wood?



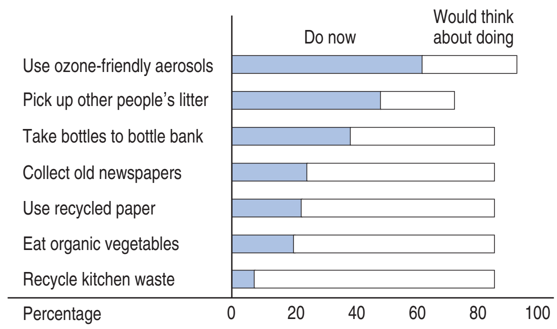
  How many of each species of tree would there be in the wood if it had 600 trees?
  Why do you think there are fewer elm trees in the wood than other species?

  b. These pie charts show some information about the ages of people in Greece and in Ireland. Roughly what percentage of people in Greece are aged 40–59?



  Dewi says: 'The charts show that there are more people under 15 in Ireland than in Greece.'
  Dewi is wrong. Explain why the charts do not show this.

- Interpret data in a simple compound bar chart.
  For example:
  In a survey people were asked about the things they did to help make the environment better. The bar chart below shows what people do now and what they would think about doing in the future.

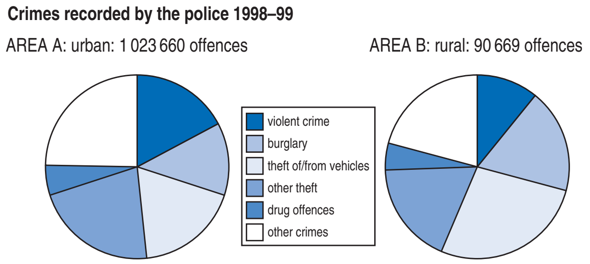


  You are going to make a television advert about the environment. Choose two issues to be in your advert using the information in the chart. Explain how you chose each issue using only the information in the chart.

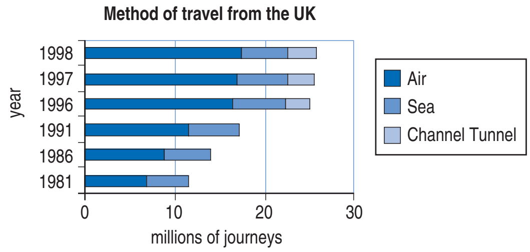
As outcomes, Year 8 pupils should, for example:

Interpret tables, graphs and diagrams, and draw inferences related to the problem; relate summarised data to the questions being explored. For example:

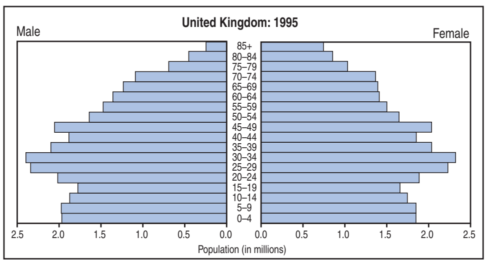
- Interpret pie charts. For example, discuss differences in crime patterns between two areas.



- Interpret data in a compound bar chart.
  For example:
  How has the method of travel changed over the last 20 years? Using the data in the graph, predict what the results will look like for this year.
  What about next year? In 10 years?



- Interpret data in a population pyramid. For example, discuss differences in the male and female populations of different countries.



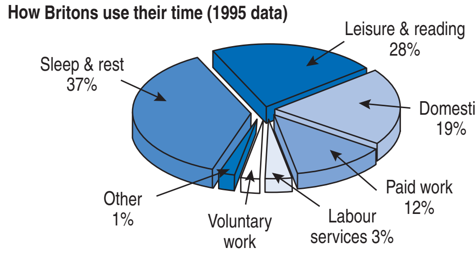
Source: US Census Bureau, International Data Base

Calculate simple statistics, such as the percentage of men and women over 70, to illustrate observations.

As outcomes, Year 9 pupils should, for example:

Interpret graphs and diagrams, and draw inferences from data representations to support and to cast doubt on initial conjectures. For example:

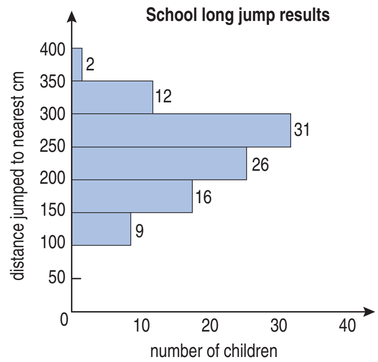
- Interpret pie charts, e.g. showing how British adults spend their time.



Source: *Economic Trends*, Office for National Statistics 1998

Criticise a claim that the pie chart shows that Britons spend too little time working. Argue that paid work amounts to 12% of 24 × 7 = 20.16 hours per week, which suggests that 1 in 2 British adults works about 40 hours a week, about right.

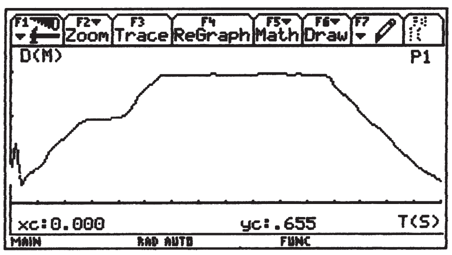
- Interpret frequency diagrams. For example:
  Here are the long jump results for a school.
  They are measured to the nearest centimetre, and classified in intervals $0 \le d < 50$, $50 \le d < 100$, etc.



a. Steve jumped 315 cm.
   He says: 'Only two people jumped further than me.' Could he be correct? Tick the correct box, then explain your answer.
   ☐ Yes ☐ No

b. Ruby says: 'The median jump was 275 cm.'
   She is not correct. Explain how the graph shows she is not correct.

- Interpret a distance–time graph, e.g. generated on a **graphical calculator** using a **CBR** (calculator-based ranger).

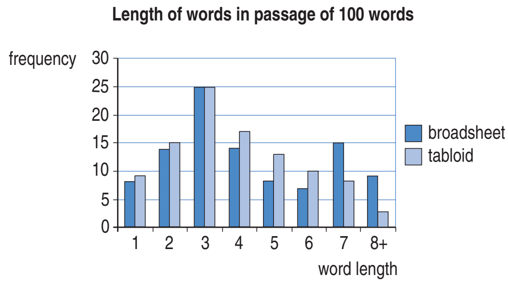


Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Interpret diagrams and graphs, and draw inferences (continued)

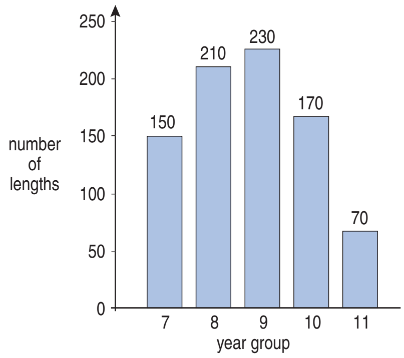
- Interpret a bar chart (discrete data). For example:

  a. This chart shows the lengths of 100 words in two different newspaper passages. Compare the two distributions.



Observe that the differences are not great, but there may be slightly greater word length and variety of word length in the broadsheet newspaper.

b. A school has five year groups.
Eighty pupils took part in a sponsored swim.
Lara drew this graph.



Look at the graph.
Did Year 10 have fewer pupils taking part than Year 7?
Tick the correct box.

☐ Yes ☐ No ☐ Cannot tell

Explain your answer.

See Y456 examples (pages 114–17).

As outcomes, Year 8 pupils should, for example:

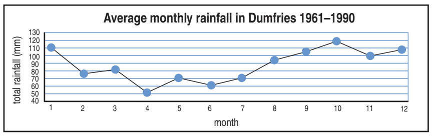
- Interpret data in a table from a secondary source. For example, describe the relationship between the number of cigarettes smoked and when smokers have their first cigarette of the day.

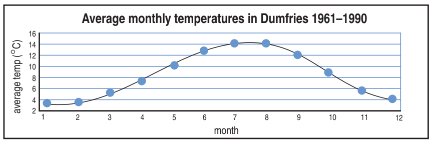
Time after waking first cigarette smoked	No. of cigarettes smoked per day		
	20 or more	10–19	0–9
Less than 5 minutes	31	12	2
5–14 minutes	28	16	3
15–29 minutes	19	17	6
30 minutes to 1 hour	14	23	12
1–2 hours	6	18	15
More than 2 hours	2	15	63

Source: *Smoking Statistics: who smokes and how much*, ASH

- Interpret line graphs, e.g. weather data.

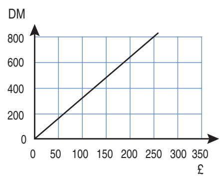
a.

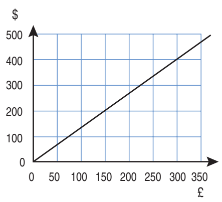




When would you visit Dumfries? Why?
The driest month in Dumfries is normally April, when temperatures are around 7 °C. June is considerably warmer, and only a little wetter.

b. These two graphs convert pounds (£) to Deutschmarks (DM) and pounds (£) to dollars ($).

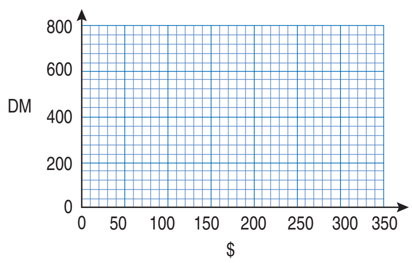




Use the graphs to complete the table.

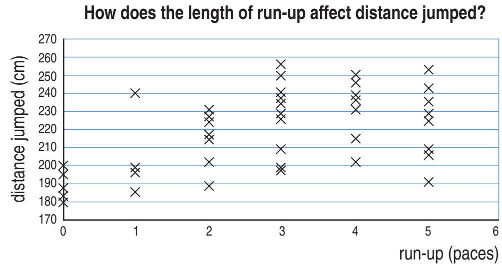
Number of £	Approximate number of DM	Approximate number of $
0	0	0
200		

Use the information in your table to draw a conversion graph for $ into DM.



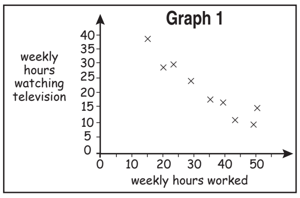
As outcomes, Year 9 pupils should, for example:

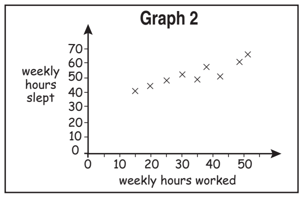
- Interpret scatter graphs, e.g. showing the effect of length of run-up on long jump distance.

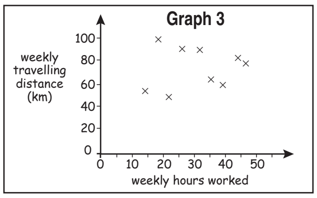


Is there enough evidence to show that increasing the number of paces before take-off improves the distance jumped?

- Develop basic understanding of correlation. For example, some students plotted three scatter graphs.







a. What does graph 1 show about the relationship between the weekly hours spent watching TV and the weekly hours worked?
b. What does graph 2 show about the relationship between the weekly hours slept and the weekly hours worked?
c. What does graph 3 show about the relationship between the weekly travelling distance and the weekly hours worked?
d. One student works for 30 hours a week or more. Estimate the weekly hours spent watching TV and the weekly hours slept by this student. Explain how you decided on your estimates.

Analyse data to find patterns and exceptions, look for cause and effect, and try to explain anomalies.

- In a study of engine size and acceleration times, observe that in general a larger engine size leads to greater acceleration. However, particular cars do not fit the overall pattern, perhaps because they are much heavier than average, or are built for rough terrain rather than normal roads.

Recognise that in controlled scientific conditions it may be possible to deduce cause and effect, but that in statistical situations establishing a connection does not necessarily imply causality.

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Compare two simple distributions using the range, mode, mean or median

Compare the distributions of two sets of data, and the relationships between them, using the range and one of the mode, mean or median. For example:

- How do pupils travel to school?
  Compare the median and range of the times taken to travel to school for two groups of pupils, such as those who travel by bus and those who travel by car.

- Which newspaper is easiest to read?
  In a newspaper survey of the numbers of letters in 100-word samples, compare the mean and the range.

Newspaper type	Mean	Range
tabloid	4.3	10
broadsheet	4.4	14

- Which is your better catching hand?
  Use data from practical experiments to compare results for the writing hand with the non-writing hand, e.g. the mode and range for the number of cubes caught.

- Do First Division or Second Division teams score more goals?
  Use data from secondary sources to compare goals scored in a season by teams in the First Division and teams in the Second Division.
  Calculate the range, mean, median and mode.
  Write two sentences comparing the results, using the range and the mean, median or mode.

Communicate methods and results

Write a short report of a statistical enquiry. Illustrate with diagrams, graphs and charts, using **ICT** as appropriate; justify the choice of what is presented. For example:

- How do pupils travel to school?
  Draw conclusions based on the original questions and the data analysis. Indicate which diagrams and statistics have proved informative and why. Note difficulties or ambiguities that arose and how they were dealt with. Summarise conclusions: for example, 80% of pupils who live within one mile walk to school; many of those that travel by car do so because it fits in with a parent's journey to work.

- Do newspapers use words or sentences of different length? If so, why?
  Explain why the analysis might be misleading.
  Does the use of technical vocabulary, names of people, conjunctions and pronouns… have an effect in making word length similar?

- How will the population of different countries change over the next 50 years?
  Write an account based on population pyramids. Use terms such as birth rate, death rate and natural increase.

As outcomes, Year 8 pupils should, for example:

Compare two distributions using the range and one or more of the mode, mean or median.
For example:

- Which type of battery lasts longer?
  Use data from an experiment to calculate the range, median and mean of each type.
  Conclude, for example, that one brand is generally of higher quality, and one has less consistent manufacturing standards, as evidenced by a greater range.

- Compare and contrast weather patterns in England and Greece.
  Use secondary data to calculate the range, mean, mode, median of temperature, rainfall, hours of sunshine… in each country. Conclude, for example, that Greece is warmer on average, but also experiences a greater variety in weather patterns.

- Compare and contrast TV viewing patterns for different age groups.
  Compare teenagers with an adult sample. Infer, for example, that teenagers watch more TV, but adults have more consistent viewing patterns.

Communicate the results of a statistical enquiry and the methods used, using **ICT** as appropriate; justify the choice of what is presented. For example:

- Prepare and present a statistical report comparing the methods of transport to an out-of-town shopping centre and a town centre. Indicate the types of shopping customers use each centre for and their reasons.

- As part of a cross-curricular project with science, produce and present a report on how the communities in two habitats differ. Compare relevant factors such as light intensity and plant or animal diversity.

- After an experiment to simulate the cooling rates of penguins, present information that establishes the result: the two graphs plotted on the same diagram, or selected temperature values at the same times.

As outcomes, Year 9 pupils should, for example:

Compare two or more distributions, using the shape of the distributions and appropriate statistics.
For example:

- Compare long jump performance
  Use frequency diagrams to compare the overall performance of Year 7 and Year 9 pupils.
  Conclude, for example, that because the two distributions are similar in shape and range, there is a similar pattern of good, average and poor jumpers in each year.
  Calculate the means of Year 7 and Year 9 pupils' jumps to be, say, 217 cm and 234cm. Conclude that Year 9 pupils generally jump between 15cm and 20cm further than Year 7 pupils.

- Compare the populations of the UK and Brazil
  Conclude, for example, that the similar range indicates that at least some parts of the Brazilian population live as long as people do in the UK.
  Use the median age to explain that whereas half the population of the UK is over the age of 35, in Brazil half the population is under the age of 24.

- Investigate the contents of 25 gram bags of crisps

	Mean mass (g)	Range (g)
Jones crisps		
Strollers crisps		

  From a table of summarised data, conclude that bags of Jones crisps are on average marginally lighter than bags of Strollers crisps, but that the greater range of the Jones crisps bags means that there will be quite a few heavy bags as well as quite a few light bags.

Communicate interpretations and results of a statistical enquiry using selected tables, graphs and diagrams from primary and secondary sources in support. For example:

- Describe the current incidence of male and female smoking in the UK, using frequency diagrams to demonstrate the peak age groups.
  Show how the position has changed over the past 20 years, using line graphs.
  Conclude that the only group of smokers on the increase is females aged 15–25 years.
  Suggest possible reasons, based on results from your own questionnaire.

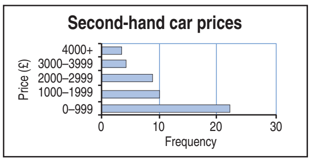
- As a joint project with geography, write about development, showing an understanding of the difficulty of defining the term, given anomalies between the various measures. Evaluate the usefulness of the indicators from scatter graphs. Refer to tables of data for particular countries to suggest reasons for differences in development on different scales and in different contexts.

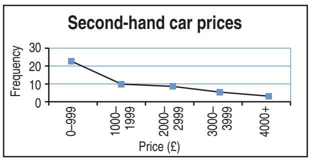
Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:
Communicate methods and results (continued)	

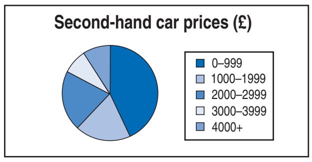
As outcomes, Year 8 pupils should, for example:

As outcomes, Year 9 pupils should, for example:

Select tables, graphs and charts to support findings.
For example, choose a bar chart to represent second-hand car prices, because it conveys the progression in value (unlike the pie chart) and has a stronger visual image than the line graph, where the joining of points to show trends could mislead.







Identify misleading graphs and statistics, such as:

- incomplete diagrams;
- inappropriate use of scale or breaking the scale on the axes to magnify differences;
- treating discrete data as continuous data, and vice versa, or joining up points with lines for a discrete distribution;
- general conclusions from very small samples, e.g. '9 out of 10 cats prefer…';
- misinterpreting lines of best fit on scatter diagrams.

Recognise that graphs produced by popular **ICT packages** often suffer from some of these faults.

Examine results critically, and justify choice of statistical representation in written presentations, recognising the limitations of any assumptions and their effect on the conclusions drawn. For example:

- Study of populations of the UK and Brazil
  Conclude that the 'bottom-heavy' shape of the Brazilian population distribution could be due to a number of factors. Observe that a significant difference between the mean and median gives a measure of the skew of the distribution.

  Note that the 'bottom-heavy' effect could be due to a rising birth rate (giving an increasing number of younger people) or to a significant death rate at all ages (reducing the number of people still alive at each higher age group). Use the high population growth rate to indicate the former, but the high infant mortality and low life expectancy to support the latter.

  Use the roughly uniform population distribution, and high life expectancy, of the UK to argue that both mortality figures and the birth rate are low.

- Study of distribution of grass and non-grass plants
  Having examined the effect of moisture content of soil on the distribution of grass, recognise that other factors may be significant.

Pupils should be taught to:

Use the vocabulary of probability

As outcomes, Year 7 pupils should, for example:

Use, read and write, spelling correctly:
fair, unfair, likely, unlikely, equally likely, certain, uncertain, probable, possible, impossible, chance, good chance, poor chance, no chance, fifty-fifty chance, even chance, likelihood, probability, risk, doubt, random, outcome...

Use vocabulary and ideas of probability, drawing on experience.
For example:

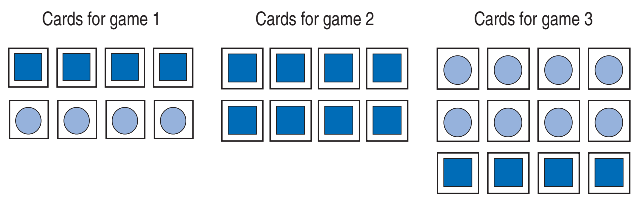
- Match one of these words to each statement below:

  CERTAIN LIKELY UNLIKELY IMPOSSIBLE

  a. I will eat a packet of crisps today.
  b. Next year, there will be 54 Fridays.
  c. I will leave the classroom through the door.
  d. The sun will rise tomorrow in the east.
  e. I will see David Beckham on my way home.

- Discuss the risk or chance of:
  a. injury in different sports;
  b. road accidents at different times of the day and year;
  c. dying before the age of 70 in different countries;
  d. a cyclone happening in England.

- A class is going to play three games.
  In each game some cards are put into a bag.
  Each card has a square or a circle on it.
  One card will be taken out, then put back.
  If it is a circle, the girls will get a point.
  If it is a square, the boys will get a point.



  a. Which game are the girls most likely to win? Why?
  b. Which game are the boys least likely to win? Why?
  c. Which game is impossible for the girls to win?
  d. Which game are the boys certain to win?
  e. Which game is it equally likely that the boys or girls win?
  f. Are any of the games unfair? Why?

- Use a line of large digit cards (1 to 10), face down and in random order.
  Turn cards over, one at a time.
  Indicate whether the next card turned is likely to be higher or lower than the card just turned.
  Give reasons for each response.

See Y456 examples (pages 112–13).

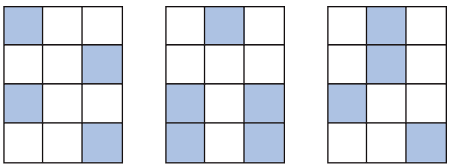
As outcomes, Year 8 pupils should, for example:

Use vocabulary from previous year and extend to:
event, theory, sample, sample space, biased…

Use the vocabulary of probability when interpreting the results of an experiment; appreciate that random processes are unpredictable. For example:

- Think of an event where:
  a. the outcome is certain;
  b. the outcome is impossible;
  c. the outcome has an even chance of occurring.

- Three different scratch cards have some hidden shaded squares. You can scratch just one square, choosing at random.
  On which card are you most likely to reveal a shaded area? Why?



- Two boxes of sweets contain different numbers of hard- and soft-centred sweets.

Box 1 has 8 hard-centred sweets and 10 with soft centres.
Box 2 has 6 hard-centred sweets and 12 with soft centres.
Kate only likes hard-centred sweets. She can pick a sweet at random from either box. Which box should she pick from? Why?

Kate is given a third box of sweets with 5 hard-centred sweets and 6 with soft centres.
Which box should Kate choose from now? Why?

As outcomes, Year 9 pupils should, for example:

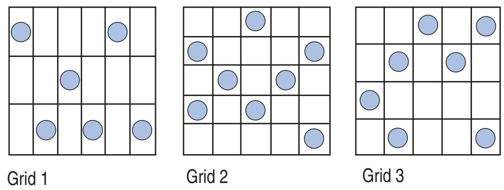
Use vocabulary from previous years and extend to:
exhaustive, independent, mutually exclusive, relative frequency, limit, tree diagram…
and the notation p(n) for the probability of event n.

Use the vocabulary of probability in interpreting results involving uncertainty and prediction.
For example:

- Discuss statements such as:
  a. As so many thousands of people play the Lottery each week somebody is **certain** to win the jackpot.
  b. Every morning I drop my toast and it has landed butter-side down for the last three mornings. It couldn't **possibly** happen again today.
  c. It can either rain or be fine, so tomorrow there is a **50% chance** of rain.
  d. To play a board game you must throw a six to start. Amin says: 'I'm **not** lucky, I'll **never** be able to start.'
  e. The risk of being killed in a road accident is about 1 in 8000, and of dying of a heart attack is 1 in 4.

- In a computer 'minefield' game, 'mines' are hidden on grids. When you land randomly on a square with a mine, you are out of the game.

  a. The circles indicate where the mines are hidden on three different grids.



  On which of the three grids is it hardest to survive?

  b. On which of these grids is it hardest to survive?
  X. 10 mines on an 8 by 8 grid
  Y. 40 mines on a 16 by 16 grid
  Z. 99 mines on a 30 by 16 grid
  Explain your reasoning.

Pupils should be taught to:

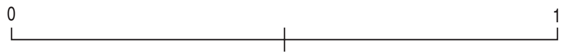
Use the probability scale; find and justify theoretical probabilities

As outcomes, Year 7 pupils should, for example:

Understand and use the probability scale from 0 to 1; find and justify probabilities based on equally likely outcomes in simple contexts.

Recognise that, for a finite number of possible outcomes, probability is a way of measuring the chance or likelihood of a particular outcome on a scale from 0 to 1, with the lowest probability at zero (impossible) and the highest probability at 1 (certain). For example:

- What fractions would you use to describe:
  a. the chance of picking a red card at random from a pack of 52 cards?
  b. the chance of picking a club card?
  Position the fractions on this probability scale.



Know that probability is related to proportion and can be represented as a fraction, decimal or percentage, e.g. discuss what is meant by a weather forecast of a 20% chance of rain.

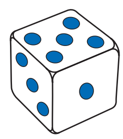
Know that if several equally likely outcomes are possible, the probability of a particular outcome chosen at random can be measured by:

$$\frac{\text{number of events favourable to the outcome}}{\text{total number of possible events}}$$

For example:

- The letters in the word **RABBIT** are placed in a tub, and a letter taken at random. What is the probability of taking out:
  a. a letter **T**? (one in six, or $\frac{1}{6}$)
  b. a letter **B**? ($\frac{2}{6}$ or $\frac{1}{3}$)

- The probability of rolling a 2 on a fair 1 to 6 dice is $\frac{1}{6}$, because 2 occurs once out of a total of 6 different possibilities.



  What is the probability of rolling:
  a. 5?
  b. an odd number?
  c. zero?
  d. a number greater than 2?
  e. a prime number?
  f. a number lying between 0 and 7?

  Mark these probabilities on a probability scale.

- A newsagent delivers these papers, one to each house.

Sun	250	Times	120
Mirror	300	Mail	100
Telegraph	200	Express	80

  What is the probability that a house picked at random has:
  a. the Times?
  b. the Mail or the Express?
  c. neither the Sun nor the Mirror?

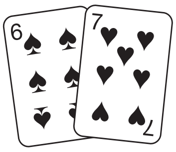
Link to problems involving probability (pages 22–3).

As outcomes, Year 8 pupils should, for example:

Know that if the probability of an event occurring is p, then the probability of it not occurring is 1 – p.

Use this to solve problems. For example:

- Consider a pack of 52 playing cards (no jokers). If the probability of drawing a club from a pack of cards is $\frac{1}{4}$, then the probability of drawing a card that is not a club is $1 - \frac{1}{4}$, or $\frac{3}{4}$.

  Calculate the probability that a card chosen at random will be:
  a. a red card;
  b. a heart;
  c. not a picture;
  d. not an ace;
  e. either a club or a diamond;
  f. an even numbered red card.

- There are 25 cars parked in a garage. 12 are red, 7 blue, 3 white and the rest black. Calculate the probability that the next car to leave the garage will be:
  a. red;
  b. blue;
  c. neither red nor blue;
  d. black or white.

- A set of snooker balls consists of 15 red balls and one each of the following: yellow, green, brown, blue, pink, black and white. If one ball is picked at random, what is the probability of it being:
  a. red?
  b. not red?
  c. black?
  d. not black?
  e. black or white?

- Imad threw a dart at a dartboard 60 times. Each time the dart hit the board. The maximum score for one dart is treble twenty. Imad scored treble twenty 12 times.

  Imrad is going to throw the dart once more. Estimate the probability that:
  a. he will score treble twenty;
  b. he will score less than 60.
  Give your reasons.

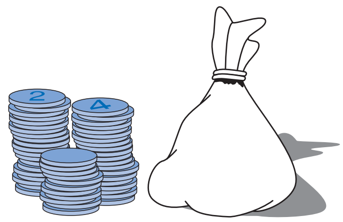
Link to problems involving probability (pages 22–3).

As outcomes, Year 9 pupils should, for example:

Know that the sum of probabilities of all mutually exclusive outcomes is 1.

Use this to solve problems. For example:

- A number of discs are placed in a bag.

  Most are marked with a number 1, 2, 3, 4 or 5. The rest are unmarked. The probabilities of drawing out a disc marked with a particular number are:

  p(1) = 0.15
  p(2) = 0.1
  p(3) = 0.05
  p(4) = 0.35
  p(5) = 0.2

  What is the probability of drawing a disc:
  a. marked 1, 2 or 3?
  b. not marked with a number?

- In an arcade game only one of four possible symbols can be seen in the final window. The probability of each occurring is:

Symbol	Probability
jackpot	$\frac{1}{16}$
moon	$\frac{1}{4}$
star	?
lose	$\frac{1}{2}$

  a. What is the probability of getting a star?
  b. What event is most likely to happen?
  c. What is the probability of not getting the jackpot?
  d. After many games, the jackpot had appeared 5 times. How many games do you think had been played?

Link to problems involving probability (pages 22–3).

Pupils should be taught to:	As outcomes, Year 7 pupils should, for example:

Understand and use the probability scale; find and justify theoretical probabilities (continued)

Identify all the possible outcomes of a single event.

For example:

- What are the possible outcomes…

  a. when a fair coin is tossed?
  There are two outcomes: heads or tails.
  The probability of each is $\frac{1}{2}$.

  b. when a letter of the alphabet is chosen at random?
  There are two outcomes: a vowel or a consonant.
  The probability of a vowel is $\frac{5}{26}$.
  The probability of a consonant is $\frac{21}{26}$.

  c. when a letter from the word HIPPOPOTAMUS is picked at random?
  There are nine outcomes: H, I, P, O, T, A, M, U, S.
  The probability of H, I, T, A, M, U or S is $\frac{1}{12}$.
  The probability of O is $\frac{2}{12}$ or $\frac{1}{6}$.
  The probability of P is $\frac{3}{12}$ or $\frac{1}{4}$.

  d. when a number is chosen at random from the set of numbers 1 to 30?
  There are two outcomes:
  prime ($\frac{10}{30}$ or $\frac{1}{3}$) or non-prime ($\frac{20}{30}$ or $\frac{2}{3}$).
  or:
  There are two outcomes:
  odd ($\frac{15}{30}$ or $\frac{1}{2}$) or even ($\frac{15}{30}$ or $\frac{1}{2}$).
  or:
  There are three outcomes:
  a number from 1–10 ($\frac{10}{30}$ or $\frac{1}{3}$),
  a number from 11–20 ($\frac{10}{30}$ or $\frac{1}{3}$),
  a number from 21–30 ($\frac{10}{30}$ or $\frac{1}{3}$).
  and so on.

Link to problems involving probability (pages 22–3).

As outcomes, Year 8 pupils should, for example:

Find and record all possible outcomes for single events and two successive events in a systematic way, using diagrams and tables.
For example:

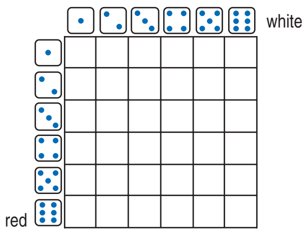
- A coin can land in two ways: head up (H) or tail up (T).

Throw a coin twice.
Record the four possible ways that the coin can land in two throws.

H	H
H	T
T	H
T	T

- What are the possible outcomes when:
  a. a mother gives birth to twins?
  b. a glazier puts red, green or blue glass in each of two windows?
  c. you can choose two pizza toppings from onion, mushroom and sweetcorn?

- One red and one white dice are numbered 1 to 6. Both dice are thrown and the scores added.
Use a sample space to show all possible outcomes.



Which score is the most likely? Why?

Using the sample space, what is the probability of:
a. getting the same number on both dice?
b. the sum of the numbers being less than 4?
c. the score on the red dice being double the score on the white dice?

- 200 raffle tickets are numbered from 1 to 200. They have all been sold. One ticket will be drawn at random to win first prize.
  a. Karen has number 125.
  What is the probability that she will win?
  b. Andrew buys tickets with numbers 81, 82, 83, 84. Sue buys tickets numbered 30, 60, 90, 120. Who has the better chance of winning? Why?
  c. Rob buys several tickets. He has a 5% chance of winning. How many tickets has he bought?
  d. Three people have each lost a ticket and do not play. What is the chance that nobody wins?

Link to problems involving probability (pages 22–3).

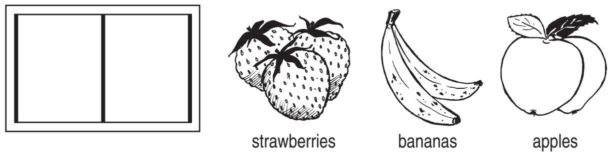
As outcomes, Year 9 pupils should, for example:

Identify all the mutually exclusive outcomes of an experiment.

For example:

- A fair coin and a fair dice are thrown.
One possible outcome is (tail, 5).
List all the other possible outcomes.

- A fruit machine has two 'windows'.
In each window, one of three different fruits is equally likely to appear.

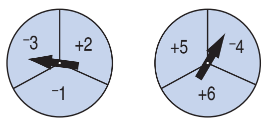


List all the possible outcomes.

What is the probability of getting:
a. two identical fruits?
b. at least one banana?
c. no bananas?

- Two coins are thrown at the same time. There are four possible outcomes:
HH HT TH TT
How many possible outcomes are there if:
  a. three coins are used?
  b. four coins are used?
  c. five coins are used?

- The hands on these two spinners are spun at the same time.



The two scores are added together.
What is the probability that the total score is negative?

- A game involves rolling 6 dice. If you get 6 sixes you win a mountain bike.
What is your chance of winning the bike?

Link to problems involving probability (pages 22–3).

Pupils should be taught to:

Collect and record experimental data, and estimate probabilities based on the data

As outcomes, Year 7 pupils should, for example:

Collect data from a simple experiment and record in a frequency table; estimate probabilities based on the data.

For example:

- Put four different coloured cubes in a bag. Shake it.
  Without looking, take a cube from the bag, but before you do so, guess its colour.
  If you are right, put a tick in the first column.
  If you are wrong, put a cross.
  Put the cube on the table.
  Carry on until you have taken out all four cubes.

  Repeat this experiment 10 times. Record your results.

Experiment number	Guesses			
	1st	2nd	3rd	4th
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

  What is the chance of being right on the 1st guess?
  On the 4th guess? Choose from: no chance, some chance, even chance, certain chance. Explain your choice.

- Use a bag containing an unknown mixture of identical, but differently coloured, counters.
  Draw one counter from the bag, note its colour, then replace it. Do this 10 times.
  Now estimate the probability of each colour.
  Check by emptying the bag.

- Make a dice (or spinner) from card in the shape of a regular solid or polygon. Weight it to make it biased, e.g. with Plasticine stuck to the inside or to the surface of the spinner.
  Throw the dice or spin the spinner 50 times.
  Estimate the probability of each score.
  Compare your estimated probabilities with what you would expect from a fair dice or spinner.

As outcomes, Year 8 pupils should, for example:

Estimate probabilities based on experimental data and use relative frequency as an estimate of probability. For example:

- Class 8C opened 20 small boxes of raisins.
  8 of the 20 boxes contained more than 28 raisins.
  What is the probability that an unopened box will contain fewer than 28 raisins?

- Throw two dice numbered 1 to 6 and sum the scores. Repeat 30 times and record each result on a frequency diagram.
  Compare results with another group.
  Are they different? Why?
  Predict what might happen if the experiment were repeated.
  Carry out the experiment again another 20 times and record the extra scores on the same diagram.
  What effect do the extra throws have on the results?
  Did the results match predictions? Why?

Understand that:

- If an experiment is repeated there may be, and usually will be, different outcomes.
- Increasing the number of times an experiment is repeated generally leads to better estimates of probability.

Solve problems such as:

- A girl collected the results of 50 European football matches:

home wins	35
away wins	5
draws	10

Use these results to estimate the probability in future European matches of:

a. a home win;
b. an away win;
c. a draw.

The girl found the results of the next 50 matches.

home wins	37
away wins	4
draws	9

Estimate, using all 100 results, the probability in future European matches of:

a. a home win;
b. an away win;
c. a draw.

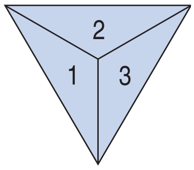
Would these probabilities be more accurate than those based on the first 50 matches? Why?

Link to comparing experimental and theoretical probabilities (pages 284–5).

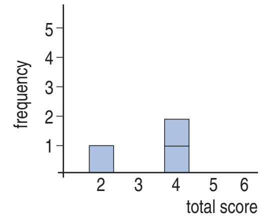
As outcomes, Year 9 pupils should, for example:

Estimate probabilities based on experimental data and use relative frequency as an estimate of probability. For example:

- Use an equilateral triangular spinner with three equal sections labelled 1, 2 and 3.
  Spin it twice. Add the two scores.
  Repeat this 40 times.



As the experiment progresses, record results in a frequency diagram.



Using the results:

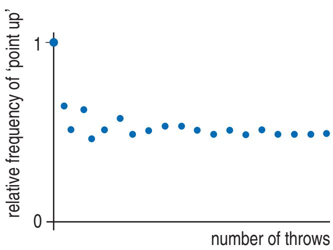
a. Which total is most likely?
b. What is the estimated probability of a total of 5? How could you make a more accurate estimate?
c. If the experiment were repeated 2000 times, how many times would you expect to get a total of 3?

Justify your answers.

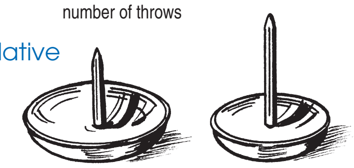
What happens if the numbers on the spinner are changed, e.g. to 1, 2, 2 or 3, 2, 2 or 1, 1, 3 or 2, 2, 2?

Recognise that, with repeated trials, experimental probability tends to a limit. Relative frequency can give an estimate of probability, independent of the theoretical probability, and may be the only realistic way of estimating probability when events are not equally likely. For example:

- Throw a drawing pin 10 times.
  Record how many times it lands 'point up'.
  Estimate the probability of 'point up' from these 10 trials.
  Repeat the experiment another 10 times. Estimate the probability based on 20 trials.
  Repeat the procedure another 80 times, calculating and plotting the probabilities (relative frequencies) after every 10 throws.



Predict and sketch relative frequency diagrams for these pins.



Pupils should be taught to:

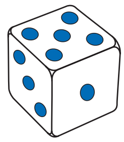
Compare experimental and theoretical probabilities

As outcomes, Year 7 pupils should, for example:

Compare experimental and theoretical probabilities in simple contexts.

For example:

- Use **computer software**, or a simple program using the RANDOM function, to simulate simple experiments, e.g. with dice or coins.



Compare these with the calculated results based on theoretical probabilities.
For example, simulate throwing a single dice a specified number of times. Observe that:
  - if the experiment is repeated, the frequencies of different scores will vary;
  - if the number of trials is large, then the frequencies settle down, each to approximately one sixth.

Answer questions such as:

- Jack, Lynn and Richard each rolled a fair dice 240 times. Here are the results they gave their teacher.
One of them had recorded their results accurately but the other two had not.

Number	Jack	Lynn	Richard
1	42	40	26
2	37	40	33
3	48	40	27
4	36	40	96
5	36	40	34
6	41	40	24

Whose results were recorded accurately?
Explain your reasons.

As outcomes, Year 8 pupils should, for example:

Compare experimental and theoretical probabilities in different contexts.

For example:

- Use **computer software** to simulate throwing two dice, recording the total score and representing different outcomes on a frequency diagram. Observe:
  a. the pattern of results, relating these to thoretical frequencies (probabilities), based on an analyisis of combinations of scores;
  b. that increasing the number of trials produces a diagram which is closer to a triangular shape.

Answer questions such as:

- Michael said: 'I bet if I drop this piece of toast, it will land jam side down!'
  What is the theoretical probability of this outcome?
  Devise an experiment to test Michael's prediction (e.g. simulate the toast with a playing card – the jam is represented by the side with spots).
  Carry out the experiment.
  Compare the experimental and theoretical probabilities.

- Yasmin bought a combination padlock for her school locker.
  The code has four digits, each from 0 to 9.
  Yasmin forgot the last digit of the code.
  What is the theoretical probability that she will choose the correct digit first time?
  Design and carry out an experiment to estimate the experimental probability.
  Compare the outcome with the theoretical probability.

- The 49 balls in the National Lottery draw are numbered from 1 to 49.
  What is the probability of the first ball:
  a. being a multiple of 5?
  b. being odd?
  c. being a prime number?
  d. not containing the digit 1?
  Design an experiment to test these theoretical probabilities.
  You could refer to the **website** to find out historical data: www.lottery.co.uk

As outcomes, Year 9 pupils should, for example:

Compare experimental and theoretical probabilities in a range of contexts, appreciating the difference between mathematical explanation and experimental evidence. For example:

- Use **computer software** to simulate tossing a coin, plotting points to show the relative frequency of heads against the number of trials. Observe that:
  a. the pattern of results is erratic at first, but settles eventually to a value of approxiamtely ½;
  b. when the experiment is repeated, a similar pattern of results is observed.

- Explore games that may or may not be 'fair'.
  For example:

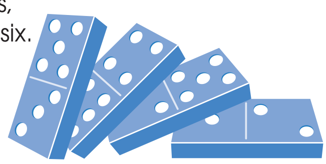
  Roll two dice 36 times. Add the two scores.
  If the outcome is EVEN, you WIN.
  If the outcome is ODD, you LOSE.

  Repeat the experiment, multiplying the two scores, rather than adding them.

  Does each game give the same probability of winning? Explain your reasons.
  Use a sample space diagram to justify results.

  What would you expect for results of EVEN, ODD and ZERO if you subtracted the two scores?

- Use a set of 28 dominoes, double blank to double six. Draw out one at a time from a bag. Record the total score and replace. Repeat many times.



  Construct a diagram to show the frequency of each score. Compare this with the diagram you would expect in theory.
  How and why is the distribution of the total of the numbers on a domino different from the totals of two dice?

- Throw two dice. Players chose one rule each and explain why they have chosen it. If the rule is satisfied the player gains a point. Predict then test the results after 20 throws.

  Suggested rules for the two scores:

A	the difference is zero
B	the total is more than 8
C	the total is a factor of 12
D	the difference is 1
E	the total is prime
F	the total is a multiple of 3
G	the product is even
H	the two numbers share a factor

  Think up and test your own rules.